ESTIMATION OF MOTION AND STRUCTURE OF 3-D OBJECTS FROM A SEQUENCE OF IMAGES

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ABSTRACT

In this article we analyze the motion of 3-D objects either in terms of image contours or intensity gradients. We make the assumption that the objects viewed are locally rigid and planar. We suppose that the projection is perspective and use the "perspective" velocity vector, which has been introduced by G.L.Scott[19]. We give spatial relations on the "perspective" velocity field both in the case of points lying on a contour and in the case of dense image points. We utilize these spatial relations to estimate the "perspective" velocity field using the normal velocity component on a contour or the motion constraint equation in the 2D image. In the first case the Kalman filter is used. In the second case a Gauss-Seidel relaxation algorithm is proposed. We demonstrate that if the "perspective" velocity vector is known at three points, one can reconstruct the 3-D motion parameters and the plane orientation. We also demonstrate that if the "perspective" velocity vector is known at four points the 3-D motion parameters and the relative depths can be reconstructed solving two systems of three linear equations and a system of four linear equations.

1. INTRODUCTION

In many applications the retinal motion results from the presence in the scene of moving 3-D objects and/or from retinal motion in a static environment. The principal task is therefore the analysis of an image sequence for estimating the motion of 3-D objects. It is well-known that the retinal motion contains also information on the structure of the 3-D objects. Thus the reconstruction of the depth map constitutes the other task in motion analysis.

There exist two approaches in motion perception and analysis. In the first one the motion is considered discrete in the time, and in the second continuous. Psychophysical experiences support the distinction of the two types of motion in biological vision[13]. In machine vision the two approaches are considered. The discrete motion analysis is based in token matching[1], while continuous motion analysis is intensity gradient based[6]. Our study is concerned with the continuous motion analysis.

It is well established that 3-D structure information can be extracted from motion measurement. In human vision local 3-D rigidity assumption is decisive for 3-D inference from 2-D motion[4]. The same assumption is generally used in machine vision and particularly in the domain of robotics. Hypotheses concerning surface structure (as planar or quadratic) do not relax not in the least the 3-D rigidity assumption.

We use a gradient-based method for determining the retinal motion. This means that we utilize the motion constraint equation in the whole image plane or the contour normal displacement as initial motion measurement. Thus we obtain at every point of the image plane or the contour a single component of the velocity field. The following stage of the analysis consists to estimate the retinal velocity field using a single velocity component. In order to make this, methods using smoothing constraints are known in the literature[5,6]. We use the "perspective" velocity vector, which is introduced by G.L.Scott[10], to represent the retinal motion. We propose a method for estimating the "perspective" velocity field. We make the assumption that the objects are locally rigid and planar.

The last stage of the analysis consists to determine the 3-D motion parameters and the structure or depth characteristics from the retinal velocity field. One can distinguish two approaches for reconstructing the 3-D objects structure and motion. The first one uses the velocity field as sparse, and the second as dense. In the second approach one must dispose the first and second derivatives of the velocity field. For the case of a sparse velocity field we give here a system of linear equations for determining the 3-D motion parameters and structure.

In Section 2 we recall the main equations for the "perspective" velocity field and we consider the hypothesis of plane surfaces. In Section 3 autoregressive spatial relations on "perspective" velocity field are given. Using these relations and the measurement of a single component we apply the Kalman filter for a recursive estimation of the 2-D velocity field on a contour and a relaxation or gradient method for estimation in the whole image plane. In Section 4 we give a system of linear equations for determining the 3-D motion parameters and structure or depth characteristics. A discussion and some conclusions constitute the last Section, where some applications are also given.

