

# Digital image

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# Image formation

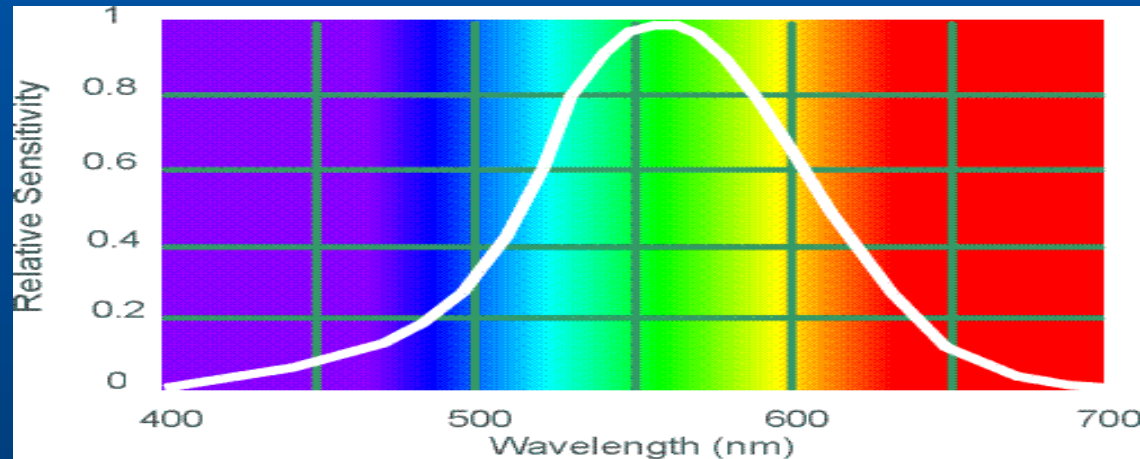
Light is an electromagnetic wave.

Its color is characterized by the wavelength content of the light.

$$L(x,y;t) = \int \Phi(x,y;t;\lambda) E(\lambda) d\lambda$$

**Monochromatic receptor**

**Trichromatic receptor**

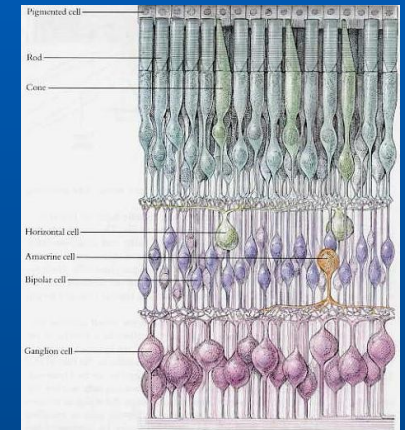
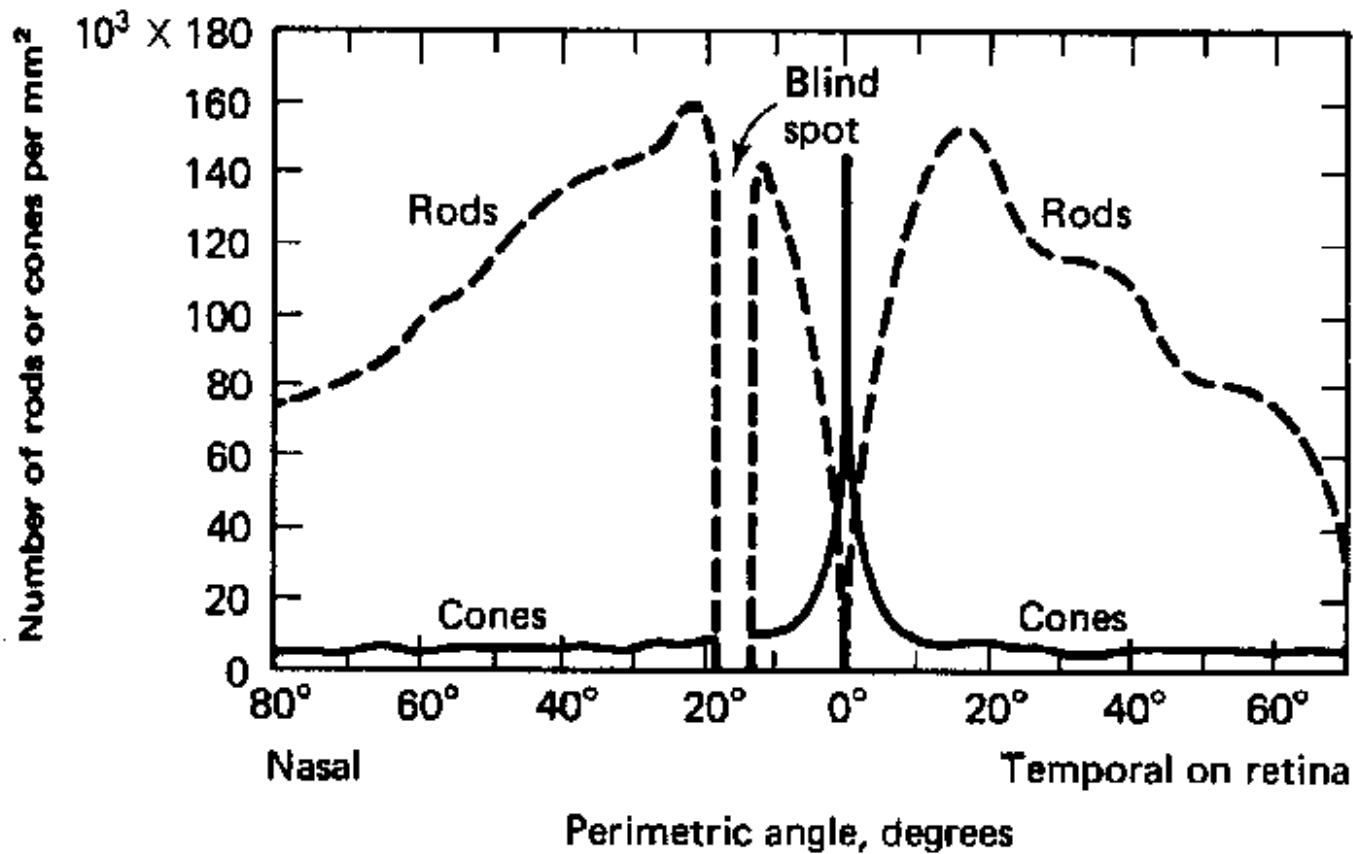


**Human vision system**

**Scotopic vision (monochromatic)**

**Photopic vision (trichromatic)**

**Visible spectrum**



The retina consists of an *array* of *rods* and three kinds of cones.

