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DISCRETE REALIZATION FOR RECEIVERS DETECTING SIGNALS OVER RANDOM DISPERSIVE CHANNELS. PART II: DOPPLER-SPREAD CHANNEL

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Abstract. We propose a new suboptimal receiver for detecting signals transmitted over random doppler-spread channels. We discuss in particular the first order channel and a transmitted signal of a constant envelope. The proposed receiver is composed of an integrator, a sampler and a quadratic filter. We give numerical results concerning the performances of this receiver. In particular, we give the evolution of error probability versus the number of samples (or sampling rate). We show the important role of the ratio of signal duration to channel coherence time and we suggest how to choose the transmitted signal duration and the sampling rate for a given signal to noise ratio.

Zusammenfassung. Vorgeschlagen wird ein neuer suboptimaler Empfänger zur Detektion von Signalen, die über Kanäle mit zufällig veränderlichen Doppler-Frequenzverschiebungen übertragen werden. Insbesondere wird ein Kanalmodell 1. Grades und ein Sendesignal mit konstanter Einhüllender untersucht. Der vorgeschlagene Empfänger besteht aus einem Integrator, einer Abtastschaltung und einem Quadrierer. Für die Wirkungsweise des Empfängers wird ein Zahlenbeispiel gegeben. Insbesondere wird die Fehlerwahrscheinlichkeit über der Zahl der Abtastwerte bzw. der Abtastrate aufgetragen. Das Verhältnis von Signaldauer und Kanalkohärenzzeit spielt eine wichtige Rolle; wir machen einen Vorschlag, wie man die Dauer des Sendesignals und die Abtastfrequenz bei einem gegebenen Signal-Rauschabstand am besten wählen sollte.

Résumé. Nous proposons un nouveau récepteur sous-optimal pour détecter des signaux ayant traversé une voie de transmission aléatoire et dispersive en fréquence. Nous étudions, en particulier, le milieu à modèle interne d'ordre 1 et un signal émis d'enveloppe constante. Le récepteur proposé est composé d'un intégrateur, d'un échantillonneur et d'un filtre quadratique. Nous donnons des résultats numériques concernant les performances de ce récepteur. En particulier, nous donnons l'évolution de la probabilité d'erreur en fonction du nombre d'échantillons (ou fréquence d'échantillonnage). Nous montrons le rôle important du rapport de la durée du signal émis sur le temps de cohérence du milieu et nous suggérons comment choisir la durée du signal émis et la fréquence d'échantillonnage pour un rapport signal à bruit donné.

Keywords. Doppler spread channel, scattering function, detection, binary communication, diversity, suboptimal receiver, sampling, quadratic filter, error probability.

1. Introduction

In part one [1], G. Tziritas and G. Hakizimana discussed a discrete realization for receivers detecting signals over range-spread channels. In this second part, using a different approach, we will study the case of a doppler-spread channel where the frequency dispersion results from the movement of a random scatterer and for which the optimal receiver [2] is composed of two parts: an estimator, essentially to track the movement, and a correlator.

We propose here a new suboptimum receiver composed of a linear filter, a sampler and a quadratic filter. The linear filter is necessary in the presence of white noise and will be associated with the doppler-dispersion. The sampler reduces the dimension of observation space and simplifies the decision problem with a certain loss of optimality. The quadratic filter realizes a quadratic form of sampled data and responds to the random phenomena. This receiver represents a further development of the classical receiver [3].

In Section 2, we will describe the channel model, the hypotheses concerning both the channel and the scattering function characterization. We will also discuss the binary decision problem which we are trying to resolve. In Section 3, we will define the concept of "coherence time" of the channel and we introduce the transmission diversity parameter [4]. These elements play an important role in determining the receiver proposed here, especially in as concerns the sampler, and its performances. In Section 4, we will describe the method for determining the filter processing composed of an integrator throughout the sampling period, which is followed by a periodic sampler and finally the classical quadratic filter [2]. In Section 5, we will study the case of a first order doppler scattering function and a transmitted signal of a constant envelope.

We will give numerical results which exhibit the parameters of transmission and of the receiver: the signal to noise ratio, the ratio of transmission duration to coherence time of the channel and the number of samples. We will finish this paper by discussing the numerical results obtained.

