CS578 - Speech Signal Processing

Lecture : Quasi-Harmonic Models of Speech

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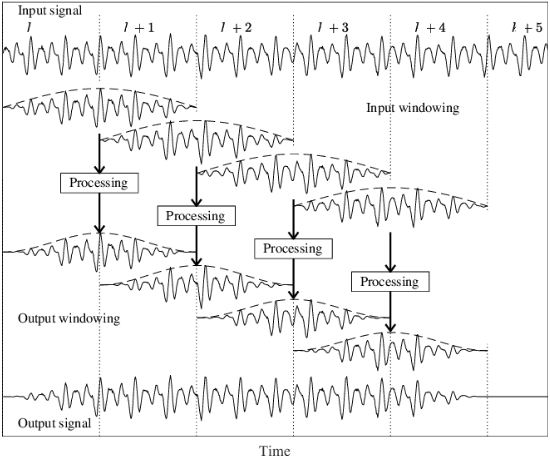
(based on material from Prof. Stylianou)

Computer Science Department - University of Crete

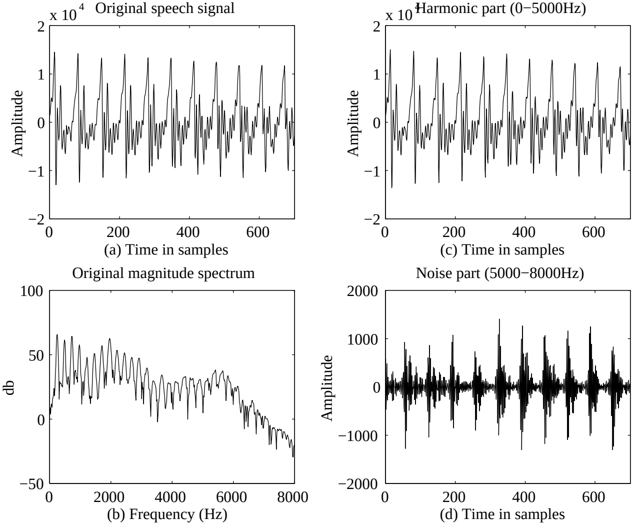
18 October 2023

Outline

Frame-by-frame analysis



Motivation for HNM



Brief overview of HNM

- HNM (Stylianou 1995 [1]) is a pitch-synchronous harmonic plus noise representation of the speech signal.
- Speech spectrum is divided into a low and a high band delimited by the so-called *maximum voiced frequency*, $F_{\text{max}}^{(l)}$.
- The *lower* band of the spectrum (below the maximum voiced frequency) is represented solely by harmonically related sine waves.
- The *upper* band is modeled as a noise component modulated by a time-domain amplitude envelope.
- HNM allows high-quality copy synthesis and prosodic modifications.

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HNM IN EQUATIONS

- Harmonic part:

$$h^{(l)}(t) = \sum_{k=-K^{(l)}}^{K^{(l)}} A_k^{(l)} e^{j2\pi k f_0^{(l)} t}$$

where $A_k^{(l)}$ and $f_0^{(l)}$ are the complex amplitude and fundamental frequency for the l-th frame, respectively

- Noise part:

$$n^{(l)}(t) = e^{(l)}(t) \left[v^{(l)}(t) \star g^{(l)}(t) \right]$$

where $e^{(l)}(t), v^{(l)}(t), g^{(l)}(t)$ are a time envelope, an estimation of the power spectral density (i.e., a filter), and white gaussian noise for the l-th frame, respectively

- Speech:

$$s(t) = h(t) + n(t)$$

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Outline

Estimating Sinusoidal parameters

- Sinusoidal representation for a speech/signal frame:

$$x(t) = \left(\sum_{k=-K}^{K} a_k e^{j2\pi f_k t} \right) w(t)$$

- Parameter estimation methods:
  - FFT-based methods (i.e., QIFFT [Abe et al., 2004-05], [2], [3])
  - Least Squares (LS) method
- Frequency mismatch (eg, $\hat{f}_k := k\hat{f}_0$ in HNM):

$$\hat{f}_k = f_k + \eta_k$$

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Quasi-Harmonic Model, QHM [5]

