Lectures 16, 17: Dataflow Analysis

Polyvios Pratikakis

Computer Science Department, University of Crete

Type Systems and Programming Languages

Based on slides by Jeff Foster

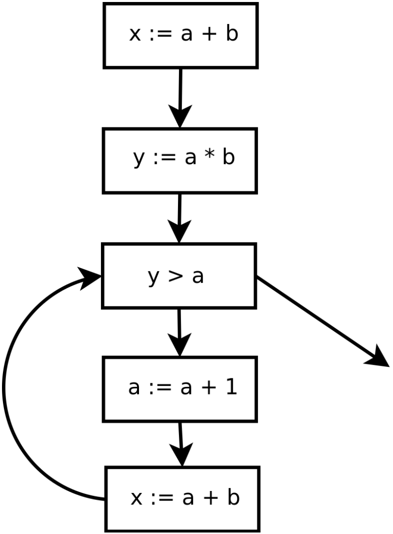
Abstract syntax trees

- ASTs are *abstract*
  - They don't contain all information in the program
    - E.g., spacing, comments, brackets, parentheses
  - Any ambiguity is resolved
    - E.g., $a + b + c$ produces the same AST as $(a + b) + c$
- but not great for analysis
  - An AST has many similar forms
    - E.g., for, while, repeat..until, ...
    - E.g., if, switch, ...
  - AST expressions might be complex, nested
    - E.g., $(10 * x) + (y > 3?5 * z : z)$
- We want a simpler representation for analysis
  - ...at least for dataflow analysis

Control-flow graph (CFG)

- A directed graph, where:
  - Each node represents a statement
  - Each edge represents control flow (i.e. what happens after what)
- Statements may be
  - Assignments `x := y` *op* `z` or `x :=` *op* `y`
  - Copy statements `x := y`
  - Branches `goto L` or `if x` *relop* `y goto L`
  - etc.

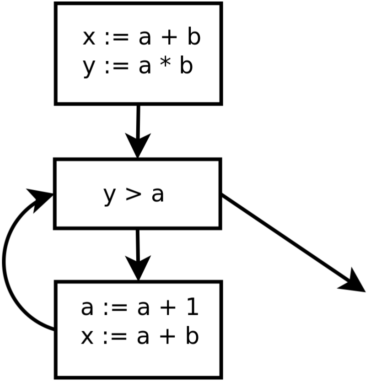
Control-flow graph example



Kinds of CFGs

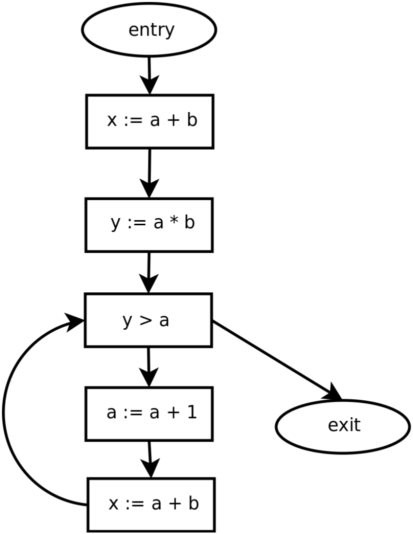
- We usually don't include declarations (e.g., `int x`)
  - Some CFG implementations do
- We may add special, unique "enter" and "exit" nodes
- We can group "straight-line" code into basic blocks
  - Straight-line: without branches, simple instructions one after the other

Control-flow graph with basic blocks



- Can lead to more efficient implementations
- But, is more complicated
  - We will use single-statement blocks here

Control-flow graph with entry/exit



CFG versus AST

- CFGs are simpler than ASTs
  - Fewer forms, less redundancy, simpler expressions
  - Capture flow of control better, easier to see execution paths
- But, AST is a more faithful representation
  - CFGs introduce temporary variables
  - CFGs lose the block-structure of the program
- AST benefits
  - Easier for reporting errors and other compiler messages
  - Easier to explain to the programmer
  - Easier to unparse and produce code closer to the original

