

CS578- SPEECH SIGNAL PROCESSING

LECTURE 1: DISCRETE-TIME SIGNAL PROCESSING FRAMEWORK

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OUTLINE

- 1 DISCRETE-TIME SIGNALS AND SYSTEMS
- 2 DISCRETE-TIME FOURIER TRANSFORM
- 3 z-TRANSFORM
- 4 LTI SYSTEMS IN THE FREQUENCY DOMAIN
- 5 PROPERTIES OF LTI SYSTEMS
- 6 DISCRETE FOURIER TRANSFORM
- 7 A/D AND D/A

DISCRETE-TIME SIGNALS

- Unit sample or “impulse”:

$$\begin{aligned}\delta[n] &= 1, & n = 0 \\ &= 0, & n \neq 0\end{aligned}$$

- Unit step:

$$\begin{aligned}u[n] &= 1, & n \geq 0 \\ &= 0, & n < 0\end{aligned}$$

- Exponential sequence:

$$x[n] = A\alpha^n$$

- Sinusoidal sequence:

$$x[n] = A \cos(\omega n + \phi)$$

- Complex exponential sequence:

$$x[n] = Ae^{j\omega n}$$

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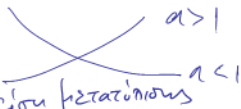
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$$x[n] = A \cos(\omega n + \phi)$$



- Complex exponential sequence:

$$\begin{aligned} \text{Im}\{x[n]\} &= A \sin(\omega n + \phi) \\ \text{Re}\{x[n]\} &= A \cos(\omega n + \phi) \\ x[n] &= Ae^{j(\omega n + \phi)} = A \cos(\omega n + \phi) + j A \sin(\omega n + \phi) \end{aligned}$$

DISCRETE-TIME SYSTEMS



Discrete-time System:

$$y[n] = T\{x[n]\}$$

Important class of systems: Linear and Time Invariant (LTI):

- Linearity:

$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\}$$

- Time-Invariant:

$$\begin{aligned} \text{if } y[n] &= T\{x[n]\} \\ \text{then } y[n - n_0] &= T\{x[n - n_0]\} \end{aligned}$$

Important property of LTI:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n - k] \\ &= x[n] \star h[n] \end{aligned}$$

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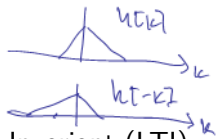
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Diagram: A block labeled 'T' with an input $x[n]$ and output $h[n]$. Below it, a sequence of arrows shows $h[n] \rightarrow h[k] \rightarrow h[n-k]$. A blue arrow points from the convolution sum to the word 'convolution' written in blue.

STABILITY AND CAUSALITY FOR LTI

Necessary and sufficient conditions for:

- Stability:

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

- Causality:

$$h[n] = 0, \text{ for } n < 0$$

STABILITY AND CAUSALITY FOR LTI

$$x[n] \xrightarrow{\text{LTI}} \boxed{h[n]} \rightarrow y[n]$$

$$y[n] = h[n] * x[n]$$

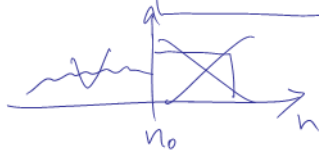
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DISCRETE-TIME FOURIER TRANSFORM, DTFT

Discrete-Time Fourier Transform pair:

- Direct:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

- Inverse:

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)e^{j\omega n} d\omega$$

Example:

$$Ae^{j\omega_0 n + \phi} \leftrightarrow 2\pi Ae^{j\phi} \delta(\omega - \omega_0)$$

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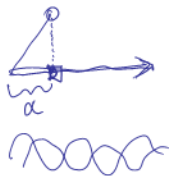
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$$x[n] \xleftrightarrow{F} X(\omega)$$

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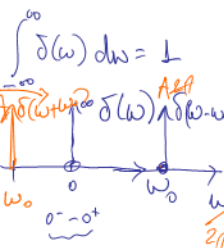


- Inverse:

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

Example:

$$Ae^{j(\omega_0 n + \phi)} \xleftrightarrow{F} 2\pi Ae^{j\phi} \delta(\omega - \omega_0)$$



$$F\{A \cos(\omega_0 n + \phi)\} = F\{A e^{j\phi} e^{j\omega_0 n} + A e^{-j\phi} e^{-j\omega_0 n}\} = A e^{j\phi} 2\pi \delta(\omega - \omega_0) + A e^{-j\phi} 2\pi \delta(\omega + \omega_0)$$

$$x[n] = \sum_k A_k \cos(\omega_k n + \phi_k)$$

DTFT PROPERTIES

- Fourier transform is complex:

$$\begin{aligned}X(\omega) &= X_r(\omega) + jX_i(\omega) \\ &= |X(\omega)|e^{j\angle X(\omega)}\end{aligned}$$