2. Channel model and detection problem

The doppler-spread channel corresponds to a "single scatterer" random medium. We suppose that the reflective characteristics of the scatterer change during the signalling interval and that the ratio of wave propagation velocity to scatterer's radial velocity is large enough compared to twice the product bandwidth duration of the transmitted signal. We also suppose that the bandwidth of the pulse is relatively small compared to the carrier frequency. With these assumptions, we can say that the random medium modulates in amplitude the transmitted pulse. Let $\tilde{f}(t)$ be the complex envelope of the transmitted signal $f(t)$ of energy E_t and duration T, as it is given in (1) of Part I.

The complex envelope of the signal at the output of the channel is therefore

$$\tilde{s}(t)=\tilde{a}(t)\tilde{f}(t); \quad 0 \leqslant t \leqslant T+L \tag{1}$$

where $\tilde{a}(t)$ is the random amplitude modulation.

In what follows we suppose that the second order statistical properties are given:

i) the expected value of $\tilde{a}(t)$ is zero,

$$E\{\tilde{a}(t)\}=0; \quad \forall t$$

and

ii) there exists a relationship between the real and imaginary parts of $\tilde{a}(t)$, which is expressed as follows

$$\begin{aligned} &E\{\tilde{a}(t)\tilde{a}(u)\}=0; \quad \forall t, u, \\ &E\{\tilde{a}(t)\tilde{a}^*(u)\}=\tilde{K}_{\tilde{a}}(t, u). \end{aligned} \tag{2}$$

This means that the covariance function of the real part is the same as that of the imaginary part, and that the cross-covariance function between the two parts is an antisymmetrical function. In this paper we suppose that the random function $\tilde{a}(t)$ is a wide-sense stationary (WSS) and gaussian. Then one can define the power spectrum $\tilde{S}(\nu)$ of the envelope $\tilde{a}(t)$ as the Fourier transform of $\tilde{K}_{\tilde{a}}(t-u)$. $\tilde{S}(\nu)$ is the scattering function of the medium and characterizes the frequency spreading of the transmitted signal energy. That is, if a signal at frequency ν_0 is transmitted, then at the channel output, a random signal with power spectrum $\tilde{S}(\nu-\nu_0)$ is received. It is a doppler-spread channel, because the scatterer is randomly fluctuating.

We shall now study the detection problem. This problem can be formulated as testing the hypothesis of signal presence in additive white noise (cf (5) of Part I).

We suppose that the complex envelope $\tilde{n}(t)$ of the noise is gaussian, zero-mean, and white of power spectral density N_0, that is

$$\begin{aligned} &E\{\tilde{n}(t)\}=0, \quad \forall t \\ &E\{\tilde{n}(t)\tilde{n}^*(u)\}=N_0\delta(t-u), \\ &E\{\tilde{n}(t)\tilde{n}(u)\}=0, \quad \forall t, u, \end{aligned} \tag{3}$$

where $\delta(\cdot)$ is the Dirac distribution.

Theoretical results concerning this problem are well-known [2]. One must solve a Fredholm integral equation which is generally very difficult. In this paper we study the scattering function as follows

$$\tilde{S}(\nu)=\frac{2k}{(2\pi\nu)^2+k^2}, \tag{4}$$

which is normalized $\int_{-\infty}^{\infty}\tilde{S}(\nu)\,\mathrm{d}\nu=1$.

This means that $\tilde{a}(t)$ is a first order process, with a cutoff frequency at kHz. The envelope of the transmitted signal is as follows

$$\tilde{f}(t)=\sqrt{\frac{E_t}{T}}, \quad 0\leq t\leq T. \tag{5}$$

Thus the average received energy is

$$\bar{E}_r=E\left\{\int_0^T|\tilde{a}(t)|^2|\tilde{f}(t)|^2\,\mathrm{d}t\right\}=\frac{E_t}{T}\int_0^T\tilde{K}_{\tilde{a}}(0)\,\mathrm{d}t=E_t. \tag{6}$$

For the scattering function in (4), there exists a state variable representation of the channel [2], and the Fredholm integral equation is replaced by a Riccati differential equation. H.L. Van Trees [2] also gives the optimum receiver, containing a Kalman filter, which is simplified by H. Cherifi, G. Tziritas and G. Jourdain [5] in the sense of a heuristic determination of the gain of the Kalman filter.

