Development and Experimental Validation of an Adaptive Extended Kalman Filter for the Localization of Mobile Robots

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Abstract—A basic requirement for an autonomous mobile robot is its capability to elaborate the sensor measures to localize itself with respect to a coordinate system. To this purpose, the data provided by odometric and sonar sensors are here fused together by means of an extended Kalman filter. The performance of the filter is improved by an on-line adjustment of the input and measurement noise covariances obtained by a suitably defined estimation algorithm.

Index Terms—Adaptive filtering, localization systems, sensor fusion, wheeled mobile robots.

I. INTRODUCTION

An accurate determination of location is a fundamental requirement when dealing with control problems of mobile robots. Two different kinds of localization exist: relative and absolute. The first one is realized through the measures provided by sensors measuring the dynamics of variables internal to the vehicle. Typical internal sensors are optical incremental encoders which are fixed to the axis of the driving wheels or to the steering axis of the vehicle. At each sampling instant the position is estimated on the basis of the encoder increments along the sampling interval. A drawback of this method is that the errors of each measure are summed up. This heavily degrades the position and orientation estimates of the vehicle, especially for long and winding trajectories [1]. In [2] practical methods are proposed to reduce odometry errors due to uncertainty about the effective wheelbase and unequal wheel diameter.

Absolute localization is performed processing the data provided by a proper set of sensors measuring some parameters of the environment in which the vehicle is operating. A set of sonars is generally used as external sensors device. Sonars are fixed to the vehicle and measure the distance with respect to parts of the known environment [3]–[10]. These sensors are also widely utilized for the guidance of autonomous vehicles with obstacle avoidance in unknown environment [11]–[14]. The characterization of sonar measures and/or the rejection of unreliable sonar readings have been widely investigated [6], [15]–[17].

The main drawback of absolute measures is their dependence on the characteristics of the environment. Possible changes to environmental parameters may give rise to erroneous interpretation of the measures provided by the localization algorithm.

The actual trend is to use sensors of different nature and to properly weigh the relative data according to their reliability. For this purpose Kalman filtering techniques represent a powerful tool [8], [10], [18]–[20].

In this paper, the data provided by odometric and sonar sensors are combined together through an adaptive extended Kalman filter (AEKF) providing on-line estimates of robot position.

The use of Kalman filtering techniques requires to derive a stochastic state-space representation of the robot model and of the measure process. Formally this can be readily performed by applying the kinematic model of the robot and the knowledge of measure equipment. The extended Kalman filter (EKF) techniques proposed in the literature [8], [10], [18]–[20], for the estimation of robot localization are based on some fixed values of the input and measurement noise covariance matrices. In many practical applications an “a priori” information of this kind is often unavailable. This is especially true if the trajectory is not elementary and if changes in the environment occur and/or if it has a complex structure. On the other hand it is well known how poor estimates of noise statistics may seriously degrade the Kalman filter performance. The main novelty of the AEKF here proposed is its capability of adaptively estimating such unknown statistical parameters.

In developing the algorithm, a particular attention has been paid to prevention of filter divergence and to the simplicity of implementation in view of on-line applications. Experimental validation shows the effectiveness of the proposed algorithm.

The paper is organized in the following way. The robot model and the sensor device equipment are described in Section II. The adaptive algorithm is reported in Section III. Section IV contains a detailed description of the experimental tests performed on the mobile base.

II. THE SENSOR EQUIPMENT

Consider an unicycle-like mobile robot with two driving wheels, mounted on the left and right sides of the robot, with their common axis passing through the center of the robot (see Fig. 1). Localization of this mobile robot in a two-dimensional (2-D) space requires knowledge of the coordinates x and y of
the midpoint between the two driving wheels and of the angle \( \theta \) between the main axis of the robot and the \( x \)-direction. The kinematic model of the unicycle robot is described by the following equations:

\[
\begin{align*}
\dot{x}(t) &= v(t) \cos \theta(t) \quad (1) \\
\dot{y}(t) &= v(t) \sin \theta(t) \quad (2) \\
\dot{\theta}(t) &= \omega(t) \quad (3)
\end{align*}
\]

where \( v(t) \) and \( \omega(t) \) are, respectively, the displacement and angular velocities of the robot. The encoders placed on the driving wheels provide a measure of the incremental angles over a sampling period \( \Delta t_k := t_{k+1} - t_k \). The odometric measures are used to obtain an estimate of the displacement and angular velocities \( \Delta x(t_k) \) and \( \Delta \theta(t_k) \), respectively, which are assumed to be constant over the sampling period. Numerical integration of (1) and (2) based on the computed \( \Delta x(t_k) \) and \( \Delta \theta(t_k) \) results in an estimate of the position and orientation increments over each sampling period of the unicycle robot. The above processing is generally performed by an odometric device connected with the low level control equipment of the robot imposing the desired \( v(t) \) and \( \omega(t) \).

