CS578- Speech Signal Processing

Lecture 3: Acoustics of Speech Production

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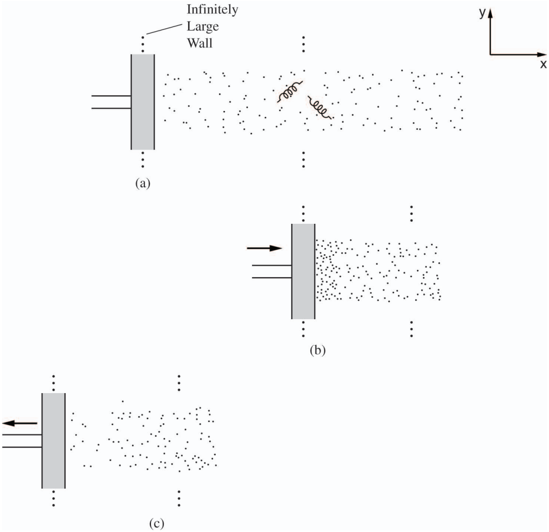
Outline

Compression and rarefaction of air particles



Some definitions

- **Sound wave:** propagation of disturbance (local changes in pressure, displacement, and velocity) of particles through a medium, creating the effect of compression or rarefaction.
- **Wavelength:** distance between two consecutive peak compressions, λ
- **Frequency:** number of cycles of compressions per second, f
- **Speed of sound:** $c = f\lambda$ (at sea level and at $70^\circ F$, $c = 344 m/sec$)
- **Isothermal process:** a slow variation of pressure where the temperature in the medium remains constant
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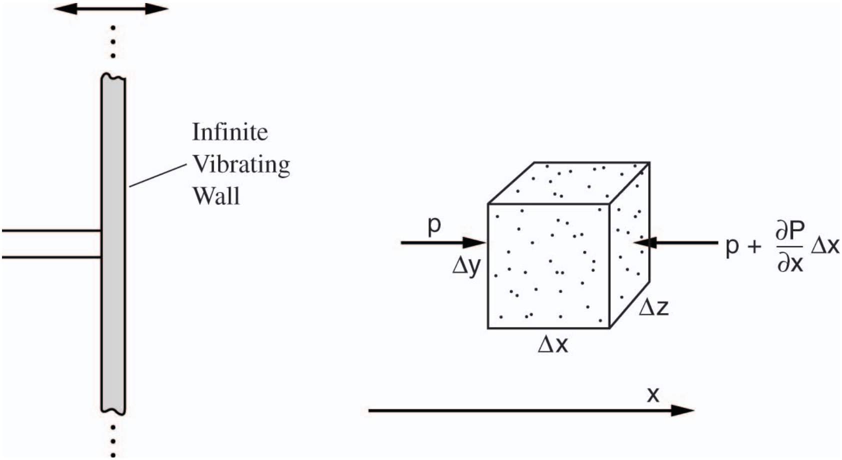
Cube configuration



Notation

Assuming *planar propagation*, and within the cube:

- $p(x, t)$ fluctuation of pressure about an ambient or average pressure P_0.
  - ▷ Threshold of hearing: $2\ 10^{-5}$ *newtons*/m^2
  - ▷ Threshold of pain: 20 *newtons*/m^2
- $v(x, t)$ fluctuation of particles' velocity about zero average velocity.
- $\rho(x, t)$ fluctuation of particles' density about an average density ρ_0.