Dataflow analysis

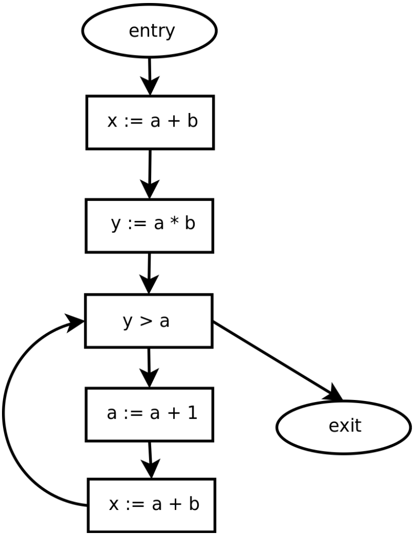
- A framework for proving facts about programs
- Reasons about lots of little facts
- Little or no interaction between different facts
  - Works best on properties about *how* the program computes
- Based on all paths through the program control-flow
  - Including infeasible paths

Available expressions

- An expression e is available at a program point p if:
  - e is computed on every path leading to p, and
  - the value of e has not changed since it was last computed
- Used in compiler optimization
  - If an expression is available don't recompute its value
  - Instead, save it in a register the first time, and use that
  - ...if possible

Dataflow facts

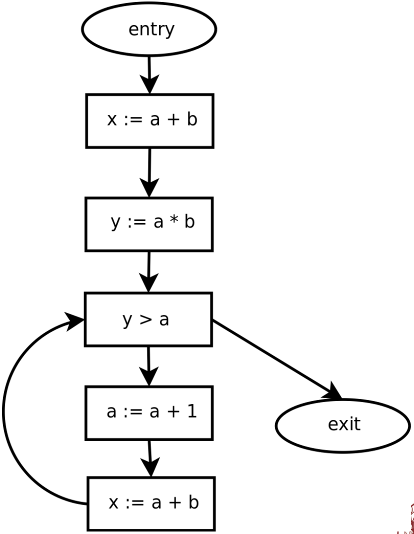
- Is expression e available?
- Possible facts:
  - $a + b$ is available
  - $a * b$ is available
  - $a + 1$ is available



Gen and kill

- What is the effect of each statement on the set of facts?

Stmt	Gen	Kill
$x := a + b$	$a + b$	
$y := a * b$	$a * b$	
$a := a + 1$		$a + 1$ $a + b$ $a * b$



Terminology

- A *joint point* is a program point where two branches meet
- Available expressions is a *forward must* problem
  - *Forward* means the facts flow from “in” to “out” at every node, follow the edge arrows
  - *Must* means at every joint point, the property must hold on *all* paths joined
- There are also *backward* and *may* problems
  - *Backward* means the facts flow from “out” to “in” at every node, backwards on the edges
  - *May* means at every joint point, the property must hold on *any* of the joined paths
- All combinations:
  - Forward may, backward must, etc.

Dataflow equations

- If s is a statement
  - $succ(s)$ is the set of all immediate successor statements of s
  - $pred(s)$ is the set of all immediate predecessor statements of s
  - $In(s)$ is the set of facts at the program point just before s
  - $Out(s)$ is the set of facts at the program point just after s
- Forward must:
  - $In(s) = \bigcap_{s' \in pred(s)} Out(s')$
  - $Out(s) = Gen(s) \cup (In(s) \setminus Kill(s))$

Live variables

- A variable x is *live* at a program point p if:
  - x will be used on some execution path starting at p
  - before x is overwritten
- Compiler optimization
  - If a variable is not live, there's no need to keep it in a register
  - If a variable is dead at an assignment, we can eliminate the assignment