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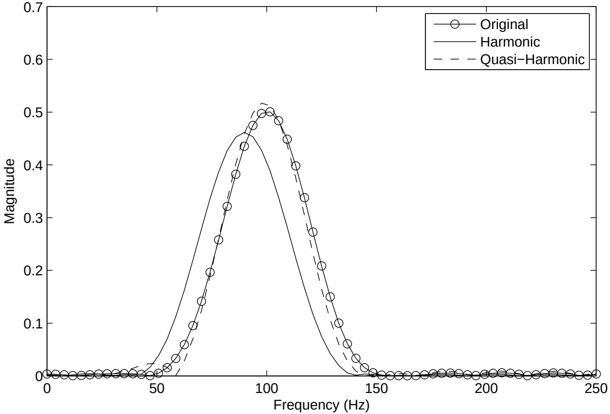
- HM versus QHM in frequency estimation - pure tone @ 100 Hz
- Given frequency for both models: 90 Hz



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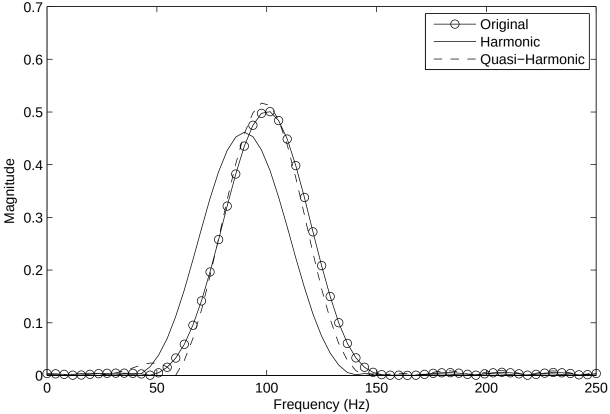

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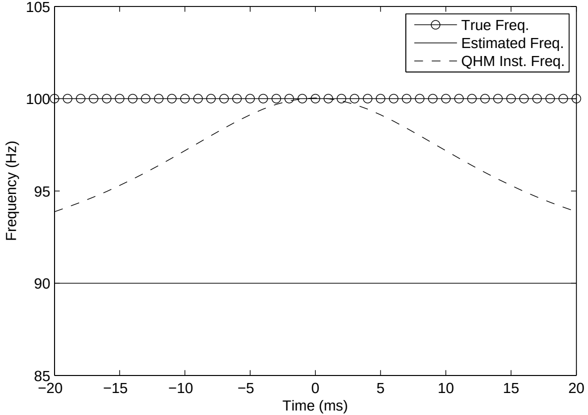

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- Let's discuss a bit on that...

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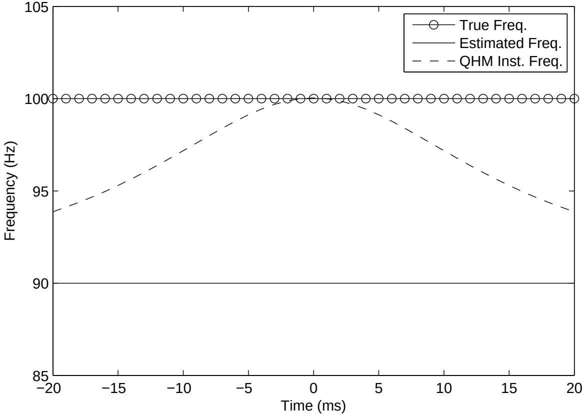

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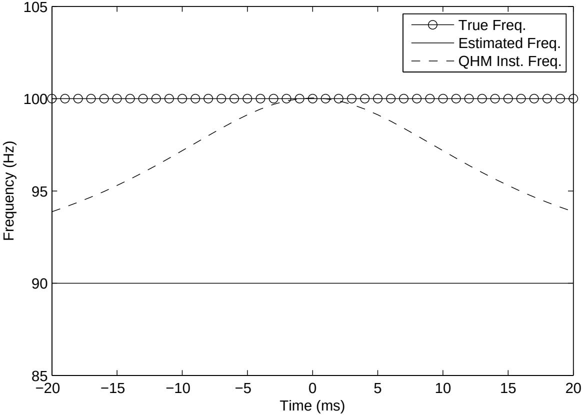