An analysis of the accuracy of the estimation procedure implemented by an odometric equipment has been developed in [1]. The encoders introduce incremental errors, which especially affect the estimate of the orientation \( \theta(t) \). This limits the applicability of the odometric localization to short trajectories only.

The localization of a mobile robot in an indoor environment can also be performed with sonar measures [3]–[8], [10]. In this context, the environment can be represented by its elementary geometric parts as corners, angles, cylinders and planes [6].

In the following, the sonar readings will be easily related to the model of the indoor environment and the configuration of the mobile robot. Consider a planar distribution of \( n_s \) sonar sensors. Denote by \( x_i^s, y_i^s, \theta_i^s \) the planar position and orientation of the \( i \)-th sonar, \( i = 1, 2, \ldots, n_s \), referred to the coordinate system \( (O', X', Y') \) fixed to the mobile robot, as reported in Fig. 2.

The position \( x_i, y_i \) and orientation \( \theta_i \) at the sampling time \( t_k \) of the \( i \)-th sonar referred to the inertial coordinate system \((O, X, Y)\) have the following form:

\[
\begin{align*}
x_i(t_k) &= x(t_k) + x_i^s \sin \theta(t_k) + y_i^s \cos \theta(t_k) \quad (4) \\
y_i(t_k) &= y(t_k) - x_i^s \cos \theta(t_k) + y_i^s \sin \theta(t_k) \quad (5) \\
\theta_i(t_k) &= \theta(t_k) + \theta_i^s \quad (6)
\end{align*}
\]

The walls and the obstacles in an indoor environment of a mobile robot can be well represented by a proper set of planes orthogonal to the plane \( XY \) of the inertial coordinate system. Each plane \( P^j, j = 1, 2, \ldots, n_p \), where \( n_p \) is the number of planes which describe the indoor environment, can be represented by the triplet \( P_i^j, P_i^j, P_i^j \), where \( P_i^j \) is the normal distance of the plane from the origin \( O \), \( P_i^j \) is the angle between the normal line to the plane and the \( x \)-direction and \( P_i^j \) is a binary variable, \( P_i^j \in \{ -1, 1 \} \), which defines the face of the plane reflecting the sonar beam. Using the above notation, the expectation \( d_i^j(t_k) \) for the distance measure of the sonar \( i \) in the position \( x_i(t_k), y_i(t_k) \) and with orientation \( \theta_i(t_k) \) from the plane \( P^j \) represented by \( P_i^j, P_i^j, P_i^j \) has the following expression (see Fig. 3):

\[
d_i^j(t_k) = P_i^j(P_i^j - x_i(t_k) \cos P_i^j - y_i(t_k) \sin P_i^j) \quad (7)
\]

if the \( P_i^j \in [\theta_i(t_k) - \delta/2, \theta_i(t_k) + \delta/2] \), where \( \delta \) is the beamwidth of the sonar sensor. The vector composed of geometric parameters \( P_i^j, P_i^j, P_i^j \), \( j = 1, 2, \ldots, n_p \), is denoted by \( \Pi \).

To simplify the position estimation algorithm without appreciably reducing its accuracy, the sonar echoes traveling along the cone edges, i.e., for \( P_i^j \in [\theta_i(t_k) - \delta/2 - \epsilon, \theta_i(t_k) + \delta/2] \), and \( P_i^j \in [\theta_i(t_k) - \delta/2 + \epsilon, \theta_i(t_k) + \delta/2 + \epsilon] \), where \( \epsilon \) is an angle...
depending on the surface roughness, have been omitted. In fact, the measures along the cone edges require an a priori model of the environment also including the different roughness of the walls and are less accurate of the distance measures given by (7).

III. ADAPTIVE ESTIMATION OF ROBOT LOCATION

The proposed AEKF for the fusion of odometric and sonar measures, providing on line estimates of robot position and orientation, is derived in this section.