In this paper, we propose a different approach. We study sampling techniques for detecting signals, which are spread by the channel. For a better comprehension of the obtained results, we present in the following section some important parameters of the transmission.

3. Transmission parameters

We introduce firstly the "coherence bandwidth" of the channel according to A. Ishimaru [6]. Let us consider an input signal of a constant envelope. The correlation of the output signal at two different times t_1 and t_2 decreases as the time difference t_1-t_2 increases. The time difference Δt at which the correlation practically disappears or decreases to a specified level is called the "coherence time". For the scattering function in (4), the "coherence time" is approximately

$$\Delta t = \frac{1}{k}.$$

Thus kT can be interpreted as the number of independent realizations of the channel characteristics during the transmission time T. In other words, at the channel output, a diversity or a number of degrees of freedom is generated during the transmission time. G. Tziritas [4] has proposed a more convenient approach to the transmission diversity, which takes into account the envelope of the transmitted signal. This approach permits the characterization of the channel and the transmitted signal simultaneously in their interaction. Hence, we define the "transmission diversity"

$$a = \frac{\left(\int_0^T \tilde{K}_{\tilde{s}}(t,t)\,\mathrm{d}t\right)^2}{\int_0^T\int_0^T |\tilde{K}_{\tilde{s}}(t,u)|^2\,\mathrm{d}t\,\mathrm{d}u} = \frac{\left(\int_0^T |\tilde{f}(t)|^2 \tilde{K}_{\tilde{a}}(t,t)\,\mathrm{d}t\right)^2}{\int_0^T\int_0^T |\tilde{f}(t)|^2|\tilde{f}(u)|^2|\tilde{K}_{\tilde{a}}(t,u)|^2\,\mathrm{d}t\,\mathrm{d}u}. \tag{7}$$

If $\tilde{a}(t)$ is a wide-sense stationary process and normalized in power $\tilde{K}_{\tilde{a}}(0)=1$, then the transmission's diversity a is given by

$$a = \frac{E_t^2}{\int_0^T\int_0^T |\tilde{f}(t)|^2|\tilde{f}(u)|^2|\tilde{K}_{\tilde{a}}(t-u)|^2\,\mathrm{d}t\,\mathrm{d}u}. \tag{8}$$

For the scattering function of (4), the covariance function is $\tilde{K}_{\tilde{a}}(\tau)=\exp(-k|\tau|)$ and the transmission's diversity for a constant envelope is

$$a = \frac{2(kT)^2}{2kT-1+\mathrm{e}^{-2kT}}. \tag{9}$$

If kT is small, the transmission diversity differs only a little from 1. If kT is large, the asymptote of the transmission diversity is $kT+0.5$ (Fig. 1).

In the same paper [4], we studied the performances of the optimum receiver for the detection and the binary symmetric communication problems. For each signal to noise ratio ($\bar{E}_r/N_0$) an optimal value for the transmission diversity and the parameter kT were obtained.

Figure 2 gives these optimal values for a binary FSK (Frequency Shift Keying) communication problem formulated in (7) of Part I and it is considered as symmetric for evaluating optimal kT. This important result will be rediscovered using sampling techniques for realizing the receiver.

4. Use of sampling

The object of using sampling is to obtain simple receiver configurations and to have a finite dimension of the observation space. T. Kadota and L. Shepp [7] prove that the better choice for the second requirement is a subspace of the space associated with the output signal covariance, if the additive noise is white. But the first requirement is not satisfied. The problem remains difficult and the solution is complicated. For this reason we shall choose the subspace generated by sampling the output signal. We want to make

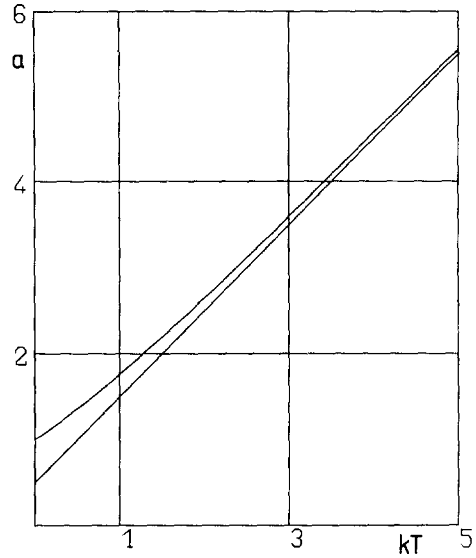

Fig. 1. Transmission diversity for a first order amplitude modulation and a constant envelope signal, and its asymptote.