2. THE "PERSPECTIVE" VELOCITY FIELD

Let us consider a 3-D coordinate system O(X,Y,Z) that is fixed with respect to the retina, OZ being the line of sight[8]. The camera focal length is normalized (f=1). Let

$V_t=(V_X,V_Y,V_Z)$ be the translational velocity and $\Omega=(\Omega_X,\Omega_Y,\Omega_Z)$ be the angular velocity of a point P(X,Y,Z) in an observer-relative decomposition. If the observer is moving through a static environment the velocity of P is $-V_t$ and $-\Omega$. For a point P(X,Y,Z) the velocity components are given by

$$\begin{aligned} \dot{X} &= V_X + Z\,\Omega_Y - Y\,\Omega_Z \\ \dot{Y} &= V_Y + X\,\Omega_Z - Z\,\Omega_X \\ \dot{Z} &= V_Z + Y\,\Omega_X - X\,\Omega_Y \end{aligned} \qquad (2.1)$$

The "perspective" velocity vector is obtained by dividing by Z[10]

$$\begin{aligned} u &= V_X/Z + \Omega_Y - y\Omega_Z \\ v &= V_Y/Z - \Omega_X + x\Omega_Z \qquad (2.2) \\ w &= V_Z/Z + y\Omega_X - x\Omega_Y \end{aligned}$$

Let (φ,ψ) be the retinal velocity field. It is related to the "perspective" velocity field by (2.3)

$$\varphi = u - xw$$
$$\psi = v - yw$$

Formulae (2.2) are valid in the whole image plane for a moving observer. Otherwise their validity is limited in the image of the moving object, and in the motion analysis a segmentation will be necessary or the detection of discontinuities on the 3-D motion parameters.

Formulae (2.2) show that the "perspective" velocity components are depending on the depth Z of P. If one wants obtain expressions depending only on two-dimensional coordinates, one must make a hypothesis concerning the surface shape. A simple and plausible hypothesis is that the surface is locally plane. In other words one takes in consideration only the surface orientation. Let us define a plane (excluding degenerate case in which the plane contains the nodal point) $n_XX+n_YY+n_ZZ=1$. For a perspective projection we have $1/Z = n_Xx + n_Yy + n_Z$ and we obtain from (2.2)

$$\begin{aligned} u &= (n_ZV_X+\Omega_Y)+n_XV_Xx+(n_YV_X-\Omega_Z)y \\ v &= (n_ZV_Y-\Omega_X)+(n_XV_Y+\Omega_Z)x+n_YV_Yy \qquad (2.4) \\ w &= n_ZV_Z+(n_XV_Z-\Omega_Y)x+(\Omega_X+n_YV_Z)y \end{aligned}$$

G.L.Scott[10] indicates that if there exists a first order "perspective" velocity field, like this of (2.4), there exists a one-parameter family of such fields, consistent with the same motion: (u-cx, v-cy, w-c).

3. ESTIMATION OF THE "PERSPECTIVE" VELOCITY FIELD

3.1. Spatial relations on the "perspective" velocity field

The expressions (2.4) for the 2-D velocity field suggest spatial relations, which are locally valid, depending on the validity of planarity assumption. We develop in detail the discrete case. For the continuous case we give only the general spatial relations. For the component u we have

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial x \partial y} = 0 \ . \qquad (3.1)$$

All second order derivatives are also vanishing for the other components v and w.

We consider now the case of discrete points lying on a contour. Let us put on again the expression for the component u

$$u = a_1x+b_1y+c_1 .$$

We consider a series of points (x_k,y_k). We can write

$$\begin{aligned} u_{k+1}-u_k &= a_1(x_{k+1}-x_k)+b_1(y_{k+1}-y_k) \\ u_k-u_{k-1} &= a_1(x_k-x_{k-1})+b_1(y_k-y_{k-1}) \qquad (3.2) \\ u_{k-1}-u_{k-2} &= a_1(x_{k-1}-x_{k-2})+b_1(y_{k-1}-y_{k-2}) \end{aligned}$$

We can write the same equations for the components v and w. A consequence of (3.2) is that

$$\begin{vmatrix} u_{k+1}-u_k & x_{k+1}-x_k & y_{k+1}-y_k \\ u_k-u_{k-1} & x_k-x_{k-1} & y_k-y_{k-1} \\ u_{k-1}-u_{k-2} & x_{k-1}-x_{k-2} & y_{k-1}-y_{k-2} \end{vmatrix} = 0 \qquad (3.3)$$

and the same for v and w. One can then write $D_{1,k}(u_{k+1}-u_k)-D_{2,k+1}(u_k-u_{k-1})+D_{1,k+1}(u_{k-1}-u_{k-2}) = 0$ where $D_{1,k}$ and $D_{2,k+1}$ are calculated from (3.3). Thus one can obtain an autoregressive relation on the velocity.