Rods (scotopic vision) : 100 millions

Cones (photopic vision) : 6 millions

Central vision : space

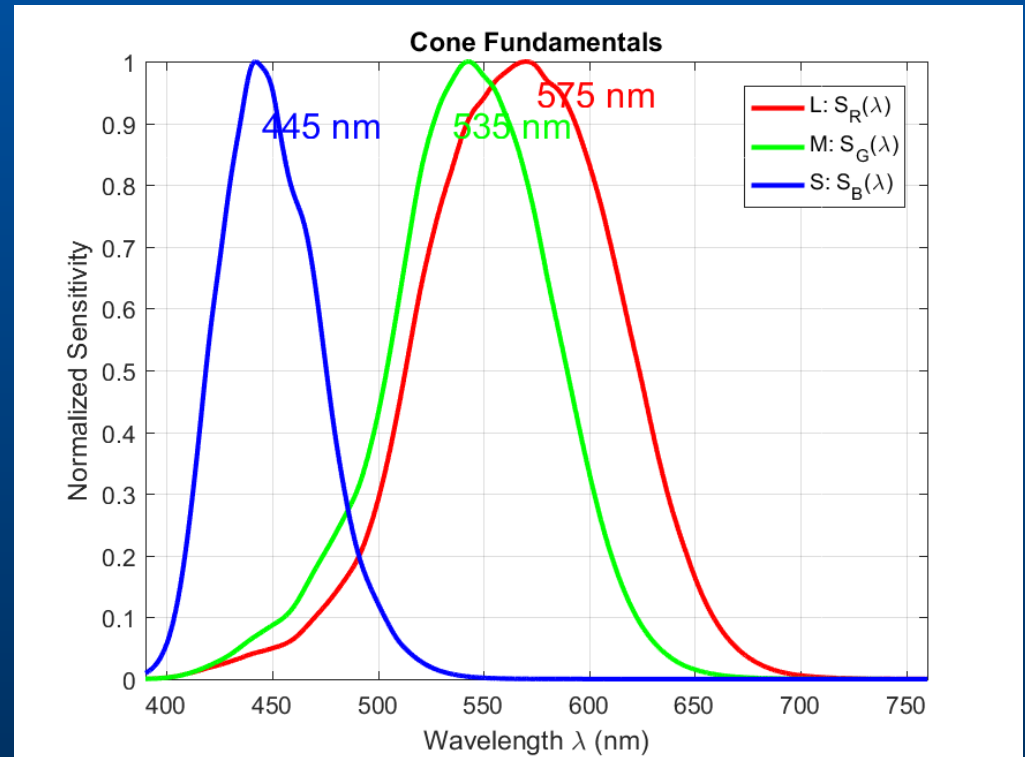
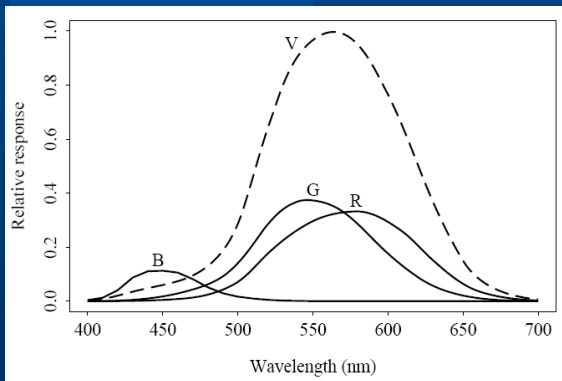
Peripheral vision : motion

# Eye sensitivity

The sensitivity of our receptors is a function of wavelength

Primary colours

Red	R	575 nm
Green	G	535 nm
Blue	B	445 nm

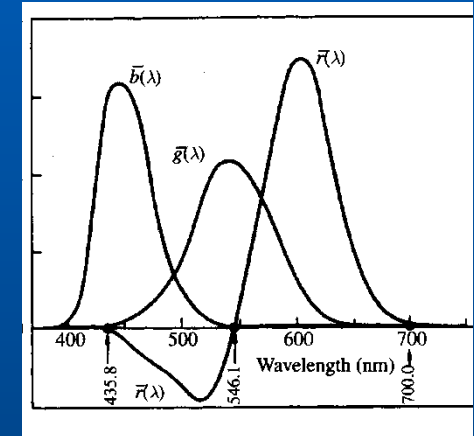


# Color definition (C.I.E.)

A linear mixing of three colors gives every spectral distribution

Three reference stimuli

Red	R	700 nm
Green	G	546 nm
Blue	B	436 nm

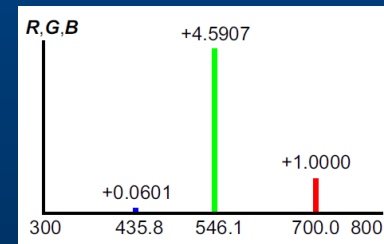
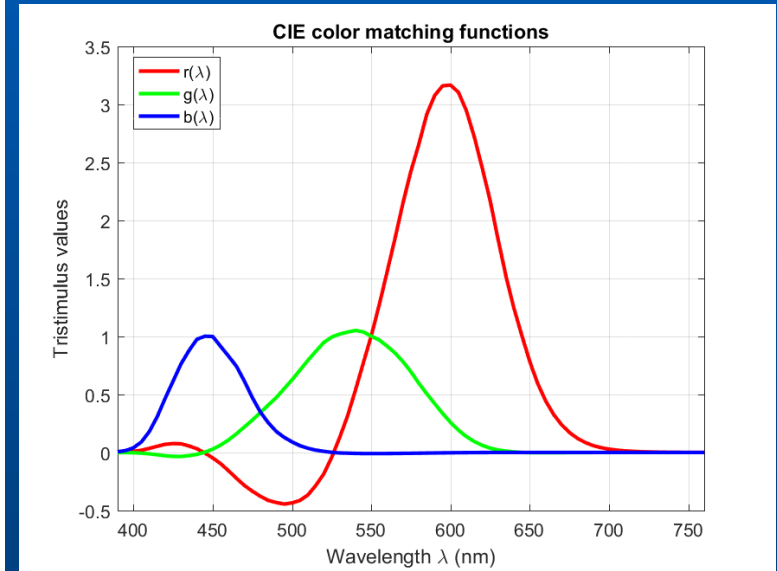
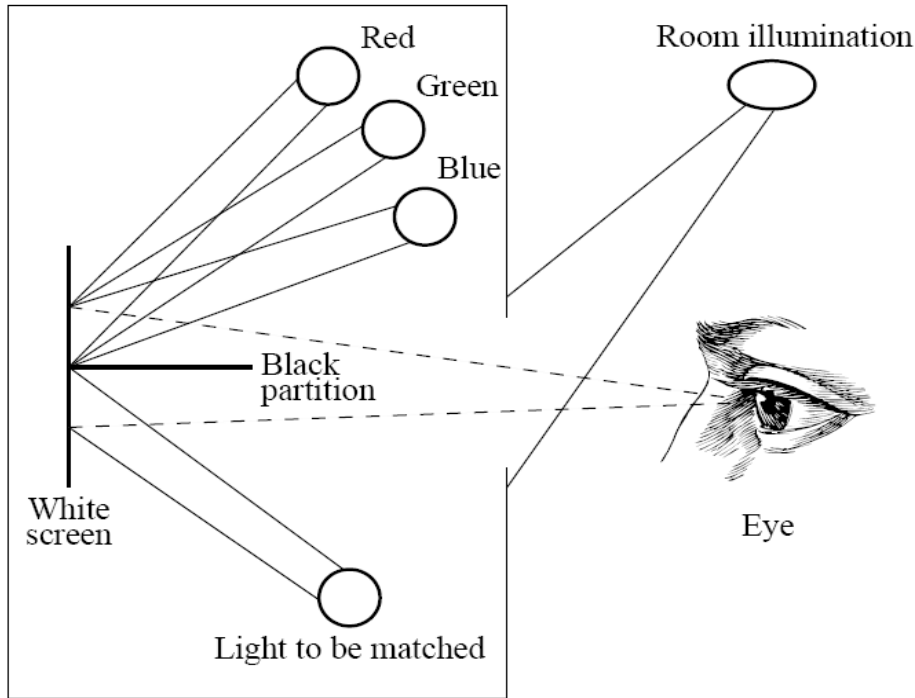