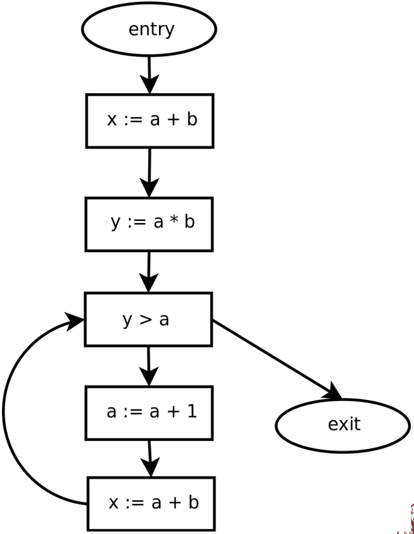
Dataflow equations

- Liveness is a *backward may* problem
  - To decide if a variable is live at a program point p, we need to look at the paths starting at p
  - The variable is live if it is used on *any* future program point
- Backward may:
  - $Out(s) = \bigcup_{s' \in succ(s)} In(s')$
  - $In(s) = Gen(s) \cup (Out(s) \setminus Kill(s))$

Gen and kill

- All possible facts:
  - a is live
  - b is live
  - x is live
  - y is live
- What is the effect of each statement on the set of facts?

Stmt	Gen	Kill
$x := a + b$	a, b	x
$y := a * b$	a, b	y
$y > a$	a, y	
$a := a + 1$	a	a



Very busy expressions

- An expression e is very busy at a program point p if:
  - On every path from p, expression e is evaluated before its value is changed
- Compiler optimization
  - The compiler can lift very busy expression computation
- What kind of problem?
  - Forward or backward?
  - May or must?

Reaching definitions

- A *definition* of a variable x is an assignment to x
- A definition of a variable x *reaches* a program point p if:
  - There is no intervening assignment to x between the definition and p
- Also called "def-use" information
- What kind of problem?
  - Forward or backward?
  - May or must?

Dominators

- A program point p *dominates* another program point p' if:
  - p occurs in all paths from the start of the program to p'
- What kind of problem?
  - Forward or backward?
  - May or must?

Space of dataflow analyses

	May	Must
Forward	Reaching definitions	Available expressions
Backward	Live variables	Very busy expressions

- Most dataflow analyses can be classified this way
  - A few cannot: e.g., bidirectional analyses
- Lots of literature on dataflow analysis

So far

- ASTs are very *abstract*, not ideal for program analysis
- Control-flow graph is an alternative representation of the program
  - Captures flow of control, all execution paths
  - Better represents computation steps
  - But, not as close to the original source
- Dataflow analysis: computes a solution to dataflow equations for a program property
  - Depending on property: forward/backward, may/must analysis
  - Worklist algorithm, computes solution per program point
- Examples: available expressions, liveness, very busy expressions, etc.

Formalizing it

- Some algebra background
- Formalization of dataflow analysis
- Properties of dataflow algorithms
  - Termination
  - Solving algorithms
  - Fixpoints
  - Accuracy
- Implementation issues

Partial orders

- A partial order is a pair $(P, \leq)$ of a set P and a relation $\leq$ such that:
  - $(\leq) \subseteq (P \times P)$: The relation $\leq$ is defined only over elements of P
  - $\leq$ is reflexive: $x \leq x$, for all $x \in P$
  - $\leq$ is anti-symmetric: if $x \leq y$ and $y \leq x$ then $y = x$
  - $\leq$ is transitive: if $x \leq y$ and $y \leq z$ then $x \leq z$