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The Wave Equation

Under the assumptions:

- If there is no friction of air particles in the cube with those outside the cube (no *viscosity*),
- Cube is very small,
- The density of air particles is constant in the cube (i.e.,$\rho_0 = \rho$)

then, one form of the *Wave Equation* is given by:

$$
\begin{aligned}
-\frac{\partial p}{\partial x} &= \rho \frac{\partial v}{\partial t} \\
-\frac{\partial p}{\partial t} &= \rho c^2 \frac{\partial v}{\partial x}
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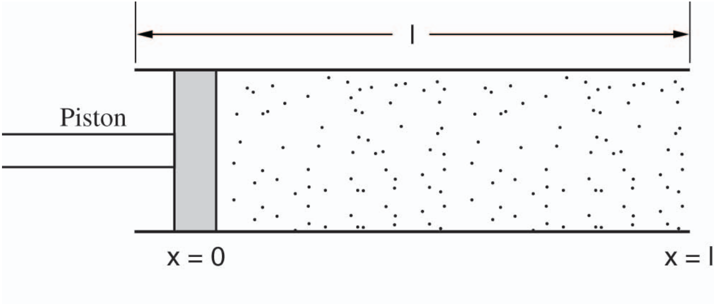
$$\begin{aligned} -\frac{\partial p}{\partial x} &= \rho \frac{\partial \upsilon}{\partial t} \\ -\frac{\partial p}{\partial t} &= \rho c^2 \frac{\partial \upsilon}{\partial x} \end{aligned}$$

Outline

Lossless Case of cross section A



$$
\begin{aligned}
-\frac{\partial p}{\partial x} &= \frac{\rho}{A}\frac{\partial u}{\partial t} \\
-\frac{\partial p}{\partial t} &= \frac{\rho c^2}{A}\frac{\partial u}{\partial x}
\end{aligned}
$$

where $u(x,t) = A\upsilon(x,t)$

Solution for a Lossless Tube

Under the assumptions/conditions:

- No friction along the walls of the tube
- At the open end of the tube, there are no variations in air pressure, i.e. $p(l,t) = 0$
- Volume velocity at $x = 0$: $u(0,t) = U_g(\Omega)e^{j\Omega t}$

▷ Volume velocity:

$$u(x,t) = \frac{\cos\left[\Omega(l-x)/c\right]}{\cos\left(\Omega\ l/c\right)} U_g(\Omega)e^{j\Omega t}$$

▷ (Incremental) Pressure:

$$p(x,t) = j\frac{\rho c}{A}\frac{\sin\left[\Omega(l-x)/c\right]}{\cos\left(\Omega\ l/c\right)} U_g(\Omega)e^{j\Omega t}$$

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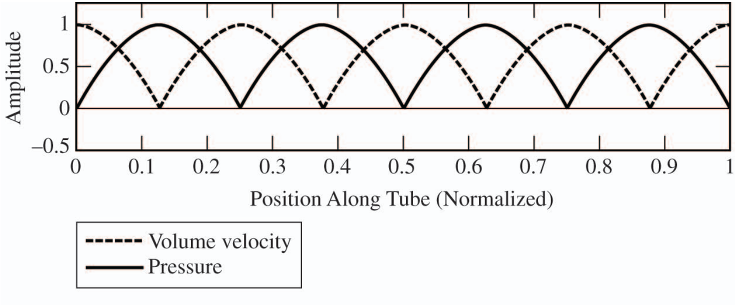
Velocity and Pressure are "orthogonal"



Input/Output volume velocity

At $x = l$

$$u(l,t) = \frac{1}{\cos(\Omega\ l/c)} U_g(\Omega) e^{j\Omega t}$$

Then, the frequency response $V(\Omega)$ is:

$$V(\Omega) = \frac{U(l,\Omega)}{U_g(\Omega)} = \frac{1}{\cos(\Omega\ l/c)}$$

providing resonances of infinite amplitudes at frequencies:

$$\Omega_k = (2k+1)\frac{\pi c}{2l}, \quad k = 0, 1, 2, \cdots$$

Example: if l= 35cm, c = 350 m/s, then $f_k = 250, 750, 1250, \cdots$ Hz.