$$R = \int \bar{r}(\lambda) d\lambda$$

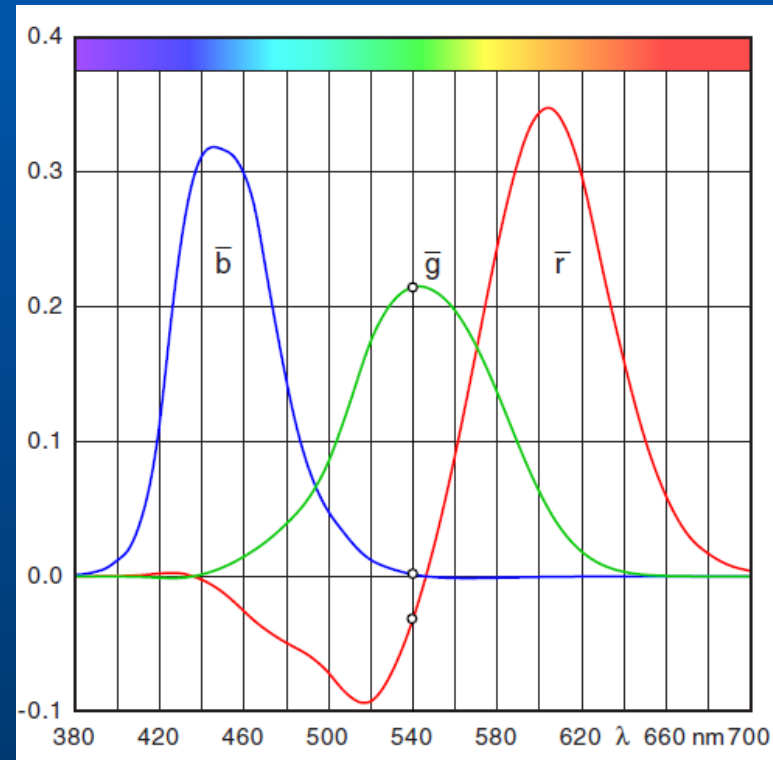
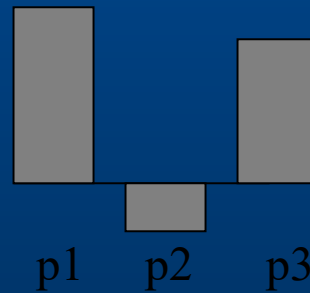
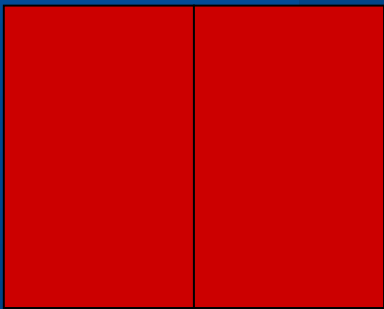
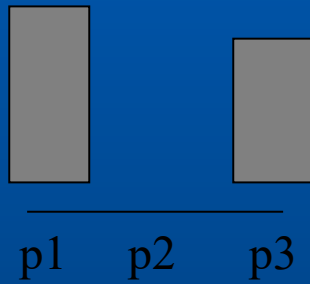
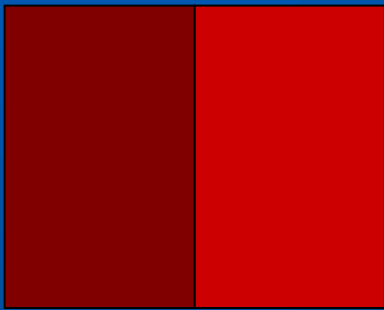
$$G = \int \bar{g}(\lambda) d\lambda$$

$$B = \int \bar{b}(\lambda) d\lambda$$

# Color matching



# Color matching

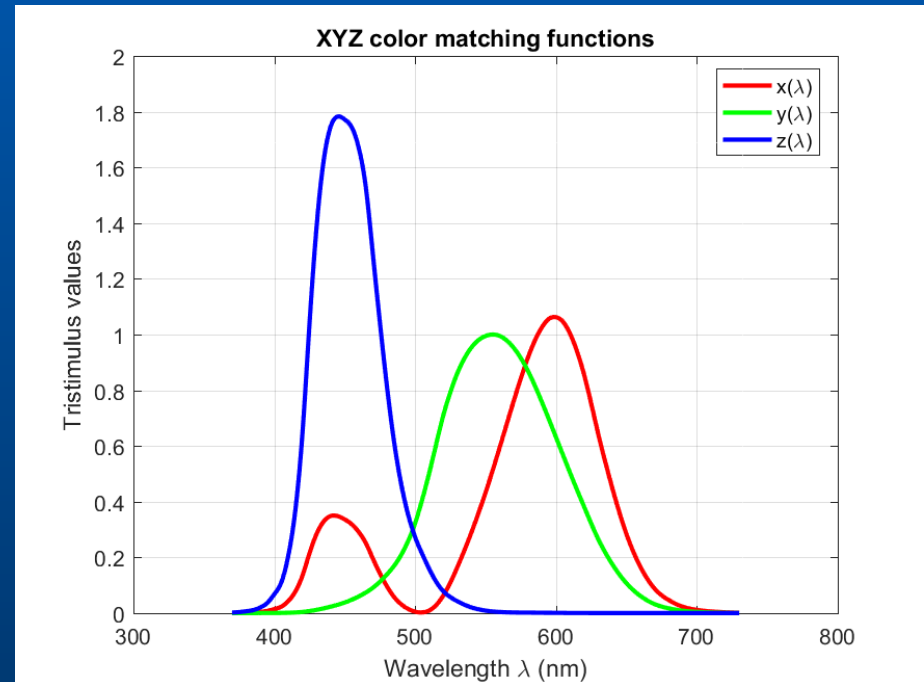


# Color system XYZ (C.I.E.)

Y : luminance

<b>X</b>	<b>0,490</b>	<b>0,310</b>	<b>0,200</b>	<b>R</b>
<b>Y</b>	<b>0,177</b>	<b>0,813</b>	<b>0,011</b>	<b>G</b>
<b>Z</b>	<b>0</b>	<b>0,010</b>	<b>0,990</b>	<b>B</b>

White :  $X=Y=Z$





# Chromaticity diagram (C.I.E.)

Trichromatic coefficients

Pure colors  
Linear mixing  
Saturated/Non-saturated

$$x = \frac{X}{X+Y+Z}$$

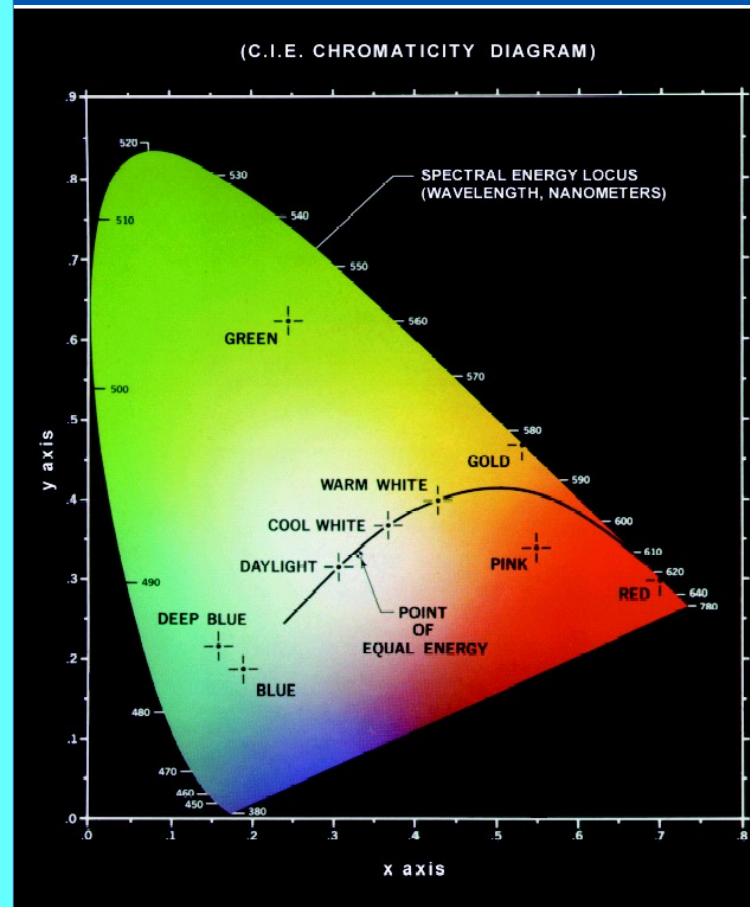
$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