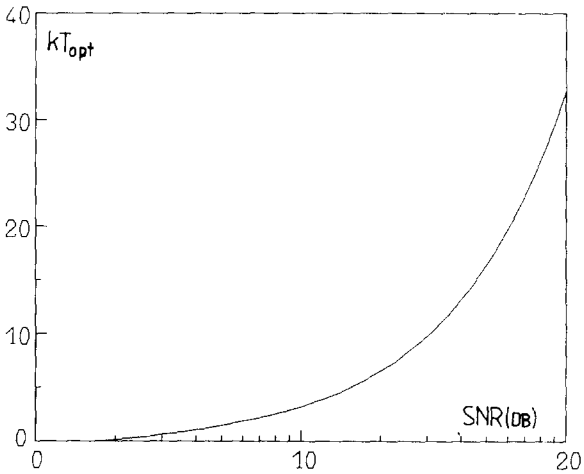

Fig. 2. The optimal kT versus signal to noise ratio for a binary FSK (Frequency-Shift-Keying) communication problem.

comparisons with the results obtained for the continuous time optimal receiver. Thus, we have to confront the singularity of white noise [8]. The advantages of white noise modelization for continuous time become a disadvantage when we want to use sampling. We have overcome this difficulty by filtering the received signal. The structure of the proposed receiver is given in Fig. 3.

After sampling, we obtain a vector $\tilde{\boldsymbol{r}}$ which is the new observation space. The detection problem can then be presented as follows:

$$\begin{array}{ll} H_1: \tilde{\boldsymbol{r}} = \tilde{\boldsymbol{s}} + \tilde{\boldsymbol{n}} & \tilde{\boldsymbol{r}} = [\tilde{r}_1, \ldots, \tilde{r}_k, \ldots, \tilde{r}_N]^{\mathrm{T}}, \\ & \tilde{\boldsymbol{s}} = [\tilde{s}_1, \ldots, \tilde{s}_k, \ldots, \tilde{s}_N]^{\mathrm{T}}, \\ H_0: \tilde{\boldsymbol{r}} = \tilde{\boldsymbol{n}} & \tilde{\boldsymbol{n}} = [\tilde{n}_1, \ldots, \tilde{n}_k, \ldots, \tilde{n}_N]^{\mathrm{T}}. \end{array} \qquad (10)$$

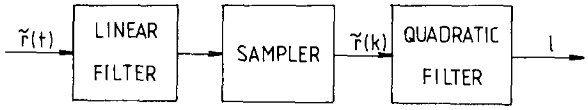

Fig. 3. Structure of the proposed receiver.

With the gaussian assumption, it is known that, for the test (10), the optimum receiver is a quadratic filter whose operation on $\tilde{r}$ is as follows:

$$l = \tilde{r}^{\dagger}(K_0^{-1} - K_1^{-1})\tilde{r} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{11}$$

The matrix K_1 (resp K_0) is the covariance of the vector $\tilde{r}$ under hypothesis H_1 (resp H_0).

The realization of the quadratic filter generally uses Choleski's procedure.

$$K_0^{-1} - K_1^{-1} = \Lambda^{\dagger}\Lambda \tag{12}$$

where Λ is a lower triangular matrix. The matrix $\Lambda^{\dagger}$ is the transposed conjugate of Λ. This decomposition is possible, because $K_0^{-1} - K_1^{-1}$ is a positive definite Hermitian matrix. Thus the obtained statistic is given by

$$l = \|\Lambda r\|^2 \tag{13}$$

where $\|\cdot\|$ is the euclidean norm.

The Fig. 4 gives the quadratic filter using a causal linear filter. Generally the filter associated with the lower triangular matrix Λ is time-variant. If the matrices K_0 and K_1 are Toeplitz a time-invariant realization is possible. T. Kailath *et al.* [10] give the following formula for Toeplitz matrices

$$R^{-1} = A_1 B_1 - B_N A_N. \tag{14}$$

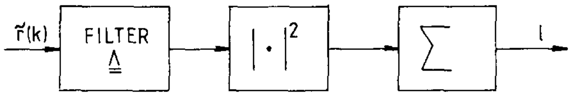

Fig. 4. The quadratic filter realization using Choleski's factorization.