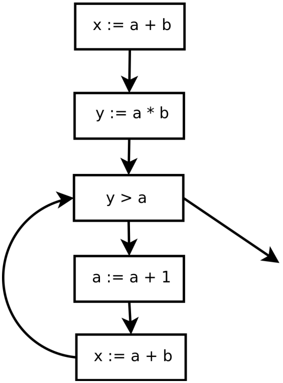
Lattices

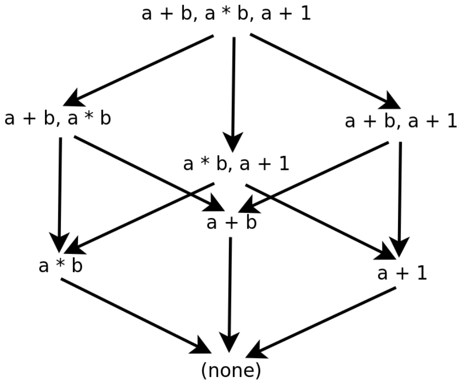
- A partial order is a lattice if $\sqcap$ and $\sqcup$ are defined such that:
  - $\sqcap$ is the *meet*, or *greatest lower bound* operation
    - $x \sqcap y \leq x$ and $x \sqcap y \leq y$
    - if $z \leq x$ and $z \leq y$ then $z \leq x \sqcap y$
  - $\sqcup$ is the *join*, or *least upper bound* operation
    - $x \leq x \sqcup y$ and $y \leq x \sqcup y$
    - if $x \leq z$ and $y \leq z$ then $x \sqcap y \leq z$

Lattices (cont'd)

- A finite partial order is a lattice if meet and join exist for every pair of elements
- A lattice has unique elements $\top$ (top) and $\bot$ (bottom) such that:
  - $x \sqcap \bot = \bot$
  - $x \sqcap \top = x$
  - $x \sqcup \bot = x$
  - $x \sqcup \top = \top$
- In a lattice
  - $x \leq y$ if and only if $x \sqcap y = x$
  - $x \leq y$ if and only if $x \sqcup y = y$
- A partial order P is a *complete lattice* if meet and join are defined on any set $S \subseteq P$

Available expressions lattice





- Typically, sets of dataflow facts form a lattice
- Top element is $\top = \{a + b, a * b, a + 1\}$
- Bottom element is $\bot = \emptyset$

Forward-must dataflow algorithm

```
Forward-Must(CFG)
  for all statements s ∈ CFG
    Out(s) := ⊤
  W := {all statements}
  while W ≠ ∅
    take s from W
    In(s) := ⋂_{s' ∈ pred(s)} Out(s')
    tmp := Gen(s) ∪ (In(s) \ Kill(s))
    if tmp ≠ Out(s) then
      Out(s) := tmp
      W := W ∪ succ(s)
    end if
  end while
```

Monotonicity

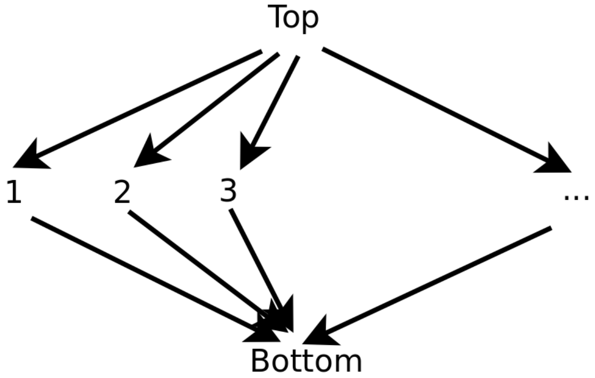
- A function f on a partial order is *monotonic* if

$$x \leq y => f(x) \leq f(y)$$

- Easy to check that operations to compute In and Out are monotonic
  - $In(s) := \bigcap_{s' \in pred(s)} Out(s')$
  - $tmp := \underbrace{Gen(s) \cup (In(s) \setminus Kill(s))}_{f_s(In(s))}$
- Putting these together
  - $tmp := f_s \left(\mathrm{d}_{s' \in pred(s)} Out(s') \right)$

Useful lattices

- $(2^S, \subseteq)$ forms a lattice for any set S
  - 2^S is the powerset of S: the set of all subsets
- If $(S, \leq)$ is a lattice, so is $(S, \geq)$
  - I.e., we can flip a lattice upside-down and still have a lattice
- The lattice for constant propagation is:



Termination

- The algorithm terminates because
  - The lattice has finite height
  - The operations to compute In and Out are monotonic
  - On every iteration:
    - We reduce the size of the worklist or
    - we move the set of facts at a statement down the lattice