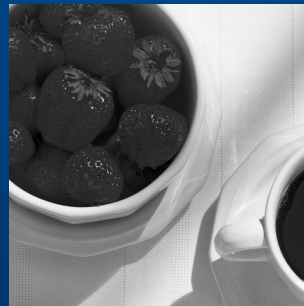
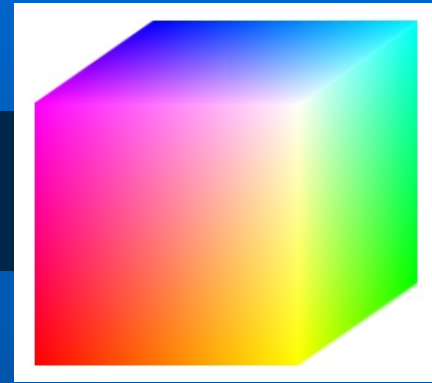
Ανάκτηση  
συνιστωσών

$$X = \frac{x}{y} Y$$

$$Z = \frac{(1-x-y)}{y} Y$$



# RGB color components



# sRGB color components

## Primary

$R_0 : x = 0.64, y = 0.33, z = 0.03$

$G_0 : x = 0.30, y = 0.60, z = 0.10$

$B_0 : x = 0.15, y = 0.06, z = 0.79$

White :  $x = 0.3127, y = 0.3290, z = 0.3583$

C.I.E. D65

$Y = 0.21 R_0 + 0.72 G_0 + 0.07 B_0$

$$R = 1.055 R_L^{(1/2.4)} - 0.055$$

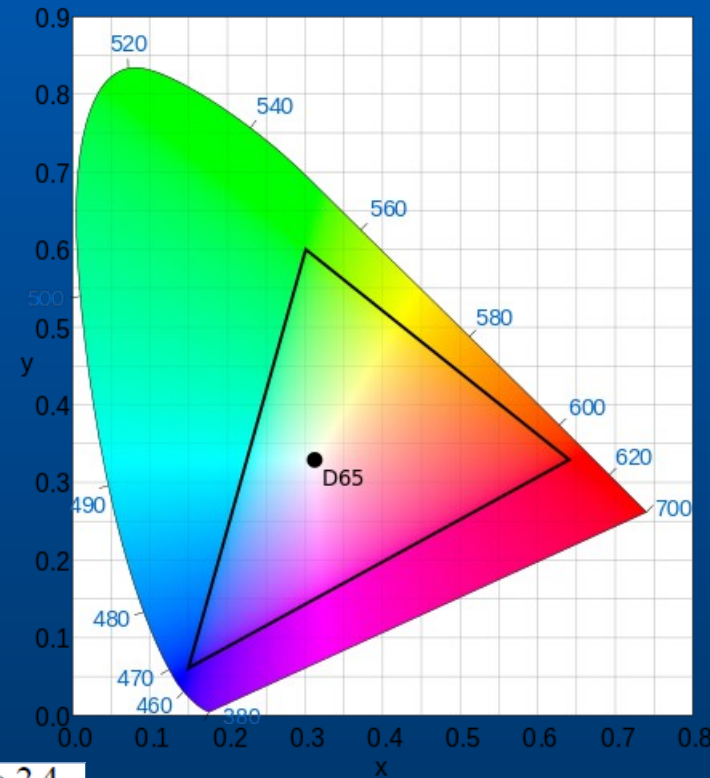
$$G = 1.055 G_L^{(1/2.4)} - 0.055$$

$$B = 1.055 B_L^{(1/2.4)} - 0.055$$

$$R_L = ((R + 0.055)/1.055)^{2.4}$$

$$G_L = ((G + 0.055)/1.055)^{2.4}$$

$$B_L = ((B + 0.055)/1.055)^{2.4}$$



# Lab color system

$$L^* = 116f\left(\frac{Y}{Y_n}\right) - 16$$

white

$$X_n, Y_n, Z_n$$

$$a^* = 500\left(f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right)$$

$$\sqrt{(a^*)^2 + (b^*)^2}$$

saturation

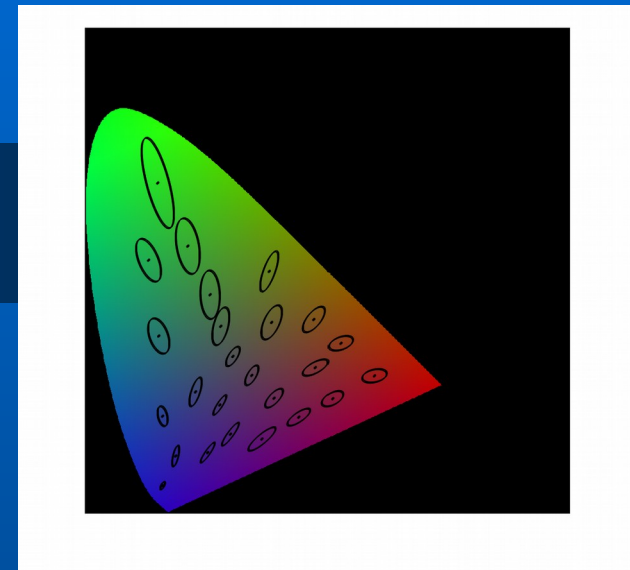
$$b^* = 200\left(f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right)$$

$$\arctan\left(\frac{a^*}{b^*}\right)$$

hue

$$f(x) = \begin{cases} x^{\frac{1}{3}} & x > 0.008856 \\ 7.787x + \frac{16}{116} & x \leq 0.008856 \end{cases}$$

Euclidean distance (just perceived distance 2,3)