Denote with $X(t) := [x(t) \ y(t) \ \theta(t)]^T$ the robot state and with $U(t) := [v(t) \ \omega(t)]^T$ the robot control input. The kinematic model of the robot can be written in the compact form of the following stochastic differential equation:

$$dX(t) = F(X(t), U(t)) \, dt + d\eta(t)$$

where $F(X(t), U(t))$ is obtained by (1)–(3) and $\eta(t)$ is a Wiener process such that $E(d\eta(t)d\eta(t)^T) = Q(t) \, dt$. Its weak mean square derivative $d\eta(t)/dt$ is a white noise process $\sim N(0, Q(t))$ representing the model inaccuracies (parameter inaccuracies, slippage, dragging). Assuming a constant sampling period $\Delta t_k = T$ and denoting $t_{k+1}$ by $(k+1)T$, the following sampled nonlinear measure equation can be associated to (8):

$$Z((k+1)T) = G(X((k+1)T), \Pi) + v(kT)$$

where $Z(kT)$ is the vector containing sonar and odometric measures and $v(kT)$ is a white sequence $\sim N(0, R(kT))$. The dimension $n_k$ of $Z(kT)$ is not constant, depending on the number of sonar sensors that are actually used at each time instant. The measure vector $Z(kT)$ is composed of two subvectors $Z_1(kT) = [z_1(kT) \ z_2(kT) \ z_3(kT)]^T$ and $Z_2(kT) = [z_1(kT) \ z_2(kT) \ \ldots \ z_{n_k+3}(kT)]^T$, where $z_1(kT) = x(kT) + v_1(kT)$, $z_2(kT) = y(kT) + v_2(kT)$, $z_3(kT) = \theta(kT) + v_3(kT)$ are the measures provided by the odometric device, and $z_{n+3}(kT) = d_{j_1}^k(kT) + d_{j_2}^k(kT) + \ldots + d_{j_{n_k}}^k(kT)$, $j_1, j_2, \ldots, j_{n_k} \in \{1, n_p\}$, $i = 1, 2, \ldots, n_k$, is the distance measure provided by the $i$th sonar sensor from the $P_j^i$ plane with $j \in \{1, n_p\}$. The environment map provides the information needed to detect which is the plane $P_j^i$ in front of the $i$th sonar.

By definition of the measurement vector one has that the output function $G(X((k+1)T), \Pi)$ has the following form:

$$G(X(kT), \Pi) = \begin{bmatrix} x(kT) \\ y(kT) \\ \theta(kT) \\ d_{j_1}^k(kT) \\ d_{j_2}^k(kT) \\ \ldots \\ d_{j_{n_k}}^k(kT) \end{bmatrix}$$

where $n_k := p_k - 3$. The number $p_k$ of measures may vary from the minimum value 3 to the maximum value $n_k + 3$, where $n_k$ is the number of sonar sensors.

Assume $U(t) = U(kT)$ for $t \in [kT, (k+1)T]$. To obtain an extended Kalman filter (EKF) with an effective state prediction equation in a simple form, model (1), (2) has been linearized about the current state estimate $\hat{x}(kT, kT)$ and the control input $U((k-1)T)$ applied until the linearization instant. Subsequent discretization with period $T$ of the linearized model results in the following EKF (where explicit dependence on $T$ has been dropped for simplicity of notation)

$$\dot{X}(k+1) = \hat{X}(k+1) + L(k)U(k),$$

$$P(k+1) = A_d(k)P(k, k)A_d(k)^T + Q_d(k)$$

$$K(k+1) = P(k+1)C^T(k+1)\left[C(k+1)P(k+1, k)C^T(k+1) + R(k+1)\right]^{-1}$$

$$\dot{X}(k+1) = \hat{X}(k+1) + K(k+1)\left[Z(k+1) - G(\hat{X}(k+1), k), \Pi)\right]$$

$$P(k+1, k+1) = [I - K(k+1)C(k+1)]P(k+1, k)$$

and in (16)–(19), shown at the bottom of the page.

$$L(k) := \begin{bmatrix} T \cos \theta(k) & -\frac{1}{2}k(k-1)T^2 \sin \theta(k) \\ T \sin \theta(k) & \frac{1}{2}k(k-1)T^2 \cos \theta(k) \end{bmatrix}, \quad A_d(k) := \begin{bmatrix} 1 & -\nu(k-1)T \sin \theta(k) \\ 0 & 1 \end{bmatrix}$$

$$Q_d(k) := \sigma_d^2(k)Q(k)$$

$$C(k) := \begin{bmatrix} C_1(k) \\ C_2(k) \\ \ldots \\ C_{n_k}(k) \end{bmatrix}, \quad [C_1(k)C_2(k)C_3(k)C_{n_k}(k)]^T = I_3$$