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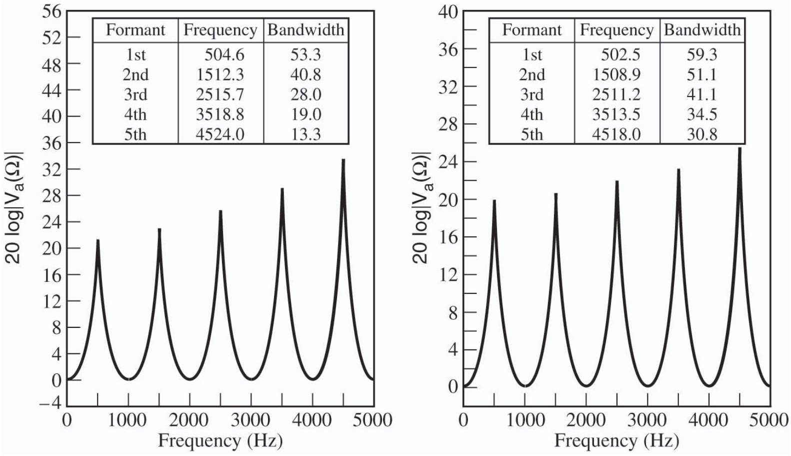
Uniform tube: being realistic

Energy loss due to the wall vibration (left) and with viscous and thermal loss (right)[1]:

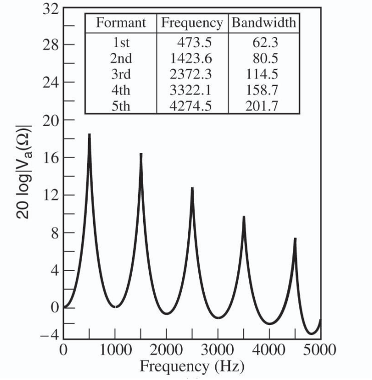


Uniform tube: being more realistic

Sound radiation at the lips, as an acoustic impedance:

$$Z_r(\Omega) = \frac{P(l,\Omega)}{U(l,\Omega)}$$

All the previous losses, plus radiation loss[1]:

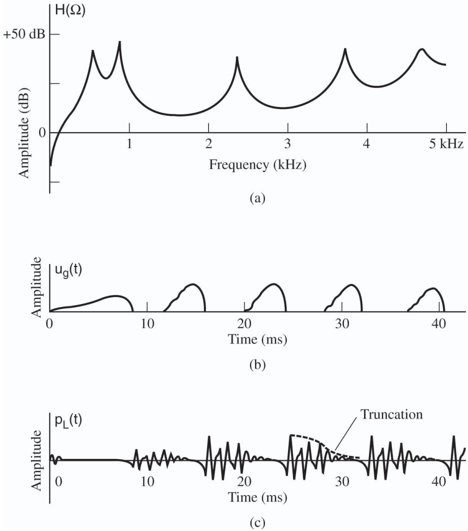


Pressure-to-volume velocity frequency response

Since we measure pressure at the lips:

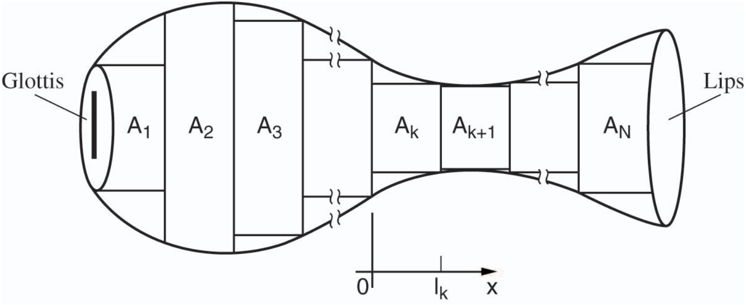
$$H(\Omega) = \frac{P(l, \Omega)}{U_g(\Omega)} = Z_r(\Omega) V(\Omega)$$

Numerical simulations for /o/[1]