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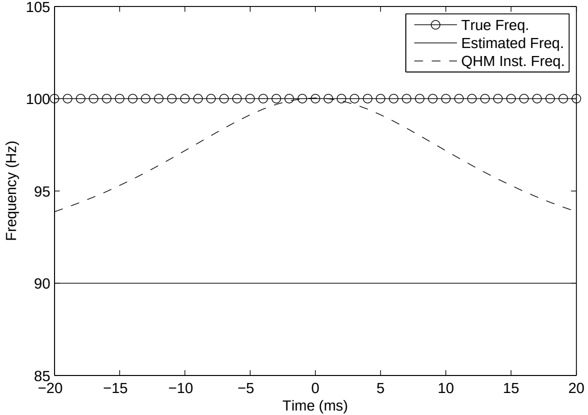

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A few details of QHM

- QHM in the frequency domain:

$$X_k(f) = a_k W(f - \hat{f}_k) + j\frac{b_k}{2\pi} W'(f - \hat{f}_k)$$

- **Decomposition of** b_k: $b_k = \rho_{1,k} a_k + \rho_{2,k} j a_k$
- Then

$$X_k(f) = a_k \left[W(f - \hat{f}_k) - \frac{\rho_{2,k}}{2\pi} W'(f - \hat{f}_k) + j\frac{\rho_{1,k}}{2\pi} W'(f - \hat{f}_k) \right]$$

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- Back to the time-domain:

$$x_k(t) \approx a_k \left[e^{j(2\pi\hat{f}_k + \rho_{2,k})t} + \rho_{1,k} t e^{j2\pi\hat{f}_k t} \right] w(t)$$

- Initially, we assumed:

$$x_k(t) = a_k \left[e^{j(2\pi(\hat{f}_k + \eta_k))t} \right] w(t)$$

- in other words, **it is suggested**:

$$\hat{\eta}_k = \rho_{2,k}/2\pi$$

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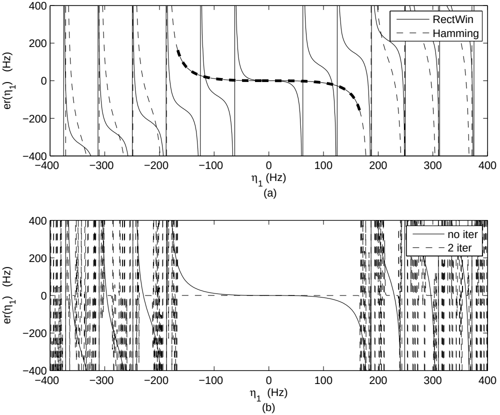
Single Sinusoid



(a)
(b)

• Iteratively, the bias can be removed when $|\eta| < B/3$, where B is the bandwidth of the squared analysis window.

QHM parameter estimation

- Frame-by-frame analysis:

$$s[n], \; n = -N, ..., N$$

- QHM:

$$h[n] = \sum_{k=-K}^{K} (a_k + nb_k) e^{j2\pi f_k n / f_s}, \; n = -N, ..., N$$

- Windowed residual error:

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$$s[n], \; n = -N, ..., N$$

- QHM:

$$h[n] = \sum_{k=-K}^{K} (a_k + n b_k) e^{j2\pi f_k n / f_s}, \; n = -N, ..., N$$