$$C_{i+3}(k) =\begin{bmatrix} -\cos P_j^i \\ -\sin P_j^i \\ x_i^j \cos(\theta(k) - P_j^i) + y_i^j \sin(\theta(k) - P_j^i) \end{bmatrix}, \quad i = 1, 2, \ldots, p_k, \quad p_k \leq n_s, \quad j \in \{1, n_p\}.$$
The form of $Q_d(k)$ expressed by (17) derives by the hypothesis that $Q(\tau) = \sigma_d^2(k)I_3$, $\tau \in [kT, (k+1)T]$. This simplification assumption has been introduced to obtain a $Q_d(k)$ which is completely known up to the unknown multiplicative scaling factor $\sigma_d^2(k)$. Moreover, the covariance matrix $R(k)$ is assumed to have the following diagonal form:

$$R(k) = \text{diag}[^2_1, \ldots, ^2_p(k)]$$

(20)

this means that no correlation is assumed between the measurement errors introduced by the sensors. As $R(k)$ is diagonal, the components of $Z(k)$ may be processed one by one reducing the inversion of the $p_k \times p_k$ matrix in (13) to the inversions of scalars [21], thus saving much computation time. The sequential processing of each component $z_i(k), \ldots, z_{p_k}(k)$ must be performed in a period of time (typically the sampling period) such that no significant change occurs in the state estimate and in its covariance matrix due to dynamics (8) [21].

The EKF can be implemented once estimates of $Q_d(k)$ and $R(k)$ are available. In general, a complete and reliable information about these matrices is not available; on the other hand it is well known how poor knowledge of noise statistics may seriously degrade the Kalman filter performance. This problem is here dealt with introducing an adaptive adjustment mechanism of $Q_d(k)$ and $R(k)$ values in the EKF equations.

**A. Adaptive Estimation of $Q_d(k)$ and $R(k)$**

A considerable amount of research has been carried out in the adaptive Kalman filtering area (see [21]–[23] and references therein), but in practice it is often necessary to redesign the adaptive filtering scheme according to the particular characteristics of the problem faced. For example, in view of real time applications, a particular attention has been here devoted to simplicity of implementation and to prevention of filter divergence, moreover, the particular structure of the input noise covariance matrix $Q_d(k)$, which is completely known save that for a multiplicative scalar, has been suitably taken into account.

The following nearly stationarity assumption is made: the parameters $\gamma_1(k), \gamma_2(k)$ and $\gamma_3(k)$ are nearly constant over $n_v \geq 1$ and over $n_\gamma \geq 2$ respectively.

Define $\gamma_i(k+1) = z_i(k+1) - G_i(X(k+1), k)$, where $z_i(k+1)$ and $G_i(X(k+1), k)$ are the $i$th component of $Z(k+1)$ and $G(X(k+1), k)$, respectively. For analogy with the linear case, residuals $\gamma_i(k), i = 1, \ldots, p_k$, are called the innovation process samples and are assumed to be well described by a white sequence $N(0, s_i(k+1))$, where $s_i(k+1), i = 1, \ldots, p_k$ can be expressed as

$$s_i(k+1) = C_i(k+1)P(k+1)A_i^T(k+1) + \sigma_{\gamma_i}(k+1)$$

$$= C_i(k+1)[A_d(k)P(k,k)A_i^T(k+1) + \sigma_{\gamma_i}^2(k)\tilde{Q}(k)]$$

$$\cdot C_i^T(k+1) + \sigma_{\gamma_i}^2(k+1).$$

(21)

This simplifying assumption is as more valid as discretization and linearization of (8) is more accurate and makes it possible to apply the methods of the adaptive filtering theory developed for the linear case.

The two above assumptions will allow us to define a simple and efficient estimation algorithm based on the condition of consistency, at each step, between the observed innovation process samples $\gamma_i(k+1), i = 1, \ldots, p_k$ and their predicted statistics $E(\gamma_i^2(k+1)) = s_i(k+1)$. Imposing such a condition, one stage estimates $\hat{\sigma}_{\gamma_i}^2(k)$ and $\hat{\sigma}_{\gamma_i}^2(k+1), i = 1, \ldots, p_k$, of $\sigma_\gamma^2(k)$ and $\sigma_\gamma^2(k+1), i = 1, \ldots, p_k$, respectively, are obtained at each step. To increase their statistical significance, the one stage estimates $\hat{\sigma}_{\gamma_i}^2(k)$ and $\hat{\sigma}_{\gamma_i}^2(k+1), i = 1, \ldots, p_k$, are averaged obtaining the relative smoothed versions $\bar{\sigma}_{\gamma_i}^2(k)$ and $\bar{\sigma}_{\gamma_i}^2(k+1), i = 1, \ldots, p_k$.