3.2. Motion measurement from 2-D images: gradient methods

From image changing intensities one can measure only a single component of the two-dimensional or "perspective" velocity field. The motion constraint equation gives a relationship between temporal and spatial gradients of image intensity g(x,y) and retinal velocities[6]

$$g_x\varphi + g_y\psi + g_t = 0 \qquad (3.4)$$

This equation is not valid at occluding edges, it assumes the intensity of any pixel does not change significantly over a short time interval and that the image irradiance changing is entirely due to motion.

Another approach that is gradient-based uses the normal component of the velocity at contour points. This supposes that a stage of edge detection is preceding[2,3]. The measurement of normal component is given by equation (3.4), but it is erroneous for occluding edges. Another method to measure the normal component is to consider a displacement on the perpendicular direction from the first contour to the second[5].

3.3. Estimation of the "perspective" velocity vector on contour points

We propose to use the autoregressive relation given in section 3.1 for estimating the "perspective" velocity field.

Let us consider the equation (3.4) and write the autoregressive relation for the component u

$$u_{k+1} = \beta_ku_k + \beta_{k-1}u_{k-1} + \beta_{k-2}u_{k-2}$$

where the identification of β_k is obvious. We can write the same relation for the other velocity components. We designate ξ_k the state vector. It is given by

$$\xi_k = [u_k \ u_{k-1} \ u_{k-2} \ v_k \ v_{k-1} \ v_{k-2} \ w_k \ w_{k-1} \ w_{k-2}]^T$$

The state equation is given below

$$\xi_{k+1} = \begin{bmatrix} \Phi_{k+1|k} & 0 & 0 \\ 0 & \Phi_{k+1|k} & 0 \\ 0 & 0 & \Phi_{k+1|k} \end{bmatrix} \xi_k + \omega_k$$

where the noise vector ω_k is zero-mean with covariance Q_k and the transition matrix $\Phi_{k+1|k}$ is

$$\Phi_{k+1|k} = \begin{bmatrix} \beta_k & \beta_{k-1} & \beta_{k-2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The equation of measurement is given by

$$y_k = [\ c_{1k}\ \ 0\ 0\ \ c_{2k}\ \ 0\ \ 0\ \ c_{3k}\ \ 0\ \ \ 0\ ]\ \xi_k + z_k$$

where y_k is the measured projection of the velocity on the normal vector of the contour

$$(c_{1k}, c_{2k}, c_{3k})\ \sqrt{f_x^2(x_k, y_k) + f_y^2(x_k, y_k)}$$
$$=(f_x(x_k,y_k), f_y(x_k,y_k), -(x_k f_x(x_k,y_k)+y_k f_y(x_k,y_k)))$$

and z_k is a measurement noise which is supposed zero-mean with variance R_k. The system and measurement noise are supposed independent between them and between different points. This is a classical linear estimation problem and the solution is the well-known Kalman filter for the discrete case[7,9]. We discuss here how to apply the filter on a contour. If the contour is closed, one can choose any point as initial and develop the filter around the contour. It is obvious that the velocity at the first points will be worst estimated than that of last points. In all cases a second application of the filter around the contour will be necessary. If the contour is not closed, the same technique can be applied inversing the sense of direction of the filter at the final point.

3.4. Estimation on the image plane

One can use the spatial relations on the "perspective" velocity field and the gradient equation (3.4) for estimating the retinal motion. We have three spatial relations for each velocity component at each point. The more natural way to use them is the minimization of a quadratic functional, which has the following form

$$\lambda^2(\|A_1U\|^2+\|A_2U\|^2+\|A_3U\|^2+\|A_1V\|^2+\|A_2V\|^2+\|A_3V\|^2$$
$$+\|A_1W\|^2+\|A_2W\|^2+\|A_3W\|^2) + \|G_xU+G_yV+G_zW+G_t\|^2$$