- Windowed residual error:

$$e[n] = w[n](s[n] - h[n]), \; n = -N, ..., N$$

QHM PARAMETER ESTIMATION

- Sum of squared error:

$$
\begin{aligned}
L(\mathbf{a},\mathbf{b}) &= \sum_{n=-N}^{N} |e[n]|^2 \\
&= \sum_{n=-N}^{N} w[n]^2 \left| s[n] - (E_0(n,:)\mathbf{a} + E_1(n,:)\mathbf{b}) \right|^2 \\
&= \left(\mathbf{s} - [E_0|E_1] \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} \right)^H W^2 \left(\mathbf{s} - [E_0|E_1] \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} \right)
\end{aligned}
$$

where

- $\mathbf{s} = [s[-N], ..., s[N]]^T \in \mathbb{R}^{2N+1}$
- $\mathbf{a} = [a_{-K}, ..., a_K]^T \in \mathbb{C}^{2K+1}$
- $\mathbf{b} = [b_{-K}, ..., b_K]^T \in \mathbb{C}^{2K+1}$
- $(E_0)_{nk} = e^{j2\pi f_k n/f_s} \in \mathbb{C}^{(2N+1)\times(2K+1)}$
- $(E_1)_{nk} = ne^{j2\pi f_k n/f_s} \in \mathbb{C}^{(2N+1)\times(2K+1)}$

QHM parameter estimation

- Least Squares solution:

$$\begin{bmatrix}\hat{\mathbf{a}}\\ \hat{\mathbf{b}}\end{bmatrix} = \left([E_0|E_1]^H W^2 [E_0|E_1]\right)^{-1} [E_0|E_1]^H W^2 \mathbf{s}$$

- A necessary condition for well-posed estimation is

$$4K + 1 < 2N + 1$$

- Typically, we consider (analysis) window length of 3 periods (with 2 periods being the bare minimum).

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ITERATIVE QHM, iQHM [6]

- This frequency updating mechanism provides frequencies which can be used in the model iteratively and result in better parameter estimation (a_k, b_k)
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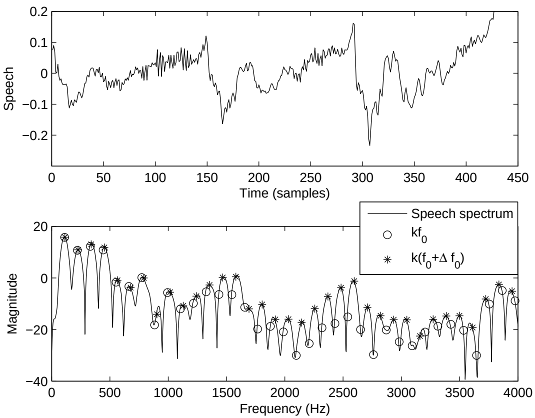
HM versus iQHM in frequency estimation - speech signal:



Robustness against additive noise

- Signal contaminated by noise:

$$y(t) = \sum_{k=1}^{4} a_k e^{j2\pi f_k} + v(t)$$

- Mean Squared Error (MSE):