Outline

Concatenating lossless Uniform Tubes



Reflection coefficient:

$$r_k = \frac{A_{k+1} - A_k}{A_{k+1} + A_k}$$

Discretizing the continuous-space tube

- Impulse response of N lossless concatenated tubes with total length l:

$$h(t) = b_0\delta(t - N\tau) + \sum_{k=1}^{\infty} b_k \delta(t - N\tau - k2\tau)$$

where $\tau = \frac{\Delta x}{c}$ and $\Delta x = \frac{l}{N}$

- Frequency response:

$$H(\Omega) = \sum_{k=0}^{\infty} b_k e^{-j\Omega 2k\tau}$$

- Observe that:

$$H(\Omega + \frac{2\pi}{2\tau}) = H(\Omega)$$

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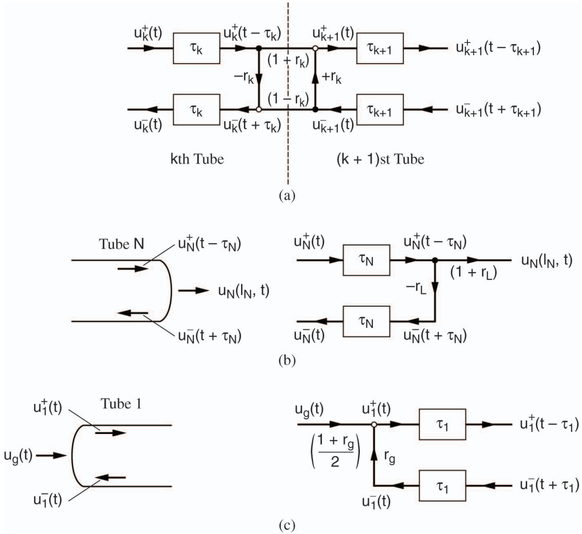
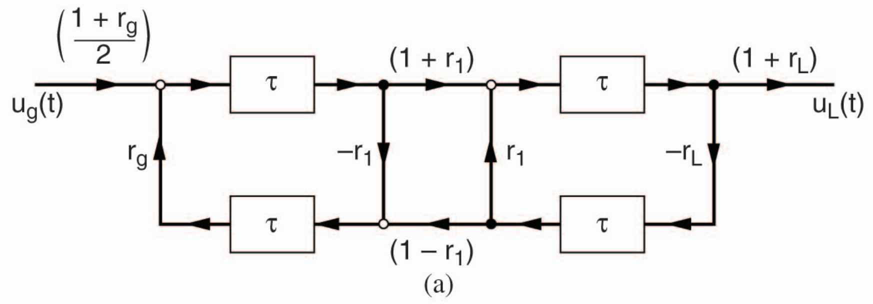
Signal flow graphs



(a) two concatenated tubes, (b) lip boundary condition, (c) glottal boundary condition

For a lossless two-tube model



Transfer function relating the volume velocity at the lips to the glottis:

$$V(s) = \frac{be^{-s2\tau}}{1 + a_1 e^{-s2\tau} + a_2 e^{-s4\tau}}$$

with $a_1 = r_1 r_g + r_1 r_L$, $a_2 = r_L r_g$ and $b = 0.5(1 + r_g)(1 + r_L)(1 + r_1)$ (*Show me this*)

- **Two cubes:** By setting $z = e^{s2\tau}$, then:

$$V(z) = \frac{bz^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

- **N cubes:**

$$V(z) = \frac{Az^{-N/2}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Discrete-time lossless models

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Choosing the number of tube elements

Question:
If a vocal tract has length $l = 17.5$ cm and the speed of sound $c = 350$ m/s, how many tubes, N, do we need to cover a bandwidth of 5000 Hz?