From (21) it is apparent that the statistical information carried by each $\gamma_i(k+1), i = 1, \ldots, p_k$, depends, at the same time, on the two unknown parameters $\sigma_{\gamma_i}^2(k)$ and $\sigma_{\gamma_i}^2(k+1)$. This indeterminateness is here dealt with using a number (say $n_\gamma$) of innovation process samples $\gamma_i(k+1), i = 1, 2, \ldots, p_k$, to estimate $\sigma_{\gamma_i}^2(k)$ and the others (say $n_{\mu}$) to estimate $\sigma_{\gamma_i}^2(k+1)$. In the light of the nearly stationary assumption, the two integers $n_\gamma$ and $n_{\mu}$ are chosen such that $n_\gamma/n_{\mu} = n_{\mu}/n_\gamma$.

Assume $n_\gamma \geq n_{\mu}$, let $\alpha$ and $\beta$ two coprime integers such that $\alpha/\beta = n_\gamma/n_{\mu}$ and let $q$ and $r$ two integers such that $\alpha = 2q + r$; then, the innovation process sequence is subdivided into intervals $I_{2q+1}$, composed of $\alpha + \beta$ samples. Each interval contains $\beta$ sequences of $q$ samples used to estimate $\sigma_{\gamma_i}^2(k)$ (the faster varying parameter), the ensembles of $q$ samples are separated by $\beta$ sequences of one sample used to estimate $\sigma_{\gamma_i}^2(k+1), i = 1, \ldots, p_k$ (the more slowly varying parameter), the last $r$ samples of each $I_{2q+1}$ interval are used to estimate $\sigma_{\gamma_i}^2(k)$. This scheme minimizes the interval of time over which either one step estimate is not updated. A symmetric situation holds if $n_{\mu} \geq n_\gamma$.

When the one step estimate $\hat{\sigma}_{\gamma_i}^2(k)(\hat{\sigma}_{\gamma_i}^2(k+1), i = 1, \ldots, p_k$ is updated, the other single stage estimate $\hat{\sigma}_{\gamma_i}^2(k+1), i = 1, \ldots, p_k, (\sigma_{\gamma_i}^2(k))$ is kept constant, so that the symbol $\hat{\sigma}_{\gamma_i}^2(k)(\sigma_{\gamma_i}^2(k+1))$ does not necessarily imply that this estimate has been computed using the last observed innovation process sample $\gamma_i(k+1), i = 1, \ldots, p_k$.

Because of the particular form (17) of $Q_d(k)$ and of the sequential scalar processing of measures, $p_k$ one stage estimates $\sigma_{\gamma_i}^2(k)$ of the unknown $\sigma_{\gamma_i}^2(k), i = 1, \ldots, p_k$, can be determined maximizing the probability of observing the corresponding $i$th component of the predicted residual $\gamma_i(k+1), i = 1, \ldots, p_k$ [21]. Namely, each $\sigma_{\gamma_i}^2(k)$ is determined by the operation

$$\text{max prob } \bar{\sigma}_{\gamma_i}^2(k+1) \geq \gamma_i^2(k+1).$$

The maximizing $\bar{\sigma}_{\gamma_i}^2(k)$ is obtained by imposing the condition of consistency between residuals and their predicted statistics, namely $\gamma_i^2(k+1) = E(\gamma_i^2(k+1)) = s_i(k+1)$. Using (21) and replacing $\sigma_{\gamma_i}^2(k+1)$ with $\bar{\sigma}_{\gamma_i}^2(k+1)$ one has (22), shown at the bottom of the page.

$$\bar{\sigma}_{\gamma_i}^2(k) = \max \left\{ \gamma_i(k+1)^2 - C_i(k+1)A_d(k)P(k,k)A_i^T(k+1) - \bar{\sigma}_{\gamma_i}^2(k+1) \right\},$$

(22)
To obtain a unique estimate of $\sigma^2_{\eta}(k)$ and to increase the statistical significance of estimators (22), which are based on only one component $\gamma_{\eta}(k+1)$, the following smoothed estimate is computed:

$$\bar{\sigma}^2_{\eta}(k) = \frac{1}{(l_{\eta}+1)p_{k}} \sum_{j=0}^{l_{\eta}} \sum_{i=1}^{p_{k}} \delta^2_{\eta,i}(k-j)$$

(23)

where $l_{\eta}$ denotes the number of one-stage estimates $\delta^2_{\eta,i}(\cdot)$ yielding the smoothed estimate.