$$MSE\{\hat{f}_k\} = \frac{1}{M}\sum_{i=1}^{M} |\hat{f}_k(i) - f_k|^2$$

$$MSE\{\hat{a}_k\} = \frac{1}{M}\sum_{i=1}^{M} |\hat{a}_k(i) - a_k|^2$$

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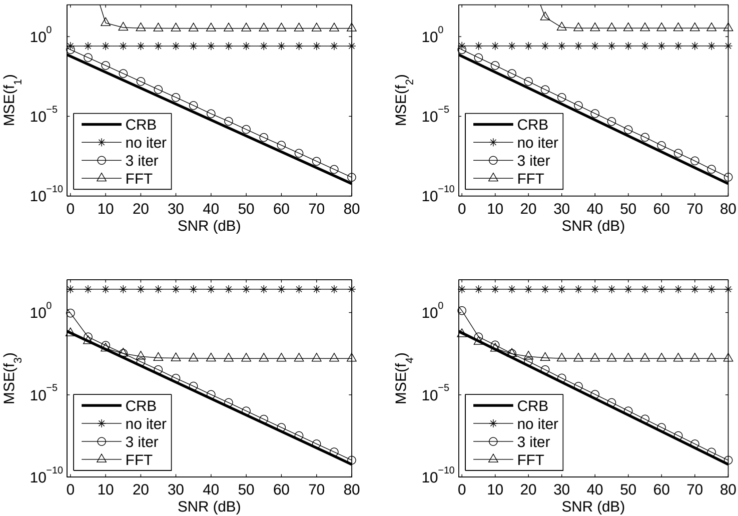
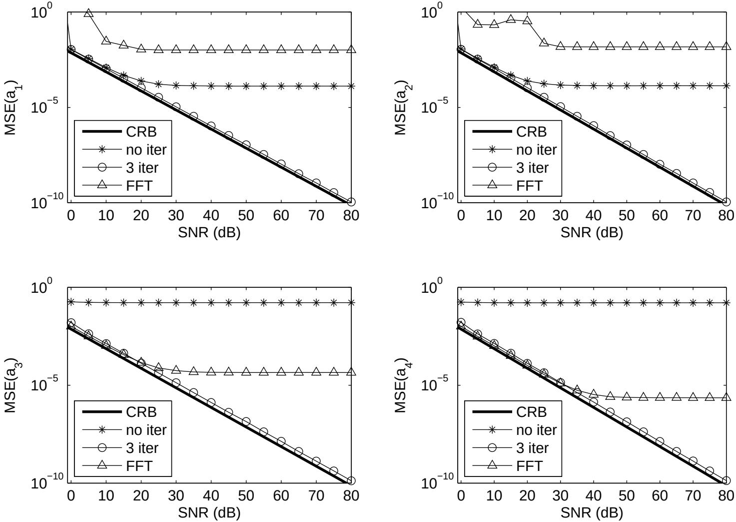
MSE of frequencies as a function of SNR.



MSE of amplitudes as a function of SNR.



Notes on QHM

- QHM has been shown to be closely related to:
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High resolution AM-FM decomposition

- AM-FM signal

$$y(t) = \sum_{k=1}^{K(t)} a_k(t) cos(\phi_k(t))$$

- Necessary assumption for unique representation: Instantaneous amplitude and instantaneous frequency, $f_k(t) = \frac{1}{2\pi}\phi'_k(t)$ are slowly varying
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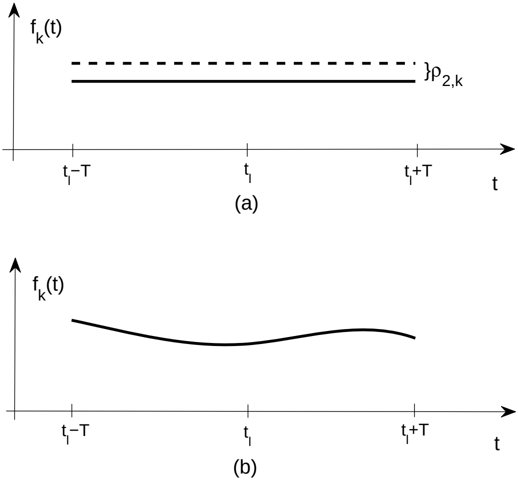
From QHM to aQHM; Graphically



From QHM to Adaptive QHM, aQHM [7]

- QHM (**stationarity assumption**):

$$x(t) = \left(\sum_{k=-K}^{K} (a_k + tb_k) e^{2\pi j f_k t} \right) w(t)$$

- Adaptive QHM (aQHM):

$$x(t) = \left(\sum_{k=-K}^{K} (a_k + tb_k) e^{j\tilde{\phi}_k(t)} \right) w(t)$$

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$$\tilde{\phi}_k(t) = 2\pi \int_0^t f_k(s) ds + \varphi, \ \ t \in [0, T]$$