where U, V and W are the complete velocity components, A_1, A_2, A_3, G_x, G_y, G_z, G_t are linear operators on the image plane, and $\|\ \|^2$ is the euclidean norm. The operators A_1, A_2 and A_3 are determined from the spatial relations. The operators G_x, G_y, G_z, where

$$g_{z;i,j} = -x_{i,j}g_{x;i,j} - y_{i,j}g_{y;i,j},$$

and G_t contain the spatial and the temporal gradient of the image intensity. The minimization of the above quadratic functional gives a system of linear equations, given below (3.5)

$$\lambda^2[(A_1)^TA_1+(A_2)^TA_2+(A_3)^TA_3]U+G_x^T(G_xU+G_yV+G_zW+G_t)=0$$
$$\lambda^2[(A_1)^TA_1+(A_2)^TA_2+(A_3)^TA_3]V+G_y^T(G_xU+G_yV+G_zW+G_t)=0$$
$$\lambda^2[(A_1)^TA_1+(A_2)^TA_2+(A_3)^TA_3]W+G_z^T(G_xU+G_yV+G_zW+G_t)=0$$

where superscript ^{T} signifies the adjoint operator. We specify in the following the operators A_1, A_2, A_3, and we give a method to solve the above system of equations.

The quadratic functional to minimize is given below

$$\iint\ [\lambda^2\ [(u_{xx})^2+2(u_{xy})^2+(u_{yy})^2+(v_{xx})^2+2(v_{xy})^2+(v_{yy})^2$$
$$+(w_{xx})^2+2(w_{xy})^2+(w_{yy})^2] + (g_xu+g_yv+g_zw+g_t)^2]\ dx\ dy$$

The smoothness constraint of this functional is to be close to the surface reconstruction constraints by a thin plate model[12].

The minimization leads to a system of linear equations of the following form

$$u_{i,j} = \bar{u}_{i,j} - g_{x;i,j}\,\gamma_{i,j}$$
$$v_{i,j} = \bar{v}_{i,j} - g_{y;i,j}\,\gamma_{i,j}$$
$$w_{i,j} = \bar{w}_{i,j} - g_{z;i,j}\,\gamma_{i,j}$$

with

$$\gamma_{i,j}[\lambda^2+(g_{x;i,j})^2+(g_{y;i,j})^2+(g_{z;i,j})^2]=(g_{x;i,j}\,\bar{u}_{i,j}+g_{y;i,j}\,\bar{v}_{i,j}+g_{z;i,j}\,\bar{w}_{i,j}+g_{t;i,j})$$

and where the averaging is given below for the component u (without loss of generality we use the same weighting factor λ^2)

$$\bar{u}_{i,j} = 0.4(u_{i+1,j}+u_{i-1,j}+u_{i,j+1}+u_{i,j-1})$$
$$-0.1(u_{i+1,j+1}+u_{i+1,j-1}+u_{i-1,j+1}+u_{i-1,j-1})$$
$$-0.05(u_{i+2,j}+u_{i,j-2}+u_{i-2,j}+u_{i,j+2}) \qquad (3.6)$$

We can use these equations in an iterative method, such as the Gauss-Seidel method, for determining the solution of the system of equations

$$u_{i,j}^{(n+1)} = \bar{u}_{i,j}^{(n)} - g_{x;i,j}\ \gamma_{i,j}^{(n)}$$
$$v_{i,j}^{(n+1)} = \bar{v}_{i,j}^{(n)} - g_{x;i,j}\ \gamma_{i,j}^{(n)}$$
$$w_{i,j}^{(n+1)} = \bar{w}_{i,j}^{(n)} - g_{x;i,j}\ \gamma_{i,j}^{(n)}$$

This method is the same utilized by B.Horn and B.Schunck[6] to determine the optical flow. The difference lies in the choice of local averaging. The local averaging used here is the discrete version of

$$u + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial y^4}$$

and that of B.Horn and B.Schunck is the discrete version of

$$u + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

This is the consequence of the different functional minimized for the two methods.