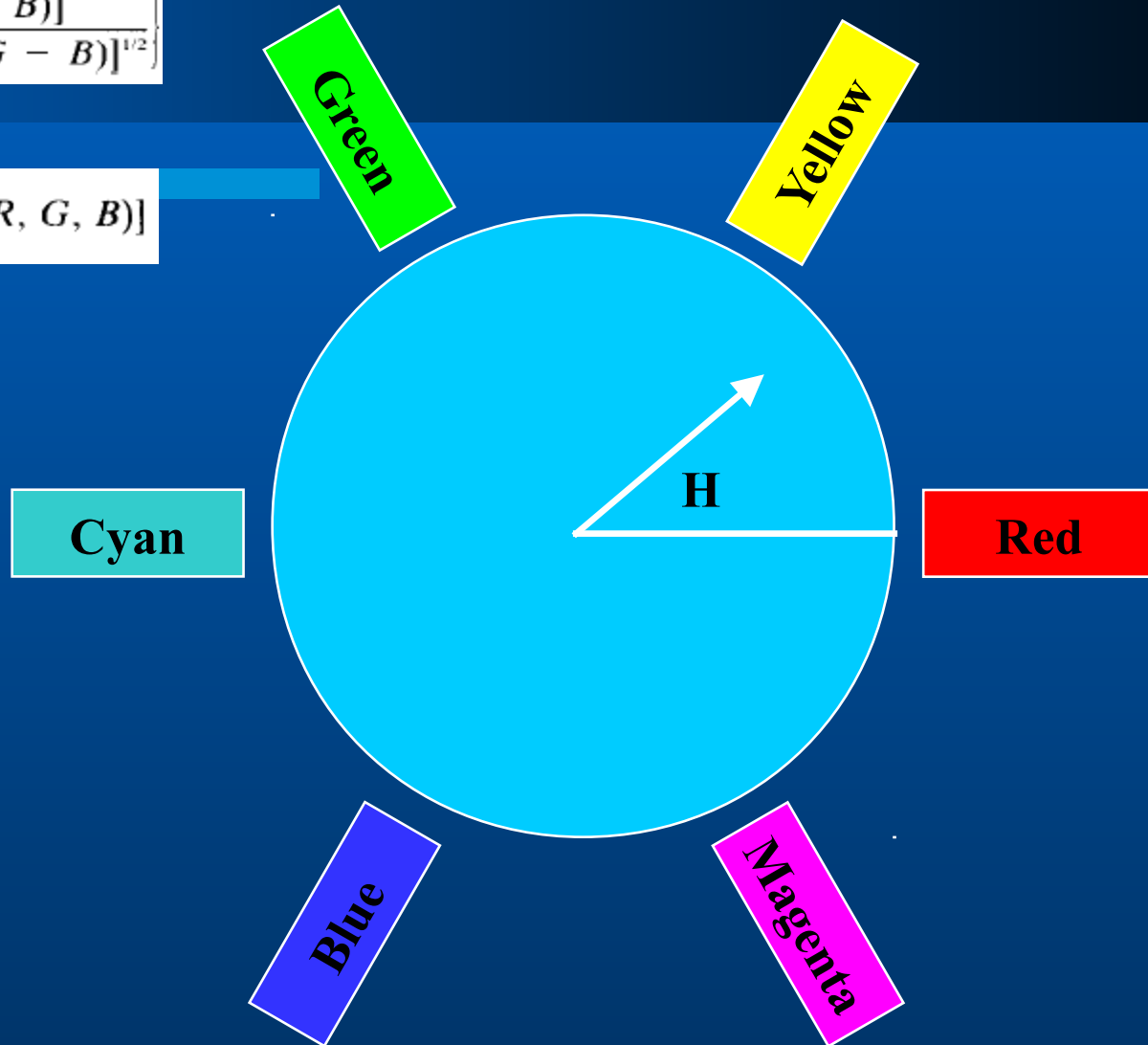
# HSL color system

$$H = \cos^{-1} \left\{ \frac{\frac{1}{2}[(R - G) + (R - B)]}{[(R - G)^2 + (R - B)(G - B)]^{1/2}} \right\}$$

$$0^\circ \leq H \leq 180^\circ$$

$$S = 1 - \frac{3}{(R + G + B)} [\min(R, G, B)]$$

$$I = \frac{1}{3}(R + G + B)$$



# YCbCr color system

$$\begin{aligned} Y &= 0,299 R + 0,587 G + 0,114 B \\ Cb &= -0,169 R - 0,331 G + 0,500 B \\ Cr &= 0,500 R - 0,419 G - 0,081 B \end{aligned}$$

Used in digital video and in standards JPEG and MPEG

Recommendation ITU-R BT.601-4

# Color quantization

**Color palette**

**Dominant colors**

**Uniform quantization (3-3-2)**

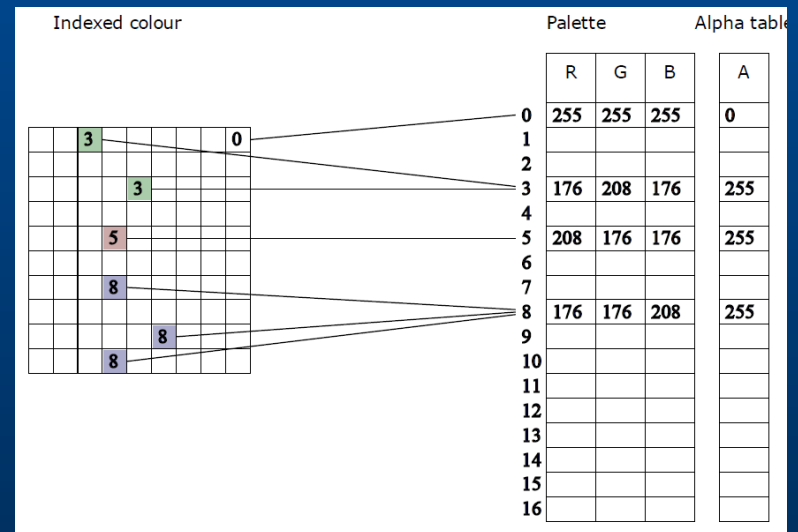
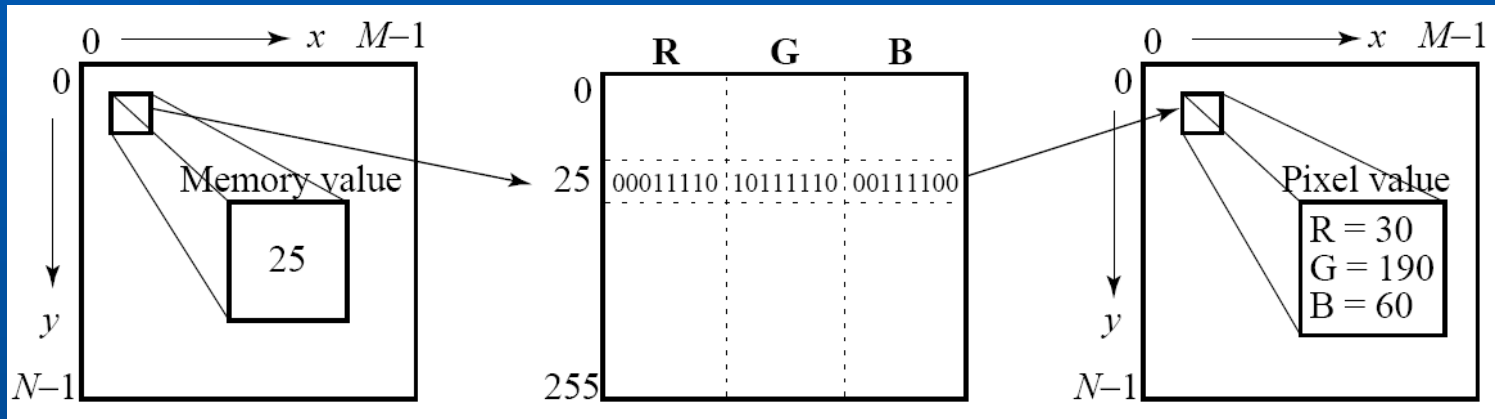
**Popular colors**