Forward dataflow

`Forward`(CFG)
  `for all statements` $s \in CFG$
    $Out(s) := \top$
  $W :=$ {`all statements`}
  `while` $W \neq \emptyset$
    `take` s `from` W
    $tmp := f_s \left(\mathrm{d}_{s' \in pred(s)}\ Out(s')\right)$
    `if` $tmp \neq Out(s)$ `then`
      $Out(s) := tmp$
      $W := W \cup succ(s)$
    `end if`
  `end while`

Lattices for known analyses

- Available expressions
  - $P = \{\text{sets of expressions}\}$
  - $S_1 \sqcap S_2 = S_1 \cap S_2$
  - $\top = \{\text{all expressions}\}$
- Reaching definitions
  - $P = \{\text{all assignment statements}\}$
  - $S_1 \sqcap S_2 = S_1 \cup S_2$
  - $\top = \emptyset$

Fixpoints

- We always start with $\top$
  - Every expression is available/no definitions reach this point
  - The most optimistic assumption
  - The strongest hypothesis possible: true at the fewest number of states
- Revise as we encounter contradictions
  - Always move down the lattice (using $\sqcap$)
- Result: greatest fixpoint

Forward vs. backward dataflow

```
Forward(CFG)
  for all statements s ∈ CFG
    Out(s) := ⊤
  W := {all statements}
  while W ≠ ∅
    take s from W
    tmp := f_s (⊓_{s' ∈ pred(s)} Out(s'))
    if tmp ≠ Out(s) then
      Out(s) := tmp
      W := W ∪ succ(s)
    end if
  end while
```

```
Backward(CFG)
for all statements s ∈ CFG
  In(s) := ⊤
W := {all statements}
while W ≠ ∅
  take s from W
  tmp := f_s (⊓_{s' ∈ succ(s)} In(s'))
  if tmp ≠ In(s) then
    In(s) := tmp
    W := W ∪ pred(s)
  end if
end while
```

Termination revisited

- How many times can we apply the step:
  - $tmp := f_s \left(\mathrm{d}_{s' \in pred(s)} \; Out(s') \right)$
  - `if` $tmp \neq Out(s)$ `then ...`
- Claim: $Out(s)$ only shrinks
  - Proof: $Out(s)$ starts as $\top$
    - so it must be $tmp \leq \top$ after the first step
  - Assume $Out(s)$ shrinks for all predecessors s' of s
  - Then $\mathrm{d}_{s' \in pred(s)} \; Out(s')$ also shrinks
  - Since f_s is monotonic, $f_s \left(\mathrm{d}_{s' \in pred(s)} \; Out(s') \right)$ shrinks

Termination revisited (cont'd)

- A *descending chain* in a lattice is a sequence
  - $x_0 \sqsubset x_1 \sqsubset \ldots$
- The *height* of a lattice is the length of the longest descending chain in the lattice
- Then, dataflow must terminate in $O(nk)$ time, where
  - n is the number of statements in a program
  - k is the height of the lattice
  - ...assuming the meet operation takes $O(1)$ time

Least vs. greatest fixpoint

- Usually in dataflow we start with $\top$, move down using $\sqcap$
  - To do this, we need a *meet semilattice with top*
    - complete meet semilattice: meet defined for all elements
    - finite height ensures termination
  - We compute the greatest fixpoint: the solution highest in the lattice
- In other settings (e.g, denotational semantics) we start with $\perp$, move up using $\sqcup$
  - Computes the least fixpoint

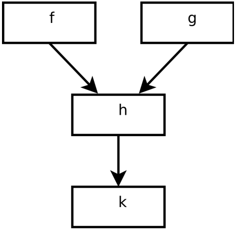
Distributive dataflow problems

- By monotonicity we have $f(x \sqcap y) \leq f(x) \sqcap f(y)$
- A function f is *distributive* if $f(x \sqcap y) = f(x) \sqcap f(y)$
- When using distributive functions, joins lose no information:

$$k(h(f(\top) \sqcap g(\top))) =$$

$$k(h(f(\top)) \sqcap h(g(\top))) =$$

$$k(h(f(\top))) \sqcap k(h(g(\top)))$$



Accuracy

- Ideally, we want the *meet over all paths* (MOP) solution
  - Assume f_s is the transfer function of statement s
  - Assume p is a path $s_1, \ldots, s_n$
  - We define $f_p = f_n; \ldots; f_1$
  - Let $path(s)$ be the set of paths from the entry to s
  - Then

$$MOP(s) = \mathop{\mathrm{d}}_{p \in path(s)} f_p(\top)$$

- If a dataflow problem is distributive then algorithm produces the MOP solution

What problems are distributive?