4. 3-D MOTION PARAMETERS AND STRUCTURE ESTIMATION

We consider now that the retinal velocity field is estimated and we will determine the 3-D motion parameters and structure using it. If the 2-D velocity field is dense, H.Longuet-Higgins and K.Prazdny[8] and A.Waxman and S.Ullman[14] give a method to calculate at each point the 3-D motion parameters and the surface orientation and curvature scaled by the depth. They use φ, ψ and their 1st and 2nd order spatial derivatives. They reconstitute all information contained in the optical flow, but one can remark that derivation amplify the errors of 2-D velocity field estimation. Nevertheless A.Waxman and S.Ullman[14] give directly the velocity field and its spatial derivatives as initial observable.

Our approach is different, but it can be considered as complementary. We use the "perspective" velocity field at distinct image points. If the retinal velocity field is sparse, this is the single method for 3-D motion and depth reconstruction. The information about structure in the case of a sparse velocity field is the relative depths, in the case of a dense field is the surface orientation and eventually the curvature.

We study separately the case of planar surfaces and that of curved surfaces.

4.1. Planar surface

If the "perspective" velocity vector is known at three non-aligned points from the expressions (2.4) results that one can obtain nine values, called the essential parameters.

$n_XV_X+c = a_1$, $n_YV_X-\Omega_Z = b_1$, $n_ZV_X+\Omega_Y = c_1$

$n_XV_Y+\Omega_Z = a_2$, $n_YV_Y+c = b_2$, $n_ZV_Y-\Omega_X = c_2$

$n_XV_Z-\Omega_Y = a_3$, $\Omega_X+n_YV_Z = b_3$, $n_ZV_Z+c = c_3$

This system of non-linear equations has been solved by M.Subbarao and A.Waxman[11] to obtain: n_X/n_Z, n_Y/n_Z (n_Z_0), n_ZV_X, n_ZV_Y, n_ZV_Z, Ω_X, Ω_Y and Ω_Z. (One can easily demonstrate that the condition n_Z_0is equivalent to: $(a_3+c_1)^2(c_3-b_2)+(b_3+c_2)^2(c_3-a_1)+(b_3+c_2)(a_3+c_1)(a_2+b_1) \neq 0$). In general there exist two solutions. The duality can be resolved using spatial or temporal coherence[11].

4.2. Non-planar surface

Let us consider that the three components of the "perspective" velocity vector are known at four image points. From (2.2) one can write

$$Z_i = \frac{V_X}{u_i - \Omega_Y + y_i\Omega_Z + x_i c} \tag{4.1}$$

for i = 1, 2, 3, 4, and a similar equation for V_Y and V_Z. We assume that V_Z _ 0 and we put $e_1 = V_X/V_Z$ and $e_2 = V_Y/V_Z$. Let us consider two points. We have (4.2) and (4.3)

$e_1(w_i-w_j)-(\Omega_Z+e_1\Omega_X)(y_i-y_j)+(e_1\Omega_Y-c)(x_i-x_j)=u_i-u_j$

$e_2(w_i-w_j)-(e_2\Omega_X+c)(y_i-y_j)+(\Omega_Z+e_2\Omega_Y)(x_i-x_j)=v_i-v_j$

Knowing the "perspective" velocity vector at four non-aligned points we obtain two systems of three linear equations (the first from (4.2) and the second from (4.3)). Solving the two linear systems we obtain:

$$e_1,\ \Omega_Z+e_1\Omega_X = e_3,\ e_1\Omega_Y-c = e_4 \tag{4.4}$$

$$e_2,\ \Omega_Z+e_2\Omega_Y = e_5,\ e_2\Omega_X+c = e_6 \tag{4.5}$$

From the two last equations of (4.4) and (4.5) we obtain a system of four linear equations

$$\begin{bmatrix} e_1 & 0 & 1 & 0 \\ 0 & e_1 & 0 & -1 \\ 0 & e_2 & 1 & 0 \\ e_2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Omega_X \\ \Omega_Y \\ \Omega_Z \\ c \end{bmatrix} = \begin{bmatrix} e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix}$$

If $e_1^2+e_2^2 \neq 0$, the solution is:

$\Omega_X = [e_1(e_3-e_5)+e_2(e_4+e_6)]/(e_1^2+e_2^2)$

$\Omega_Y = [e_1(e_4+e_6)-e_2(e_3-e_5)]/(e_1^2+e_2^2)$

$\Omega_Z = [e_1^2e_5+e_2^2e_3-e_1e_2(e_4+e_6)]/(e_1^2+e_2^2)$

$c = [e_1^2e_6-e_2^2e_4-e_1e_2(e_3-e_5)]/(e_1^2+e_2^2)$

Then from (4.1) we obtain Z_i/V_Z. If $e_1^2+e_2^2 = 0$, (4.4) gives

$\Omega_Z = e_3 = e_5$

$c = e_6 = -e_4$.