This was first obtained by Gohberg and Semencul, where A_1, B_1, A_N and B_N are lower-triangular Toeplitz matrices. We give in Appendix the method to determine the above matrices. The realization of the quadratic filter (11) using formula (14) can be implemented by time-invariant causal filters.

B. Picinbono [11] proposes a fast procedure to realize a quadratic filter associated with a Toeplitz matrix. However, it does not always correspond to our case. We thus terminate the discussion about the quadratic filter of the receiver (Fig. 3), and we now discuss the design of the linear filter of the receiver.

The sampling, which comes after the linear filter in Fig. 3, can be periodic or non periodic. Moreover, the linear filter can be time-variant, but we require that its impulse response be limited in the sampling interval in order to avoid interferences from previous filter inputs (like noise). To determine the linear filter, we introduce a criterion, for maximizing the power signal to noise ratio produced after the sampling of the filter output.

We develop these ideas in the case of the WSS channel and a constant envelope transmitted signal. Let $\tilde{g}(\tau)$ be the impulse response of the linear filter, which is non-zero only for $0 \leq \tau \leq \Delta t$, with Δt being the period of sampling. We consider here only periodic sampling, because we have a stationary signal $\tilde{s}(t)$ in $[0, T]$.

Hence we have

$$\tilde{r}_k = \int_0^{\Delta t} \tilde{g}(u)\tilde{r}(k\Delta t - u)\,\mathrm{d}u = \tilde{s}_k + \tilde{n}_k.$$

We can calculate the signal $\tilde{s}_k$ and noise $\tilde{n}_k$ power.

$$\sigma_s^2 = E\{|\tilde{s}_k|^2\} = \frac{E_t}{T}\int_0^{\Delta t} \tilde{g}(u)\tilde{g}^*(v)\tilde{K}_{\tilde{a}}(v-u)\,\mathrm{d}v\,\mathrm{d}u,$$

$$\sigma_n^2 = E\{|\tilde{n}_k|^2\} = N_0\int_0^{\Delta t} |\tilde{g}(u)|^2\,\mathrm{d}u.$$

Let $\{\lambda_n\}$ and $\{\tilde{\varphi}_n(\cdot)\}$ be respectively the eigenvalues and the eigenfunctions of the covariance function $\tilde{K}_{\tilde{a}}(\cdot,\cdot)$ in $[0, \Delta t]$, that is

$$\tilde{K}_{\tilde{a}}(t, u) = \sum_{n=1}^{\infty} \lambda_n\tilde{\varphi}_n(t)\tilde{\varphi}_n^*(u); \quad 0 \leq t, u \leq \Delta t$$

We want to determine $\tilde{g}(u)$ such that the signal to noise power ratio is maximized. This requirement gives

$$\tilde{g}(u) = \tilde{\varphi}_{\max}(u); \quad 0 \leq u \leq \Delta t$$

where $\tilde{\varphi}_{\max}(\cdot)$ is the eigenfunction which corresponds to the largest eigenvalue.

As $\tilde{K}_{\tilde{a}}$ is stationary

$$\tilde{\varphi}(u) = \tilde{\varphi}^*(\Delta t - u); \quad 0 \leq u \leq \Delta t.$$

Thus the linear filter is matched to this eigenfunction[1]. In general, the complex envelope of the filter is low-pass. We can therefore use an integrator over the sampling interval

$$\tilde{g}(u) = \frac{1}{\Delta t}; \quad 0 \leq u \leq \Delta t$$

With the above assumptions, the covariance matrix of the noise (hypothesis H_0) is given by

$$\boldsymbol{K}_0 = \frac{N_0}{\Delta t}\boldsymbol{I}, \quad \text{where } \boldsymbol{I} \text{ is the identity matrix}$$