- Fourier transform is periodic with period 2π :

$$X(\omega + 2\pi) = X(\omega)$$

- For real valued sequence $x[n]$:

$$X(\omega) = X^*(-\omega)$$

- Energy of a signal (Parseval theorem):

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

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DTFT PROPERTIES

- Fourier transform is complex:

$$\rightarrow X(\omega) = \sum_n x[n] \cdot e^{-j\omega n}$$

$$\boxed{X(\omega)} = X_r(\omega) + jX_i(\omega)$$

$$= |X(\omega)| e^{j\angle X(\omega)}$$

καρτεσιανή
μορφή

- Fourier transform is periodic with period 2π :

$$|X(\omega)| = \sqrt{X_r^2(\omega) + X_i^2(\omega)}$$

$$\angle X(\omega) = \tan^{-1} \frac{X_i(\omega)}{X_r(\omega)}$$

$$\underline{X(\omega + 2\pi) = X(\omega)}$$

$$e^{-j(\omega+2\pi)n} = e^{-j\omega n} \cdot e^{-j2\pi n} \rightarrow 1$$

- For real valued sequence $x[n]$:

$$X(\omega) = X^*(-\omega)$$

$$e^{j(\omega+2\pi)n} = e^{j\omega n} \cdot e^{j2\pi n} = e^{j\omega n}$$

$\cos(2\pi n) - j\sin(2\pi n)$
 $n \in \mathbb{Z}$

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$$\boxed{\sum_{n=-\infty}^{\infty} |x[n]|^2} = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

ενέργεια



UNCERTAINTY PRINCIPLE

Given a signal $x[n]$ we define as:

- Duration of the signal:

$$D(x) = \sum_{n=-\infty}^{\infty} (n - \bar{n})^2 |x[n]|^2$$

- Bandwidth of the signal:

$$B(x) = \int_{-\pi}^{\pi} (\omega - \bar{\omega})^2 |X(\omega)|^2 d\omega$$

where

$$\begin{aligned}\bar{n} &= \sum_{n=-\infty}^{\infty} n |x[n]|^2 \\ \bar{\omega} &= \int_{-\pi}^{\pi} \omega |X(\omega)|^2 d\omega\end{aligned}$$

Uncertainty Principle states that:

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Uncertainty Principle states that:

$$D(x)B(x) \geq 1/2$$

$$D \cdot B = \alpha$$

HILBERT TRANSFORM

For a real signal $x[n]$, we form the analytic signal:

$$s[n] = s_r[n] + js_i[n]$$

where $s_r[n] = x[n]/2$ and

$$S_i(\omega) = H(\omega)S_r(\omega)$$

$$S_r(\omega) = F\{s_r[n]\} \\ = f\{x[n]/2\}$$

where $H(\omega)$ is referred to as Hilbert transform:

$$H(\omega) = \begin{cases} -j & 0 \leq \omega < \pi \\ j & -\pi \leq \omega < 0 \end{cases}$$

$$j^2 = -1$$



INSTANTANEOUS AMPLITUDE AND FREQUENCY

The analytic signal may be written as:

$$s[n] = A[n]e^{j\theta[n]}$$

- Instantaneous amplitude:

$$A[n] = |s[n]|$$

- Instantaneous frequency:

$$\omega[n] = \left. \frac{d\theta(t)}{dt} \right|_{t=nT}$$

where

$$\theta(t) = \int_{-\infty}^t \omega(\tau) d\tau$$

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Z-TRANSFORM

z-Transform pair:

- Direct:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- Inverse:

$$x[n] = \frac{1}{2\pi j} \oint_C X(z)z^{n-1} dz$$

Example:

$$a^n u[n] \leftrightarrow \frac{1}{1 - az^{-1}} \quad |z| > |a|$$

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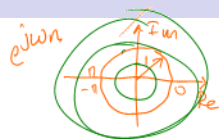
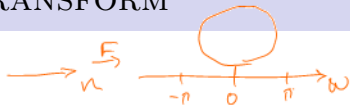
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Z-TTRANSFORM



z-Transform pair:

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$z = r e^{j\omega}$
 Fourier: $r=1$

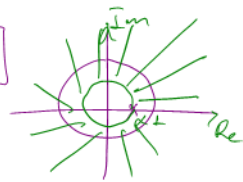
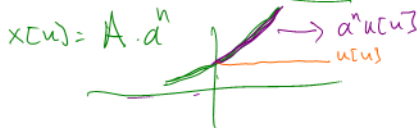
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Example:

$$a^n u[n] \xleftrightarrow{Z} \frac{1}{1 - az^{-1}}, \quad |z| > |a|$$



Z-TRANSFORM: RATIONAL FUNCTIONS

Usually:

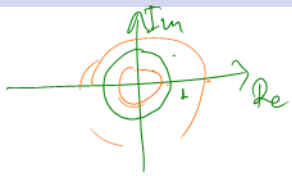
$$X(z) = \frac{P(z)}{Q(z)} = Az^{-r} \frac{\prod_{k=1}^{M_i} (1 - a_k z^{-1}) \prod_{k=1}^{M_0} (1 - b_k z)}{\prod_{k=1}^{N_i} (1 - c_k z^{-1}) \prod_{k=1}^{N_0} (1 - d_k z)}$$

No repeated poles, no poles outside the unit circle:

$$X(z) = \sum_{k=1}^{N_i} \frac{A_k}{(1 - c_k z^{-1})}$$

Z-TRANSFORM: RATIONAL FUNCTIONS

$x[n] = x_r[n] + jx_i[n]$



a^4 (pole)
 $5 + j0$ (pole)

Usually:

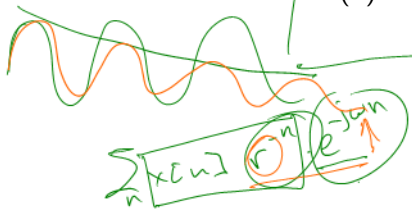
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$M = M_i + M_o$

$N = N_i + N_o$

No repeated poles, no poles outside the unit circle:

$$X(z) = \sum_{k=1}^{N_i} \frac{A_k}{(1 - c_k z^{-1})}$$



$$\left\{ \frac{x[n]}{r^n} \right\}$$

$$\sum x[n] z^{-n} \quad | \quad z = r e^{j\omega}$$

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EIGENVALUES, EIGENFREQUENCIES, AND EIGENFUNCTIONS

LTI

If $x[n] = e^{j\omega_0 n}$, then

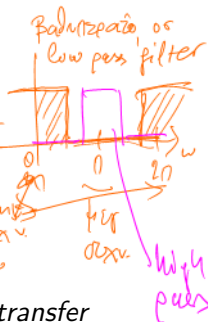
$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] = h[n] * x[n] = x[n] * h[n]$$

$$y[n] = H(\omega_0) x[n]$$

where $H(\omega)$ is referred to as *frequency response*:

$$H(\omega) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}$$

while $H(z)$ is usually referred to as *system function* or *transfer function*



CONVOLUTION THEOREM

LT I

If

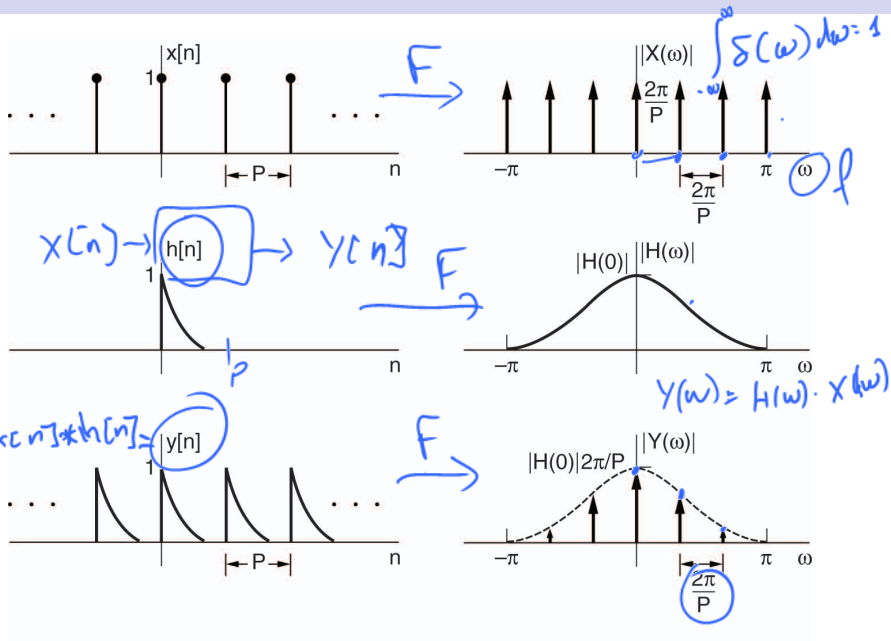
$$\begin{aligned} x[n] &\longleftrightarrow X(\omega) \\ h[n] &\longleftrightarrow H(\omega) \end{aligned}$$