**Median cut algorithm**

**Vector quantization**

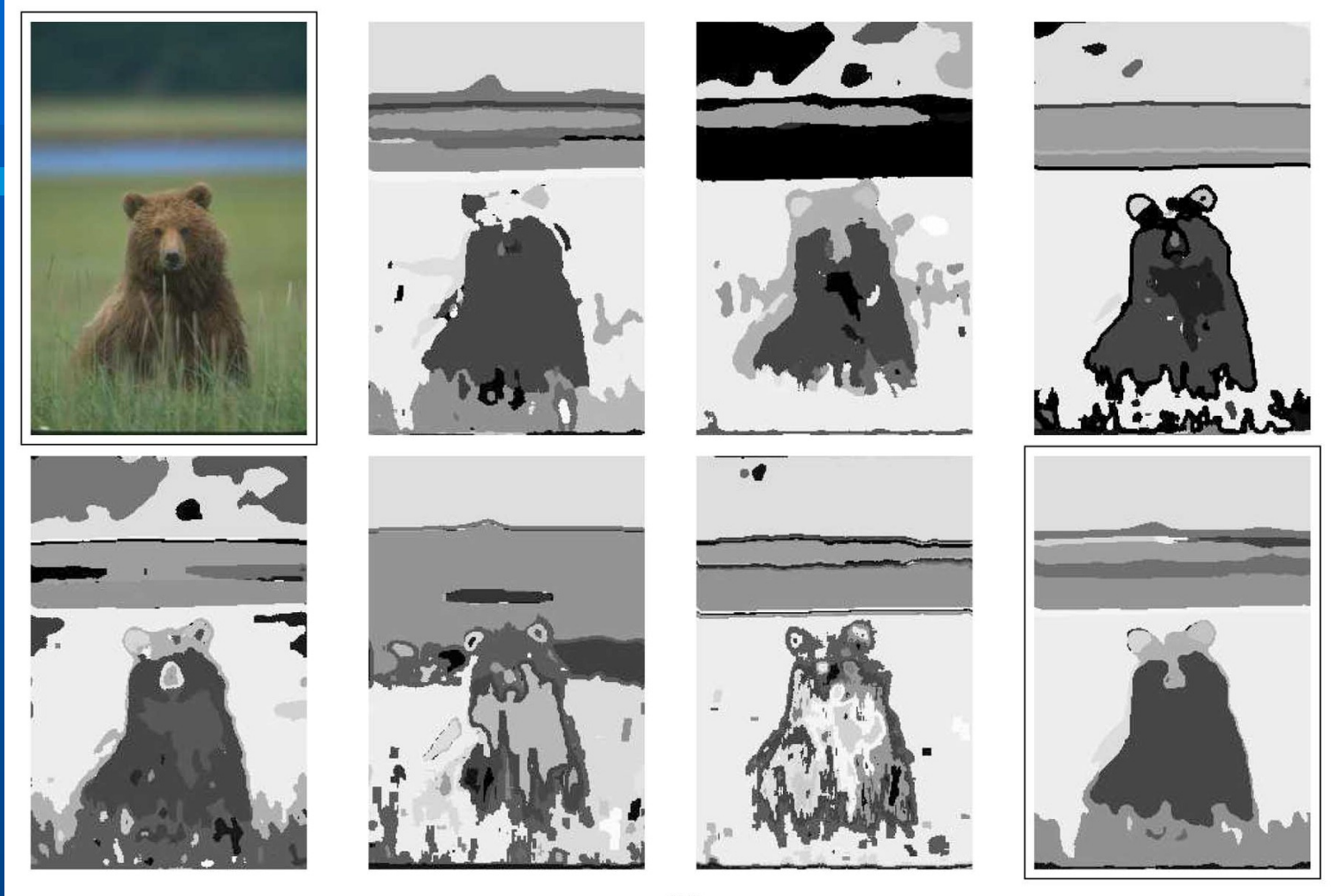
# Palette

## Look-up table





# Color quantization / color system



**RGB, HSI, YIQ, XYZ, Lab, Luv**

# Median cut algorithm

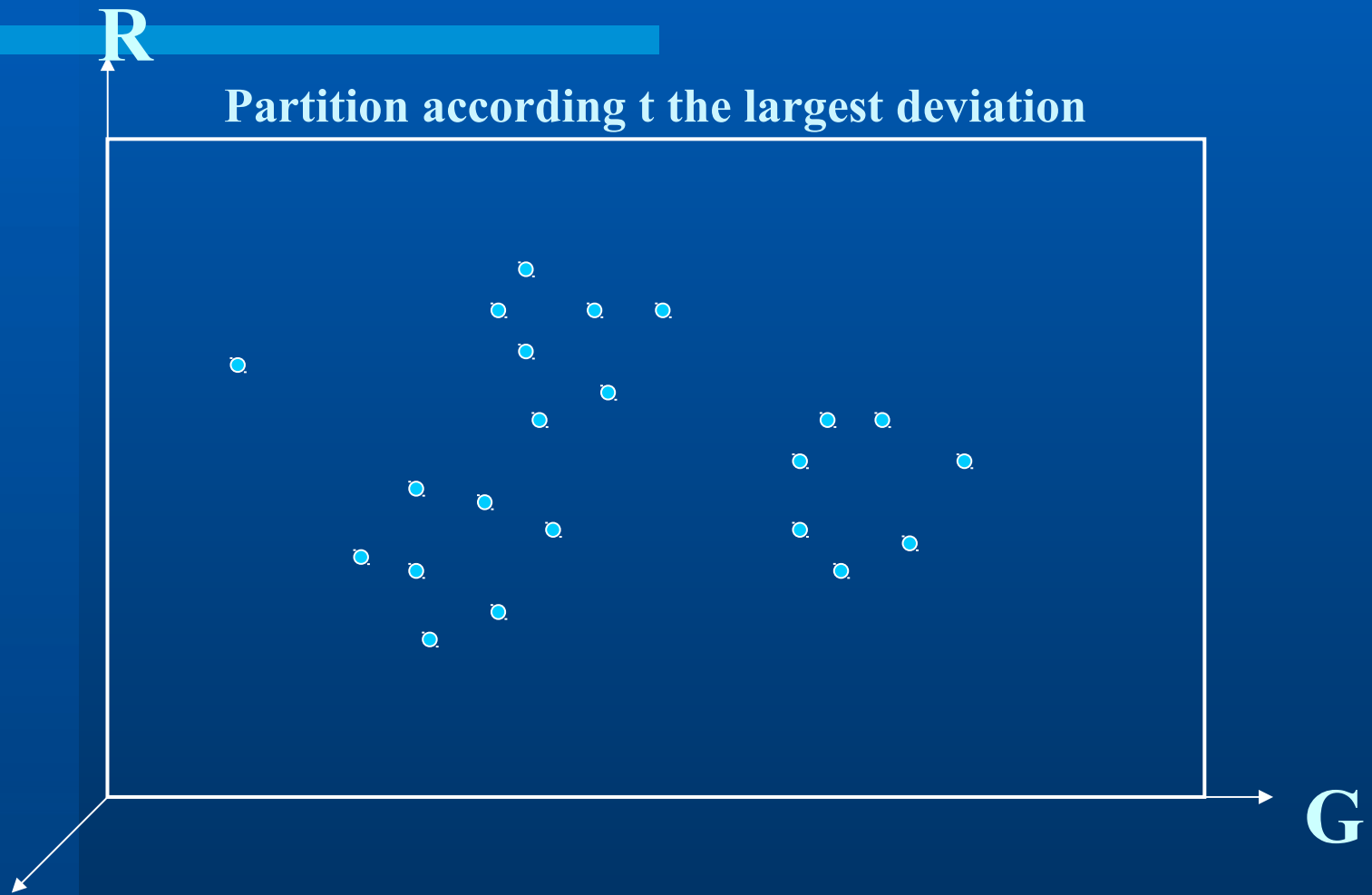
```
Color_quantization(Image, n)
{
    For each pixel in Image with color C, map C in RGB space;

    P = {RGB space partition};
    While (n-- > 0) {
        L = Heaviest (P);
        Split L into L1 and L2;
        Remove L from P, and add L1 and L2 instead;
    }

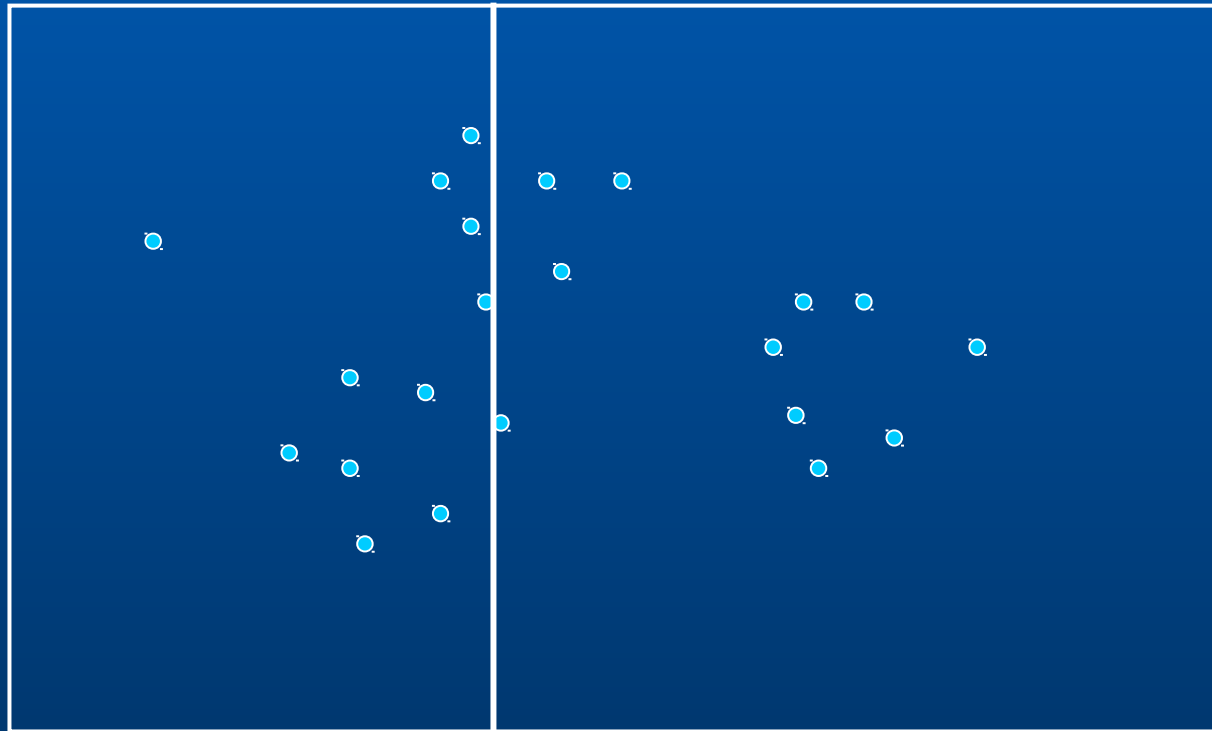
    For all boxes in B do
        assign a representative (color centroid);

    For each pixel in Image do
        map to one of the representatives;
}
```