Answer: N = 10

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Complete discrete-time model from N Tubes

Discrete-time pressure-to-volume velocity frequency response:

$$H(z) = R(z)V(Z)$$

where $R(z) \approx 1 - \alpha z^{-1}$ and $V(z)$ is an all-pole model.
And for the speech signal (voiced case):

$$X(z) = A_v G(z) H(z)$$

with A_v to control loudness and $G(z)$ being the z-transform of the glottal flow input.

or

$$X(z) = A_v G(z) \frac{1 - \alpha z^{-1}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

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Glottal waveform model

A typical glottal flow waveform over one cycle is modeled as:

$$g[n] = (b^{-n}u[-n]) \star (b^{-n}u[-n])$$

which has as z-transform:

$$G(z) = \frac{1}{(1-\beta z)^2}$$

So for a *voiced* frame:

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Modeling other states

- **For noisy inputs:**

$$X(z) = A_n U(z) V(z) R(z)$$

- **For impulsive sounds:**

$$X(z) = A_i V(z) R(z)$$

- **being more general:**

$$X(z) = A \frac{(1 - az^{-1}) \prod_{k=1}^{M_i} (1 - c_k z^{-1}) \prod_{k=1}^{M_o} (1 - d_k z)}{(1 - bz)^2 \left(1 - \sum_{k=1}^{N} a_k z^{-k}\right)}$$

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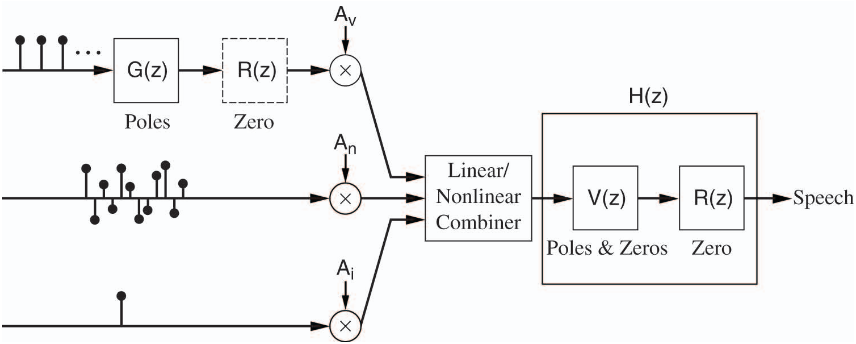
An Overview then



Outline

Glottal Flow Derivative

Since speech signals, $x(t)$ can be obtained in general by:

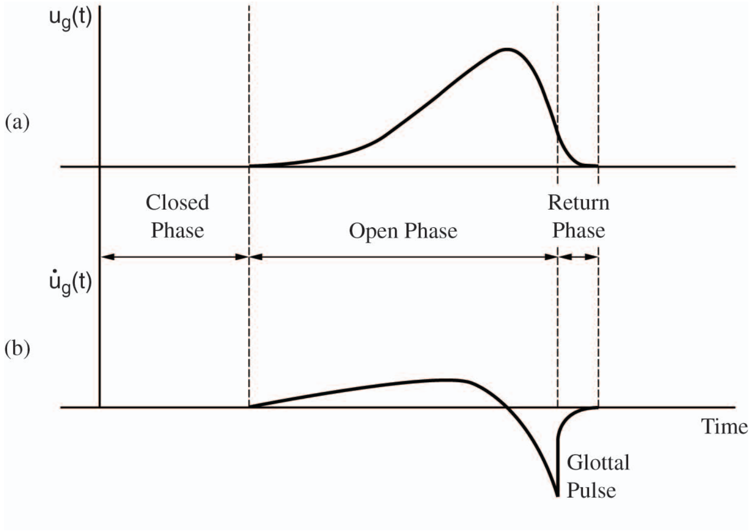
$$x(t) \approx A\frac{d}{dt}\left[u_g(t) \star v(t)\right]$$

and because:

$$A\frac{d}{dt}\left[u_g(t) \star v(t)\right] = A\left[\frac{d}{dt}u_g(t)\right] \star v(t)$$