In a recursive form the proposed estimate of $\sigma^2_{\eta}(k)$ is

$$\bar{\sigma}^2_{\eta}(k) = \bar{\sigma}^2_{\eta}(k-1) + \frac{1}{(l_{\eta}+1)p_{k}} \left[ \sum_{i=1}^{p_{k}} \left( \delta^2_{\eta,i}(k) - \delta^2_{\eta,i}(k-(l_{\eta}+1)) \right) \right].$$

(24)

Analogously, the operation

$$\max \text{prob}_{\gamma_{\eta,i}(k+1) \geq \gamma_{\eta}(k+1)}$$

and (21) give the following one stage estimate of $\sigma^2_{\eta,i}(k+1)$, $i = 1, \ldots, p_{k}$

$$\sigma^2_{\eta,i}(k+1) = \max \{ \gamma^2_{\eta}(k+1) - [C_{\eta}(k+1)A_{\eta}(k)P(k, k)A_{\eta}^T(k) \cdot C_{\eta}(k+1) + C_{\eta}(k+1)\sigma^2_{\eta,i}(k)Q(k) \cdot C_{\eta}^T(k+1)]', 0 \}$$

(25)

the smoothed version $\bar{\sigma}^2_{\eta,i}(k+1)$ is

$$\bar{\sigma}^2_{\eta,i}(k+1) = \frac{1}{l_{\eta}+1} \sum_{j=0}^{l_{\eta}} \sigma^2_{\eta,i}(k+1-j)$$

(26)

where $l_{\eta}$ denotes the number of one-stage estimates $\sigma^2_{\eta,i}(\cdot)$ yielding the smoothed estimate.

In a recursive form the proposed estimate of $\sigma^2_{\eta,i}(k+1)$ becomes

$$\bar{\sigma}^2_{\eta,i}(k+1) = \bar{\sigma}^2_{\eta,i}(k) + \frac{1}{l_{\eta}+1}(\delta^2(k+1)-\delta^2(k-l_{\eta})).$$

(27)

The proposed adaptive estimation algorithm is given by (24) and (27) and is able to prevent filter divergence. In fact, as long as the innovation samples $\gamma_{\eta}(k+1)$, $i = 1, \ldots, p_{k}$ are sufficiently small and consistent with their statistics, the filter operates satisfactorily and the noise model is kept small (or null) by (22). If a sudden increase of the absolute value of the innovation process samples is observed, (22) provides an increased $Q_{\eta}(k) = \bar{\sigma}^2_{\eta,i}(k)Q(k)$, and hence an augmented filter gain, thus preventing filter divergence.

Parameters $l_{\eta}$ and $l_{\eta}$ of estimators (24) and (27) are chosen on the basis of two antagonist considerations: low values would produce noise estimators which are not statistically significant, large values would produce estimators which are scarcely sensitive to possible rapid fluctuations of the true $\sigma^2_{\eta}(k)$ and $\sigma^2_{\eta,i}(k)$, $i = 1, \ldots, p_{k}$. During filter initialization, the starting values $(\delta^2_{\eta}(0)^2$ and $(\delta^2_{\eta,i}(0)^2$, $i = 1, \ldots, p_{k}$, of $\sigma^2_{\eta}(k)$ and $\sigma^2_{\eta,i}(k)$, respectively, must be chosen on the basis of the “a priori” available information. In the case of a lack of such information, a large value of $P(0, 0)$ is useful to prevent divergence.