- Analyses of *how* the program computes
  - Live variables
  - Available expressions
  - Reaching definitions
  - Very busy expressions
- All Gen/Kill problems are distributive
- Analyses of *what* the program computes are not distributive
  - Constant propagation

Implementation issues

- Dataflow facts are assertions of what is true at every program point
- We represent the set of facts as a bit-vector
  - Order all possible facts
  - The *i*-th bit represents the *i*-th fact
  - Intersection is bitwise and
  - Union is bitwise or
- “Only” a constant factor speedup
  - But very useful in practice!

Basic blocks

- A *basic block* is a sequence of statements such that
  - No statement except the last is a branch
  - There are no branches to any statement in the block except the first
- Practically, when implementing dataflow
  - Compute Gen/Kill for each basic block
    - By composing the transfer functions of statements
  - Store In / Out sets only for each basic block
  - Typical basic block is around 5 statements

CFG visiting order - acyclic

- Assume forward dataflow
  - Let $G = (V, E)$ be the control-flow graph
  - and k be the height of the lattice
- If G is acyclic, visit it in topological order
  - For every edge, visit the head node before the tail node
- Running time is $O(|E|)$
  - Regardless of the lattice size

CFG visiting order - cycles

- If G has cycles, visit in reverse postorder
  - Order of depth-first search
- Let Q be the max number of back-edges on a path without cycles
  - Depth of loop nesting
  - Back edge goes from descendant node to ancestor node in DFS tree
- Then if $\forall x. f(x) \leq x$ (sufficient, not necessary)
  - Running time is $O((Q+1)|E|)$
    - depends on definition of $\top$: f shrinks the fact set

Flow-sensitivity

- Dataflow analysis is *flow-sensitive*
  - The answer produced depends on the order of statements in the program
  - We keep track of facts *per program point*
- Alternative: *flow-insensitive* analysis
  - Analysis result does not depend on the statement order
  - Standard example: types
    - A variable has the same type before and after any statement

Dataflow analysis and functions

- What happens at function calls?
  - Lots of possible solutions in the literature
- Usually, analyze one function at a time
  - Called *intraprocedural* analysis
  - When analyzing multiple functions together called *interprocedural*
    - Special case: *whole-program* analysis
- Consequences of intraprocedural analysis
  - Call to function kills all dataflow facts
  - Depending on language, we may be able to save some: e.g., called function cannot affect caller's local variables

Dataflow analysis and pointers

- Dataflow is good at analyzing local variables
  - What about values in the heap?
  - Not modeled in traditional dataflow
- In practice, when $*x := e$
  - Assume it can write anywhere
  - All dataflow facts killed!
  - Better: assume it can write all variables whose address is taken
- In general: it's hard to analyze pointers

Analysis terminology

- Must vs. May
  - Definition depends on which answer is imprecise: yes/maybe, or no/maybe result
  - Not always followed in the literature
- Forward vs. Backward
- Flow-sensitive vs. flow-insensitive
- Distributive vs. non-distributive
- Intraprocedural vs. interprocedural vs. whole-program

Dataflow analysis used in practice

- Moore's law: Hardware advances double computing power every 18 months
- Proebsting's law: Compiler advances double computing power every 18 *years*
    - Costs less than making chips, but not very much worth the trouble for optimization

- Useful for other things:
    - bug-finding: memory leaks, security vulnerabilities, etc.
    - support for high-level language-features
    - program understanding
    - ...