and if: $y[n] = x[n] \star h[n]$, then:

$$Y(\omega) = X(\omega)H(\omega)$$

$$Y(z) = X(z) \cdot H(z)$$

EXAMPLE OF CONVOLUTION



WINDOWING (MODULATION) THEOREM

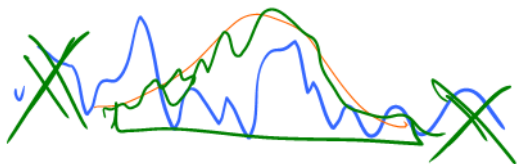
If

$$x[n] \longleftrightarrow X(\omega)$$

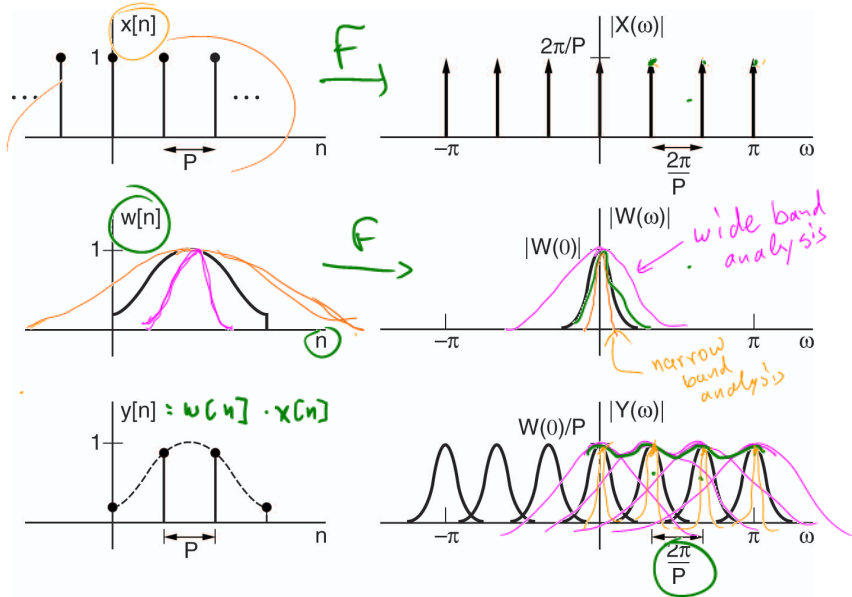
$$w[n] \longleftrightarrow W(\omega)$$

and if: $y[n] = x[n]w[n]$, then:

$$\begin{aligned} Y(\omega) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Theta)W(\omega - \Theta)d\Theta \\ &= \frac{1}{2\pi} X(\omega) \circledast W(\omega) \end{aligned}$$



EXAMPLE OF MODULATION



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DIFFERENCE EQUATIONS

In time:

$$y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

Handwritten notes: $z^{-k} Y(z)$ (with arrow pointing to $y[n-k]$), $z^{-k} X(z)$ (with arrow pointing to $x[n-k]$)

In z-domain:

$$\frac{Y(z)}{X(z)}$$

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 - \sum_{k=1}^N a_k z^{-k}} = \frac{P(z)}{Q(z)}$$
$$= A z^{-r} \frac{\prod_{k=1}^{M_i} (1 - a_k z^{-1}) \prod_{k=1}^{M_0} (1 - b_k z)}{\prod_{k=1}^{N_i} (1 - c_k z^{-1})}$$

Handwritten notes: A (with arrow pointing to the constant term), z^{-r} (with arrow pointing to the delay term), z^{-k} (with arrow pointing to the denominator terms)