Ω_X is obtained from: $v_i+\Omega_X+y_ic-x_i\Omega_Z = 0$,

and Ω_Y from: $u_i-\Omega_Y+y_i\Omega_Z+x_ic = 0$.

The solution has been obtained assuming that V_Z _ 0. This is equivalent to have

$$\begin{vmatrix} w_1-w_2 & y_1-y_2 & x_1-x_2 \\ w_2-w_3 & y_2-y_3 & x_2-x_3 \\ w_3-w_4 & y_3-y_4 & x_3-x_4 \end{vmatrix} \neq 0$$

We remark that one can separate the translational and the rotational component of the 2-D velocity field. Therefore it is possible to determine the focus of expansion that is the point

$(V_X/V_Z,\ V_Y/V_Z)$

where the translational velocity component is vanishing.

One can also determine an instantaneous time-of-collision, which is an interesting parameter in passive navigation

$$T_{col} = -\frac{Z}{\dot{Z}} = \frac{1}{x\Omega_X - y\Omega_Y - V_Z/Z}$$

5. APPLICATIONS AND CONCLUSIONS

We have applied the method presented here on a simulated motion of a quartic curve, whose the equation on the image plane is given in the following

$$x^4 + y^4 = 1$$

The 3-D parameters were: $n_X = n_Y = 0$, $n_ZV_Z = 0.1$, $V_X = V_Y = 0$, $\Omega_X = \Omega_Z = 0$, $\Omega_Y = 0.1$. Four iterations along the contour were necessary to estimate exactly the retinal velocity field, when the measurement noise is zero. In Fig. 1 we give the obtained results, which is the true 2-D velocity field and the estimated one. They are coinciding.

Fig. 1. The true and the estimated 2-D velocity field for a quartic curve lying on a plane.

We summarize in the following the principal results contained in this article.

Using the assumption of local 3-D rigidity and local planarity we have given spatial relations on the "perspective" velocity field. For points lying on a contour an autoregressive relation of order 3 for each component velocity component is given (3.4). In the whole image plane three spatial relations are given (3.1) for each velocity component.

The spatial relations on the 2-D velocity field are used to estimate it, utilizing for a contour the perpendicular component of the velocity and for the whole image the motion constraint equation. In the case of a contour we propose a recursive method utilizing a Kalman filter as estimator. In the case of the whole image plane we propose a classical relaxation method to solve the system of linear equations.

Finally we give a system of linear equations to determine the motion parameters and the relative depths. Four points and their velocities are considered. A least squares resolution may be used to reduce the effect of the retinal velocity field estimation noise on the estimated 2-D velocity field. We show also that the translational and the rotational component of the velocity can be separated, and therefore the focus of expansion may be determined. We give also the instantaneous time-of-collision.

Before closing this article we would like comment certain aspects, which merit more attention. The stage of initial local measurements from image intensities is very important. We have seen that we can obtain a good estimation of the retinal velocity field from a single component and that we can reconstruct the 3-D motion and structure exactly from the retinal velocity field. The stage that eventually introduces important errors is the initial measurement. We think that an effort must be consecrated to improve this measurement stage, or eventually a method matching contours globally must be determined.

Another important and difficult problem, which is not studied here, is that of discontinuities on the 2-D velocity field. Discontinuities arise at occluding boundaries at both cases of several moving objects and depth discontinuities. If only depth discontinuities are present, a detector at the stage of estimation of the retinal velocity field must be used to avoid the propagation of errors. If several moving objects are present in the scene, a segmentation of the retinal velocity field is necessary.

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