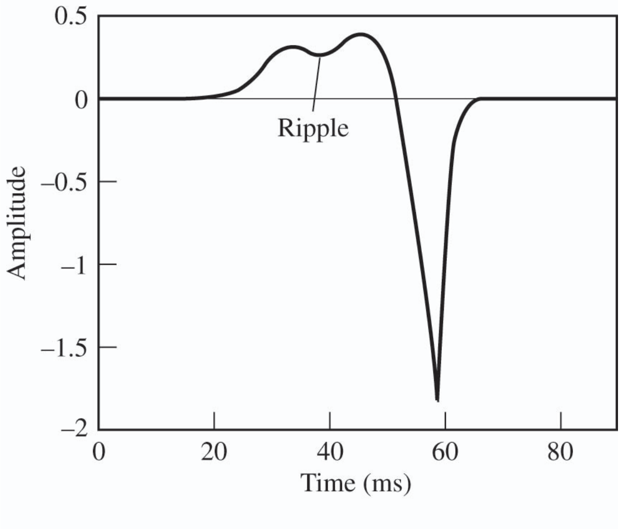
we usually consider the derivative $\frac{d}{dt}u_g(t)$ as input to the system, which is referred to as *Glottal Flow Derivative*

Glottal flow and its derivative

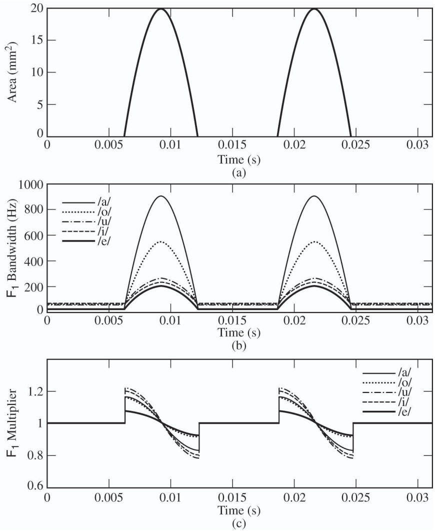


Outline

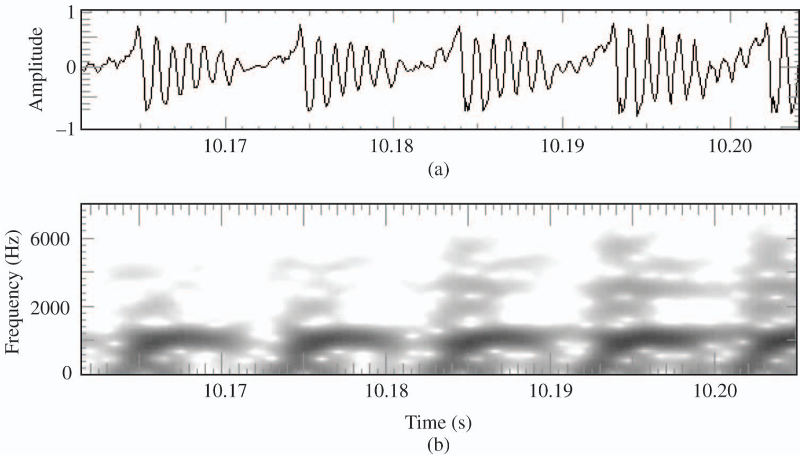
Ripple in the glottal flow derivative?



Regarding the first formant [2]



Truncation effect - again



Outline

Acknowledgments

Most, if not all, figures in this lecture are coming from the book:

T. F. Quatieri: Discrete-Time Speech Signal Processing,
principles and practice
2002, Prentice Hall

and have been used after permission from Prentice Hall

Outline

M. Portnoff, *A Quasi-One-Dimensional Digital Simulation for the Time-Varying Vocal Tract*.
PhD thesis, Massachusetts Institue of Technology, May 1973.

C. Jankowski, *Fine Structure Features for Speaker Identification*.
PhD thesis, Massachusetts Institute of Technology, Dept. of EE and CS, June 1996.