B. Sonar Readings Selection

It is known that more sensor data does not always mean better estimates. The grounds may be an inadequate interpretation of erroneous data due to sensor failures or the inconsistency of the underlying filter equations, this second reason is discussed in [24]. In the present case, sonar measurements may not only be affected by white Gaussian noise but also by crosstalk interferences and multiple echoes [5], [25]. Techniques for the treatment of these systematic errors based on the construction of a navigation map and/or on a time pulse modulation have been proposed in [16] and [17], respectively. A simple method is proposed here to deal with the undesired interferences produced by the presence of unknown obstacles. A sort of prefiltering is performed neglecting those sonar measures that are not deemed to be reliable. A simple and efficient way to perform these preliminary measure selection is to compare the actual sonar readings with their expected values. Measures are discharged if the difference exceeds a threshold. The value of the threshold has to be chosen on the basis of two opposite considerations: too large values would accept undesired interferences as reliable sonar readings, while too low values would produce an undesired loss of useful information. For this reason, it is preferable not to choose an unique fixed threshold, but to adapt the preliminary choice on line, on the basis of the reliability of the current predicted sonar measure. This is here done in the following way: at each step, the residual $\gamma_{i+k} = z_{i+k} - G_{i+k}(\hat{X}(k+1), k)$, $i = 1, 2, \ldots, p_{k} - 3$, is compared to its expected value $G_{i+k}(\hat{X}(k+1), k)$ which is computed on the basis of the estimated robot location and on the “a priori” knowledge of the environment. As $\gamma_{i+k} \sim N(0, s_{i+k}(k+1))$, the current value $z_{i+k} = \gamma_{i+k}(k+1)$ is accepted if $|\gamma_{i+k}(k+1)| \leq 2\sqrt{s_{i+k}(k+1)}$. Namely, the variable threshold is chosen as two times the standard deviation of the innovation process.

This adaptive threshold realizes a reasonable tradeoff between the aforementioned opposite considerations. If the $\hat{X}(k+1, k)$ is reliable, so is $G_{i+k}(\hat{X}(k+1, k), k)$, namely the expected sonar reading, so that a strict test must be performed by choosing a low threshold. This is automatically obtained taking into account equation that, by (21), reliability of $\hat{X}(k+1, k)$ corresponds to a low $P(k+1, k)$. The opposite occurs if $\hat{X}(k+1, k)$ has a large error covariance matrix.

The structure of the proposed localization algorithm is reported in Fig. 4.

IV. EXPERIMENTAL RESULTS

The experimental tests have been performed on the LabMate mobile base in an indoor environment with different geometry. This mobile robot is realized with two driving wheels, as reported in Fig. 1, and the odometric data are the incremental measures that at each sampling interval are provided by the encoders attached to the right and left wheels of the robot. These measures are directly acquired by the low level
controller of the mobile base. The sonar measures have been realized by the standard proximity system of the LabMate base composed by a set of nine Polaroid sonar sensors. In Fig. 5, a picture of LabMate system with the sonar sensors displacement is reported.

A preliminary reduction of crosstalk interferences has been realized by a proper distribution on the orientations of the sonar sensors. The sonars on the lateral sides of the mobile base have been mounted with a difference on the orientation of 15°, as reported in Fig. 6. A significant reduction of the wrong readings produced by the presence of unknown obstacles has been also realized selecting of the sonar measures by the procedure described in Section III-B.

The control algorithm is based on a proper discrete-time implementation of the control algorithm proposed in [26] and based on the kinematic inversion approach. The continuous-time control algorithm is explicitly represented by the following equations:

\[ v(t) = K_p(y_d(t) \sin(\theta(t)) + x_d(t) \cos(\theta(t))) + C_p \epsilon_v(t), \]
\[ \omega(t) = K_\omega(\dot{x}_d(t) - \dot{\theta}(t)) + C_\omega \epsilon_\omega(t), \]

where \( K_p, K_\omega, C_p, \) and \( C_\omega \) are the gain values and

\[ \epsilon_v(t) = (y_d(t) - y(t)) \cos(\theta(t)) - (x_d(t) - x(t)) \sin(\theta(t)), \]
\[ \epsilon_\omega(t) = (y_d(t) - y(t)) \sin(\theta(t)) + (x_d(t) - x(t)) \cos(\theta(t)) \]

and \( x_d(t), \ y_d(t), \) and \( \theta_d(t) \) represent the desired planned trajectory. The AEKF has been implemented on a personal computer with Windows 3.1 system by the developing environment described in [27]. In this development system, the planned trajectory has been computed considering the nonholonomic and environment constraints according to the algorithm proposed in [28]. The system is connected directly with the robot low level controller and the proximity system by a standard RS232 serial port. The sampling period \( T = 0.3 \) s has been chosen.