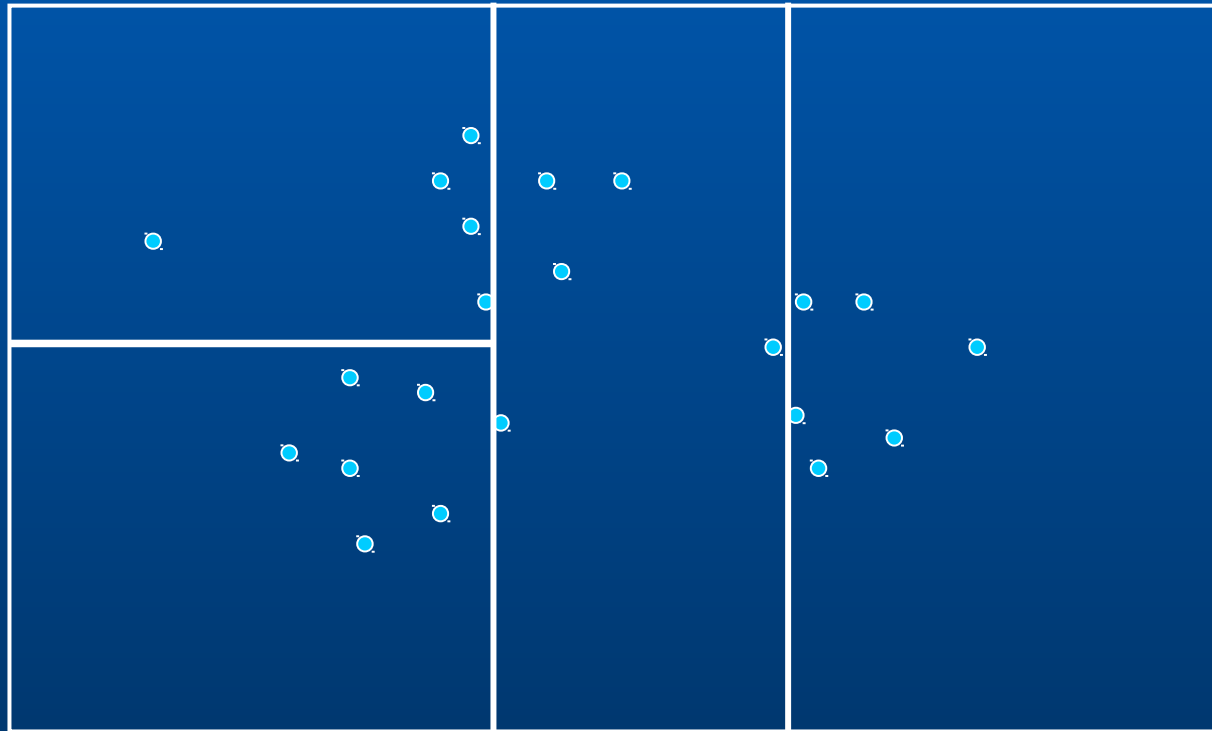
# Median cut (1/6)



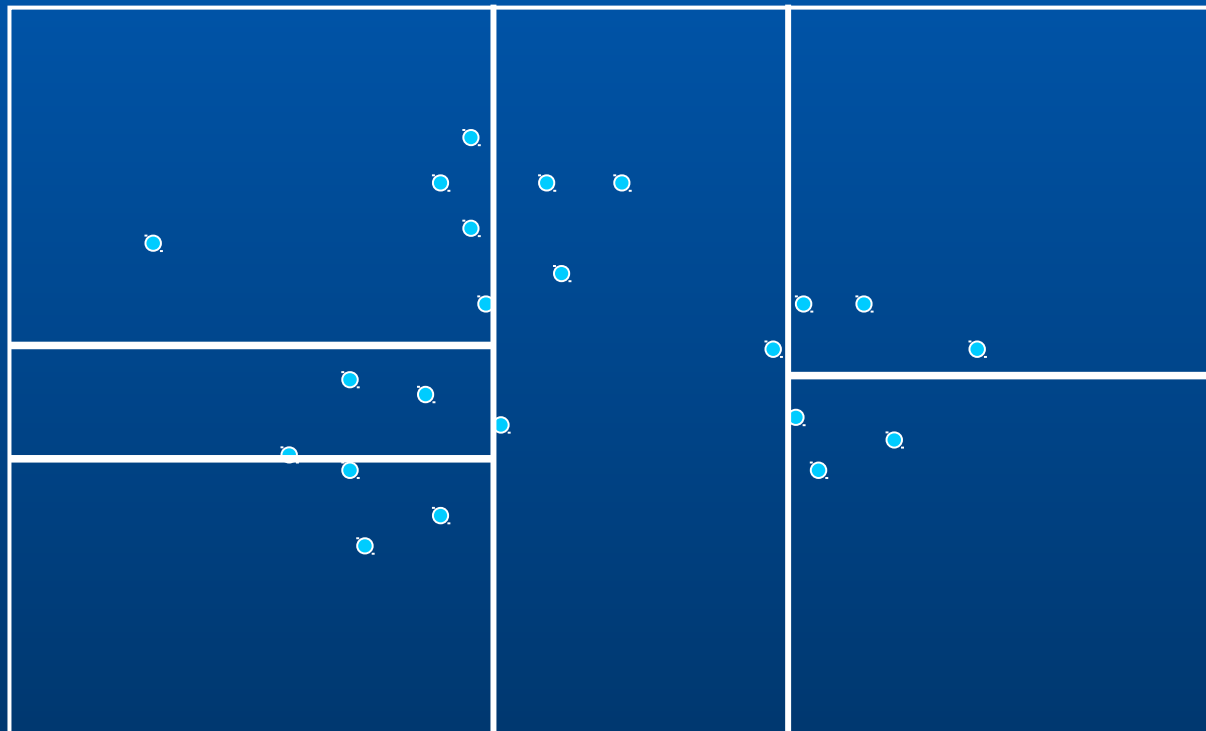
# Median cut (2/6)