For the signal component, we have

$$E\{\tilde{s}_n\tilde{s}_m^*\} = \frac{\bar{E}_r}{T}\frac{1}{(\Delta t)^2}\int_{(n-1)\Delta t}^{n\Delta t}\int_{(m-1)\Delta t}^{m\Delta t} \tilde{K}_{\tilde{a}}(t-u)\,\mathrm{d}t\,\mathrm{d}u.$$

For the scattering function given in (4), we obtain the signal component of the covariance matrix, as follows

$$E\{\tilde{s}_n\tilde{s}_m^*\} = \frac{\bar{E}_r}{T}\mathrm{e}^{-(kT/N)|n-m|}2\left(\frac{N}{kT}\right)^2\left[\cosh\left(\frac{kT}{N}\right) - 1\right]$$

[1] For a non stationary signal $\tilde{s}(t)$ the linear filter must be matched to the eigenfunction corresponding to a sampling interval, with the same criterion of optimization.

for $n \neq m$, and

$$E\{|s_n|^2\} = \frac{\bar{E}_r}{T} 2\left(\frac{N}{kT}\right)^2 \left[-1 + \frac{kT}{N} + e^{-kT/N}\right], \quad n = 1, \ldots, N. \tag{15}$$

But the receiver and the error probability depend on the sampling rate. Thus, by numerical calculation of the error probability, we are searching for a sufficient sampling rate. The Collins' method of computing error probability is used [12].

This is the subject of the following section.

5. Numerical results and comments

In Fig. 5, we give the probability of error versus the sampling rate (measured by $N = T/\Delta t$) for $kT = 5$ and for different values of the signal to noise ratio.

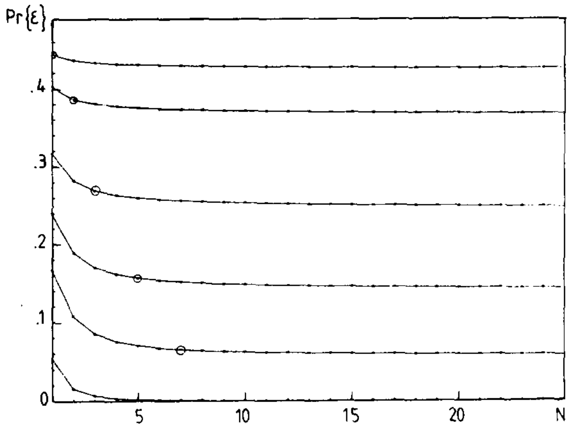

Fig. 5. Error probability versus number of samples for $kT = 5$ and $SNR = 1, 2, 5, 10, 20, 100$ (from top to bottom).

The error probability is defined by

$$Pr\{\varepsilon\} = \frac{1 - P_D + P_F}{2}$$

where P_D is the detection probability and P_F is the false alarm probability.

Figures 6 and 7 give the same curves for $kT = 10$ and 100 respectively.

The above probability of error decreases asymptotically, when the number of samples in $[0, T]$ increases. This evolution is more or less fast depending on the product kT. Then a sufficient number of samples can be defined, so that the corresponding probability of error is close enough to the asymptotic value. For a large signal to noise ratio, we state that this number grows with the signal to noise ratio up to about kT. It means that a sufficient Δt is about $1/k$, which corresponds to the channel coherence time. The first important results can be roughly explained by the fact that two samples, which belong to the same interval of channel coherence time, contain similar information, making the sampling redundant. We also notice that the sufficient number of samples can be reduced (down to $kT/2$ and even less) when the signal to

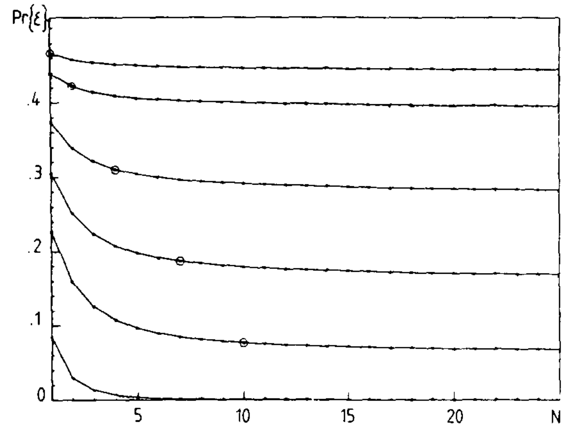

Fig. 6. Error probability versus number of samples for $kT = 10$ and $SNR = 1, 2, 5, 10, 20, 100$ (from top to bottom).