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aQHM, in practice

- One sample: no interpolation between estimations is needed
  - Taylor series expansion of the instantaneous phase of kth component:

$$\phi_k(t) = 2\pi\zeta_k t + \sum_{i=0}^{\infty} \phi_{k,i} \frac{t^i}{i!}$$

  - Instantaneous frequency of the kth component at $t = 0$:

$$\nu_k(0) = \zeta_k + \frac{\phi_{k,1}}{2\pi}$$

  - ... and previously we had:

$$F_k(0) = f_k + \frac{\rho_{2,k}}{2\pi}$$

- Higher rates (i.e., 5ms, 10ms): Interpolation between estimates is required:
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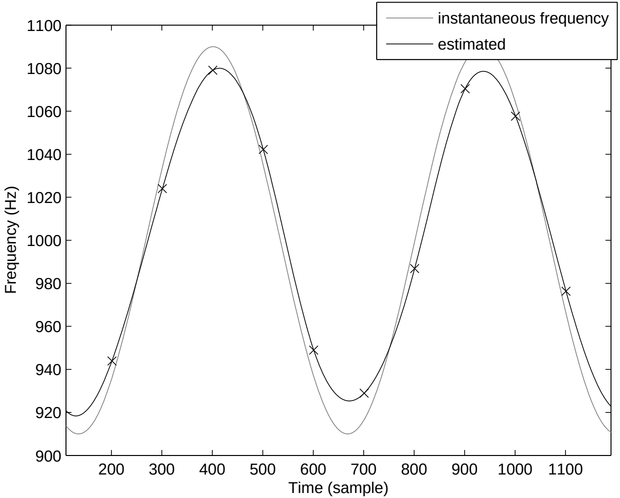
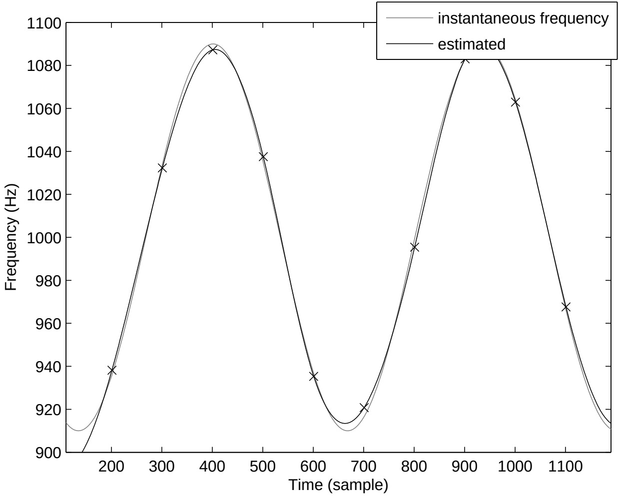
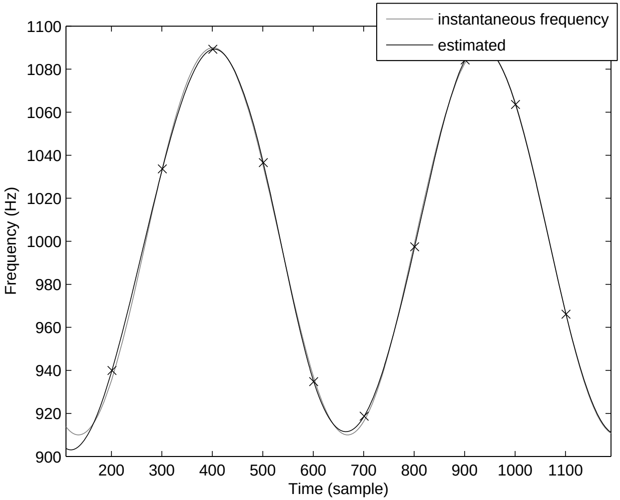
Example of estimation in aQHM: no iteration (QHM)



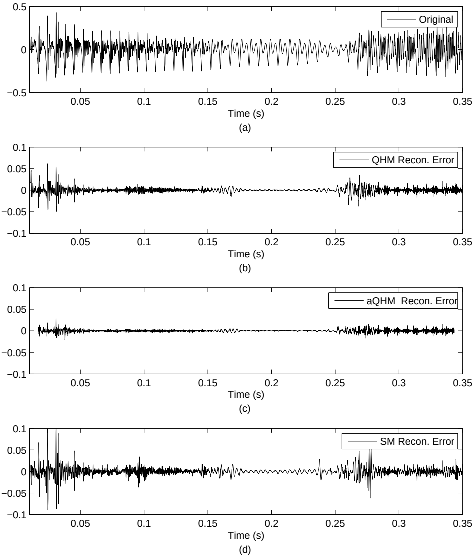
Example of estimation in aQHM: one iteration