In the developed experiments, the proposed localization algorithm has been tested with relatively long trajectories on an indoor environment represented by a suitable set of planes orthogonal to the plane \( XY \) of the inertial system. The first experiment performed is illustrated in Fig. 7. The planned trajectory from the start configuration \( S \) to the goal configuration \( G \) (the gray path) is computed by the development environment described in [27]. In order to test the capability of reducing the position error on the proposed localization algorithm, at the configuration \( S \) the robot starts with an initial position error of 0.26 m. Frame (a) shows the trajectory obtained with the \( a \) priori fixed \( \hat{\sigma}_n^{2}(k) = 2.5 \cdot 10^{-3}, \)
\[ \hat{\sigma}_{n,3+\ell}^{2}(k) = 8.2 \cdot 10^{-6}, \text{for } \ell = 1, 2, \ldots, p_k, \] for any integer \( k \). Frame (b) shows the trajectory obtained with AEKF which has been implemented with initial values \( \sigma_0^{2} = \sigma_0^{2}(0) \) equal to corresponding values used for EKF. Frame (c) shows the behavior of the estimated \( \hat{\sigma}_n^{2}(k) \) assuming \( l_n = 6 \). This figure evidences a sharp increase of \( \hat{\sigma}_n^{2}(k) \) in correspondence of the configuration \( C \) of the mobile base when the sonar measures start to be actually used. On the configuration \( C \) the effects of the sonar readings is a rapid increase of residuals \( \gamma_n(k) \), and, by (23), \( \hat{\sigma}_n^{2}(k) \) increases too. As a consequence also the filter gain increases and this allows the filter to properly weigh the incoming observations. The estimated robot position is rapidly corrected and this provokes a decrease of residual samples and hence of the estimated
The adaptive estimation of $\tilde{\sigma}_{\eta}^2(k)$ did not produce very significant changes with respect to the initial values because of the large $I_p = 60$ chosen on the basis of experimental tests. This choice was also motivated by the proximity of the initial value $\tilde{\sigma}_{1,\eta}^2(0)$ to the values provided by the off-line calibration of the sonars. This test evidences the improvement introduced by the adaptation algorithm, which is able to reduce to 0.01 m the initial position error by better exploiting the information carried by the sonar readings in correspondence of the obstacle, while the EKF yielded a final error of 0.11 m.

Fig. 8 illustrates the results of the second experiment. Part (a) of this figure represents the realized trajectory with localization deduced by odometric measures only. In order to test the limitation of the odometric measures, the planned trajectory is composed by a large set of orientation changes. The black path is the realized trajectory with only odometric measures. In this case, at the end of the test, the robot is out of the planned trajectory. Part (b) shows the same test with localization based on the EKF implemented with "a priori" chosen $Q_d(k)$ and $R(k)$. Part (c) shows the same test
with localization based on the AEKF. In all the experiments the values $n_\theta = n_\nu = 2$ have been assumed; this means that the innovation process vector samples $\Gamma(k+1)$ have been alternately used to estimate $\sigma_\theta^2(k)$ and $\sigma_\nu^2(k+1)$, $i = 1, \ldots, p_k$ according to the algorithm described in Section III-A. The plot clearly evidences the improvement introduced by the adaptation mechanism.

A long trajectory developing through three different rooms has been considered in the third experiment to test the proposed localization algorithm in realistic operating conditions. For convenience, the graphic representation of the results has been reported on a sequence of figures [Fig. 9(a)–(e)] where the goal configuration $G$ of each frame is the start configuration $S$ of the subsequent. Fig. 9 reveals that the
Fig. 9. Planned trajectory is represented in the frames sequence from (a)–(e). The gray path is the planned trajectory, the black path is the realized trajectory, and the black dots are the actually used sonar measures.
realized and planned trajectories are almost everywhere nearly coinciding.

Fig. 10 shows the effect of the sonar measure selection. An unknown obstacle of small dimensions, which is not introduced in the environment map, is placed at the side of a wall and the corresponding readings are seen as undesired interfer-ences (the considered problem is not obstacle avoidance). No perturbation affects the robot motion because these measures are eliminated by the procedure outlined in Section III-B. Of course this procedure, based on sonar measure selection is successful as far as the unknown obstacle dimensions are small enough compared with the reference wall dimensions.

V. CONCLUSION

This paper proposed a new method for the accurate localization of a mobile robot. The approach is based on the use of a linearized Kalman filter endowed with an adaptive algorithm for the adjustment of the input and measurement noise covariance matrices. The adaptation mechanism has been introduced to allow the filter to cope with realistic operating conditions. If the planned trajectory is relatively simple and not too long, some “a priori” engineered noise statistics may produce satisfactory results, but filter divergence may occur over complex trajectories and the localization algorithm provides unreliable estimates. In this latter case the introduction of an adaptive algorithm seems to be the most efficient and simple remedy to prevent filter divergence. The experiments reported in the paper confirmed that high performances of the localization algorithm are really obtainable in a wide range of experimental situations.

REFERENCES


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