MAGNITUDE-PHASE RELATIONSHIPS

- Minimum, Maximum and Mixed-phase systems

$$H(z) = H_{min}(z)H_{max}(z)$$

- Minimum-phase and All-pass system

$$H(z) = H_{min}(z)A_{all}(z)$$

Note that

$$\sum_{n=0}^m |h_{min}[n]|^2 \geq \sum_{n=0}^m |h[n]|^2, \quad m \leq 0$$

MAGNITUDE-PHASE RELATIONSHIPS

$$x[n] * y[n] \xrightarrow{F} X(\omega) \cdot Y(\omega)$$

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- Minimum-phase and All-pass system

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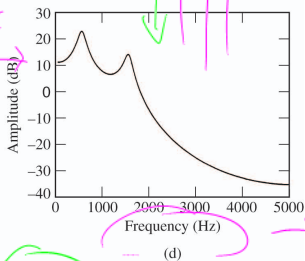
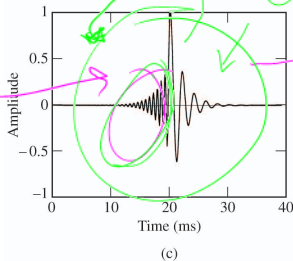
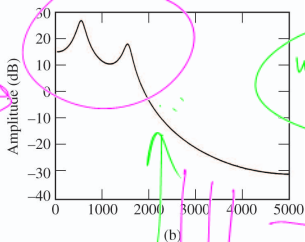
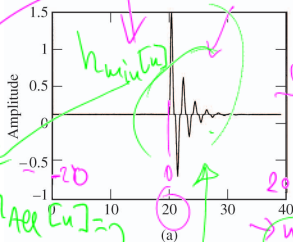
Note that

$$\sum_{n=0}^m |h_{min}[n]|^2 \geq \sum_{n=0}^m |h[n]|^2, \quad m \geq 0$$

$$h_{mix}[n] = h_{min}[n] * h_{All}[n]$$

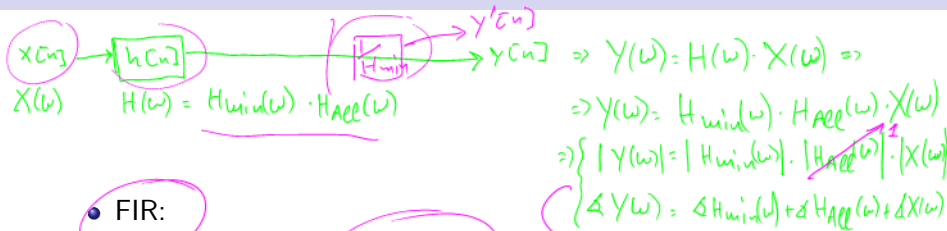
$$H(\omega) = H_{min}(\omega) \cdot H_{All}(\omega) \Rightarrow |H(\omega)| = |H_{min}(\omega)| \cdot |H_{All}(\omega)| = |H_{mix}(\omega)|$$

EXAMPLE OF MINIMUM AND MIXED PHASE



mixed = minimum. ACpass

FIR AND IIR FILTERS



- FIR:
- IIR:

$h[n] \neq 0, 0 \leq n < M$

$h[n] = \sum_{k=1}^{N_i} A_k c_k^n u[n]$

$|Y(\omega)| = |H_{min}(\omega)| \cdot |X(\omega)|$

$|Y(\omega)| = \left| \frac{1}{H_{min}(\omega)} \right| \cdot |H_{min}(\omega)| \cdot |X(\omega)|$

$\cdot |X(\omega)| \Rightarrow$

$\Rightarrow |Y(\omega)| = |X(\omega)|$

min

1/min



OUTLINE

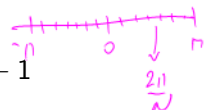
- 1 DISCRETE-TIME SIGNALS AND SYSTEMS
- 2 DISCRETE-TIME FOURIER TRANSFORM
- 3 z-TRANSFORM
- 4 LTI SYSTEMS IN THE FREQUENCY DOMAIN
- 5 PROPERTIES OF LTI SYSTEMS
- 6 DISCRETE FOURIER TRANSFORM**
- 7 A/D AND D/A

DISCRETE FOURIER TRANSFORM

Discrete Fourier Transform, DFT, pair:

- Direct:

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn}, \quad 0 \leq k \leq N-1$$

$$X(\omega) = \sum_n x[n] \cdot e^{-j\omega n}$$


- Inverse:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi}{N} kn} \quad 0 \leq n \leq N-1$$

Parseval theorem:

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2$$

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ANALOG TO DIGITAL AND DIGITAL TO ANALOG