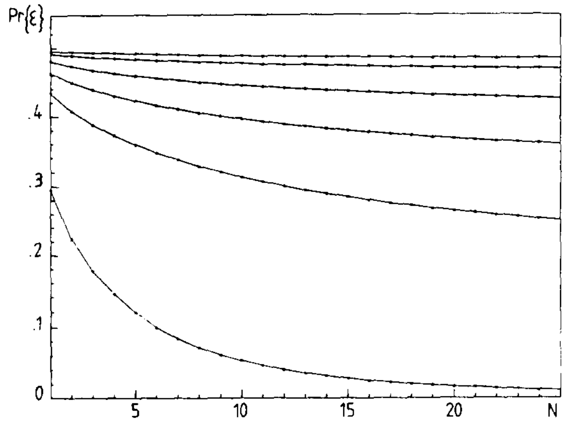

Fig. 7. Error probability versus number of samples for $kT = 100$ and $SNR = 1, 2, 5, 10, 20, 100$ (from top to bottom).

noise ratio becomes small (SNR$<$2). However, the probability of error is, in that case, not much below $\frac{1}{2}$.

The asymptotic performances of the receiver, when the number of samples becomes large, can be directly computed, using Collin's method [12] and some approximations for the eigenvalues of the signal component of the covariance matrix [13]. L. Andriot [14] makes an extensive development of the calculation of asymptotic performances. We present here the most important results.

The asymptotic performances are given in Fig. 8, for different values of product kT. A comparison with numerical results obtained without using eigenvalue approximation shows that the limit performances are valid for $kT \geqslant 10$. The asymptotic performances are also compared with the bound on error probability given by G. Tziritas [4]. The asymptotic error probability is minimal versus signal to noise ratio for an optimal product kT (in the domain of validity), which is itself an increasing function of the signal to noise ratio. We give this optimal kT versus signal to noise ratio in Fig. 9.

This important result confirms similar conclusions found for a continuous structure of the optimal receiver (for the case of binary symmetric communication in Fig. 2). It means that, the channel coherence time being fixed, it is possible to choose an optimal duration of the transmitted signal in order to minimize

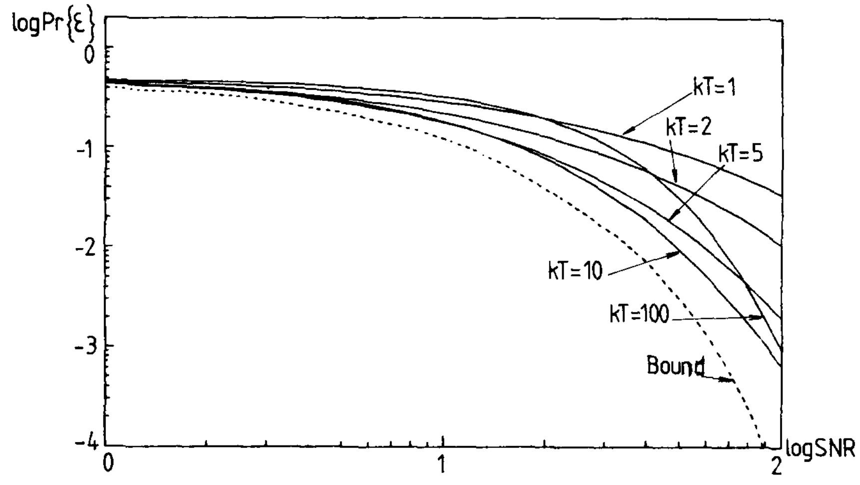

Fig. 8. Asymptotic error probability.

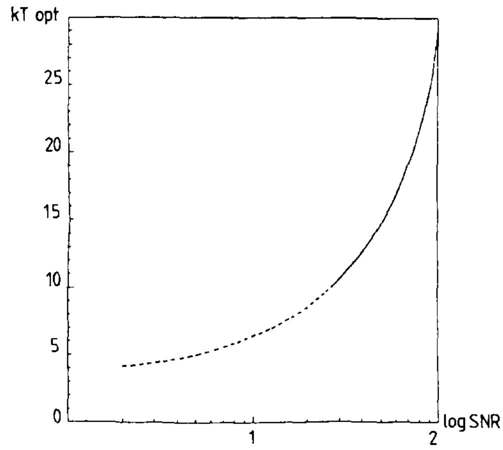

Fig. 9. Optimal kT versus SNR using asymptotic formulas.

the error probability for a given signal to noise ratio. The sampling rate, then, can be chosen, as explained before, with respect to the signal to noise ratio and the kT product.