Example of estimation in aQHM: two iterations



Reconstruction errors with QHM, aQHM, SM



Outline

Extended aQHM

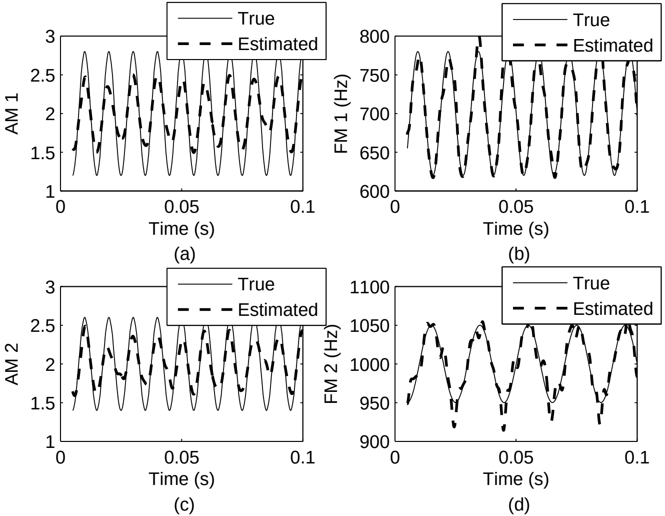
- Recall aQHM:

$$x(t) = \sum_{k=-K}^{K} (a_k + tb_k) e^{j\tilde{\phi}_k(t)}$$

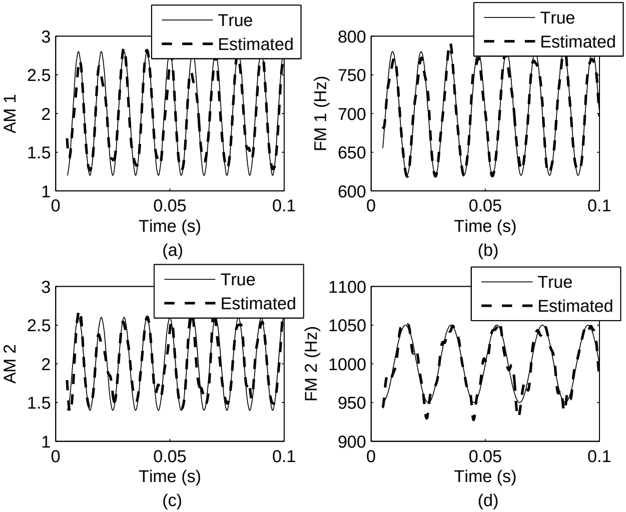
- Extended aQHM:

$$x(t) = \sum_{k=-K}^{K} (a_k + tb_k) \tilde{\alpha}(t) e^{j\tilde{\phi}_k(t)}$$

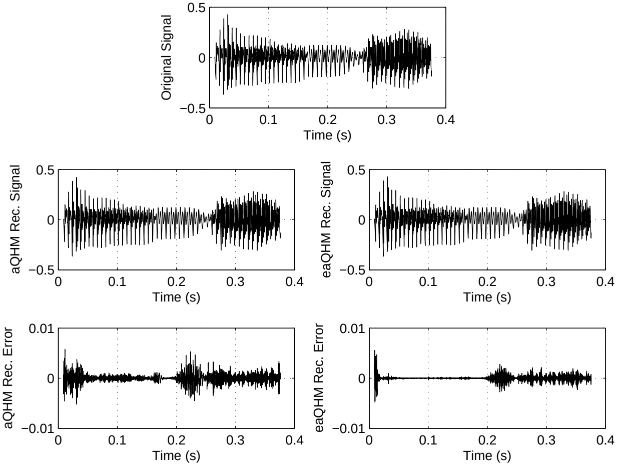
AM-FM modeling: aQHM



AM-FM modeling: extended aQHM



Comparing Adaptive Models



Outline

THANK YOU

for your attention

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