6. Conclusion

We have studied a new suboptimum receiver (Fig. 3) using sampling techniques for detecting signals over a first order doppler spread channel. We have computed the performances of the receiver depending on three parameters: the signal to noise ratio, the product kT and the number of samples or the sampling rate. We have shown that the three parameters are related. Thus for a given signal to noise ratio and for a given channel coherence time ($= 1/k$) there exists an optimal duration for the transmitted signal. This result shows that the receiver proposed here is near to optimum, because we have the same behaviour with the optimum receiver. The choice of the duration of the constant envelope transmitted signal being made, the sufficient number of samples is approximately kT for a large signal to noise ratio, and less important for a small signal to noise ratio. At this point, we emphasize the duality between the doppler-spread and the range-spread channel. We find a similar result concerning parameter kT (ratio of signal bandwidth to channel coherence bandwidth), for the doppler- and range-spread channel respectively.

Furthermore, one can see that the performances of the proposed receiver are near to those of the optimum receiver for a sufficient number of samples. We have not given numerical results for the binary symmetric communication problem, but we expect similar results to be obtained. Using another procedure, H. Cherifi [15] studied the "Rayleigh receiver" for the binary symmetric communication problem over a doppler spread channel, which is equivalent to a single sample receiver. A synthesis of the two parts remains to be done, that is the design of a receiver using a sampler for a double spread random channel.

Appendix

We give here the explicit formulae for the inverse of a Toeplitz matrix according to T. Kailath, A. Vieira and M. Morf [16].

Let us consider a symmetric (hermitean) Toeplitz positive definite matrix $\boldsymbol{R}_N$

$$\boldsymbol{R}_N = \begin{bmatrix} r_0 & r_1 & \cdots & r_{N-1} \\ r_1 & r_0 & \cdots & r_{N-2} \\ & \cdots & \cdots & \\ r_{N-1} & r_{N-2} & \cdots & r_0 \end{bmatrix}$$

Let D_N be the determinant of $\boldsymbol{R}_N$ and $\alpha_N = \beta_N = D_N / D_{N-1}$.

Consider the following equations:

$$[1 \;\; a_{1,N} \cdots a_{N-1,N}]\boldsymbol{R}_N = [\alpha_N \;\; 0 \cdots 0]$$

and

$$[b_{N-1,N} \cdots b_{1,N} \;\; 1]\boldsymbol{R}_N = [0 \cdots 0 \;\; \beta_N].$$

These equations can be effectively solved by the so-called Levinson algorithm [17]. Then the inverse of the non-singular Teoplitz matrix $\boldsymbol{R}_N$ is given by:

$$\boldsymbol{R}_N = \frac{1}{\alpha_N}\left\{\begin{bmatrix} 1 & & & 0 \\ a_{1,N} & \ddots & & \\ \vdots & \ddots & \ddots & \\ a_{N-1,N} & \cdots & a_{1,N} & 1 \end{bmatrix}\begin{bmatrix} 1 & b_{1,N} & \cdots & b_{N-1,N} \\ & \ddots & \ddots & \vdots \\ & & \ddots & b_{1,N} \\ 0 & & & 1 \end{bmatrix} - \begin{bmatrix} 0 & & & 0 \\ b_{N-1,N} & \ddots & & \\ \vdots & \ddots & \ddots & \\ b_{1,N} & \cdots & b_{N-1,N} & 0 \end{bmatrix}\begin{bmatrix} 0 & a_{N-1,N} & \cdots & a_{1,N} \\ & \ddots & \ddots & \vdots \\ 0 & & \ddots & a_{N-1,N} \\ & & & 0 \end{bmatrix}\right\}$$

The definitions of matrices $\boldsymbol{A}_1$, $\boldsymbol{B}_1$, $\boldsymbol{A}_N$ and $\boldsymbol{B}_N$ in formula (14) are then obvious.

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