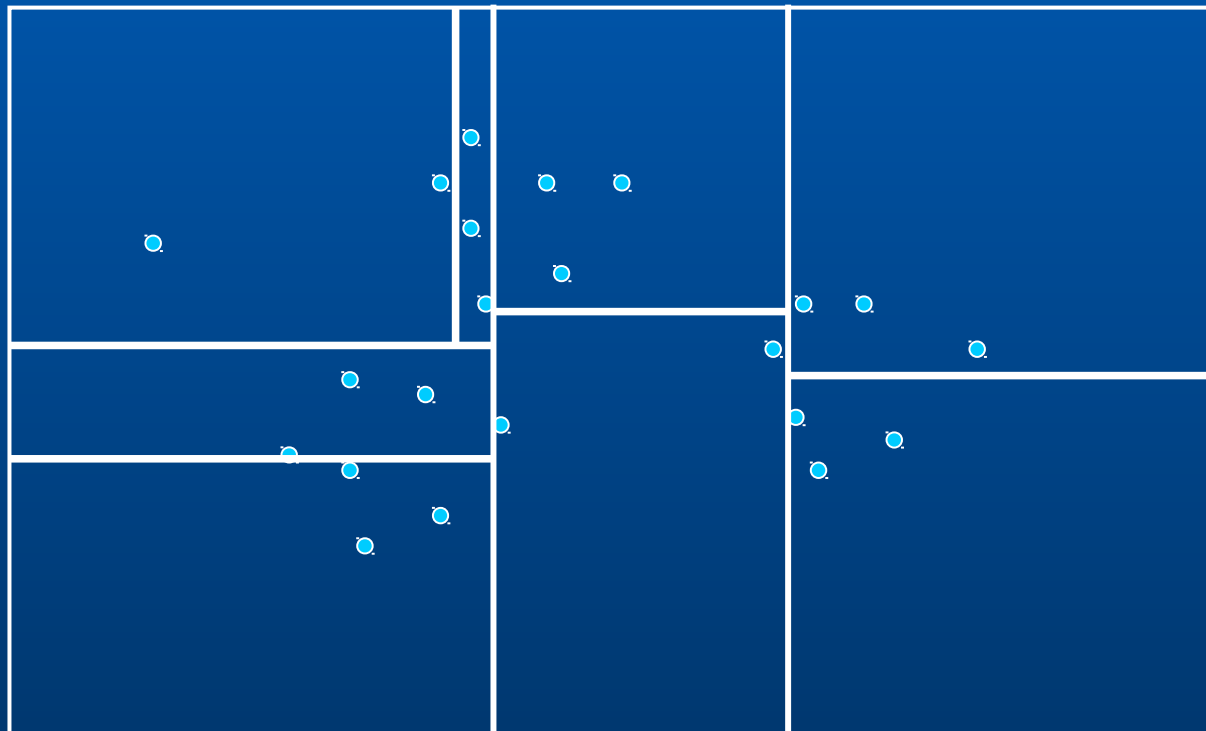
# Median cut (3/6)



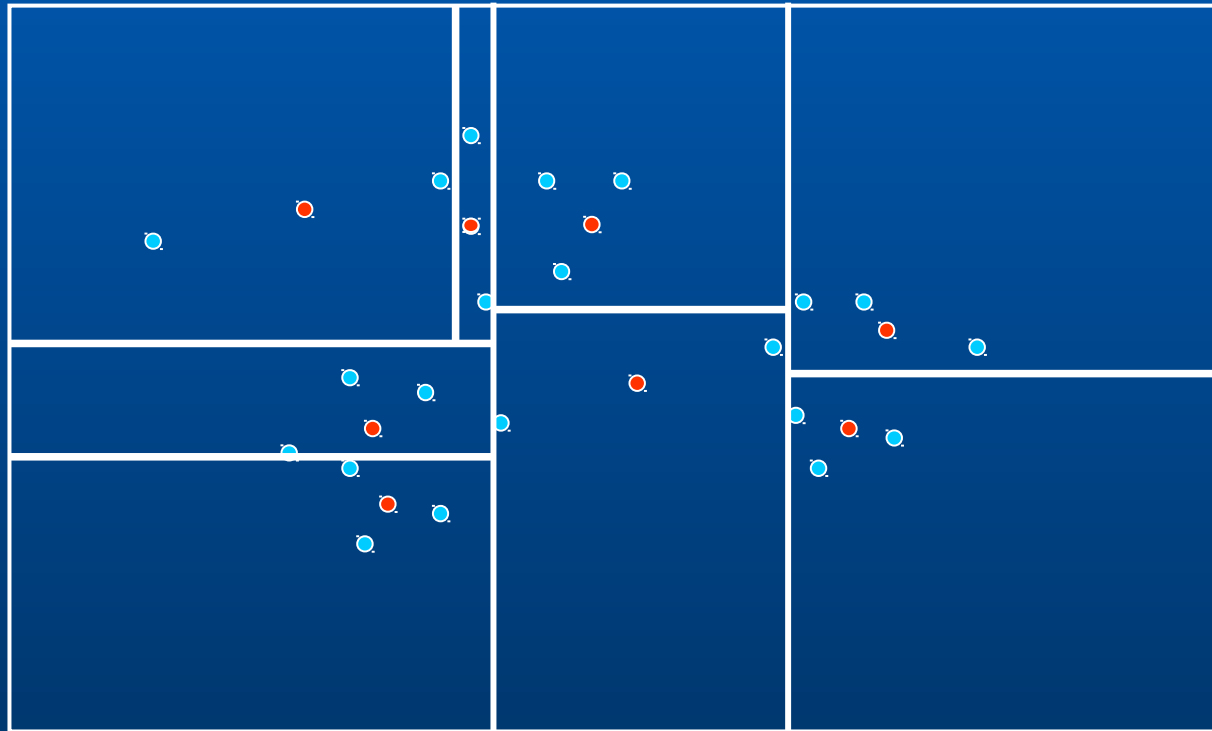
# Μεσαία τομή (4/6)



# Median cut (5/6)



# Median cut (6/6)





# Vector quantization

Minimization of the mean square quantization error

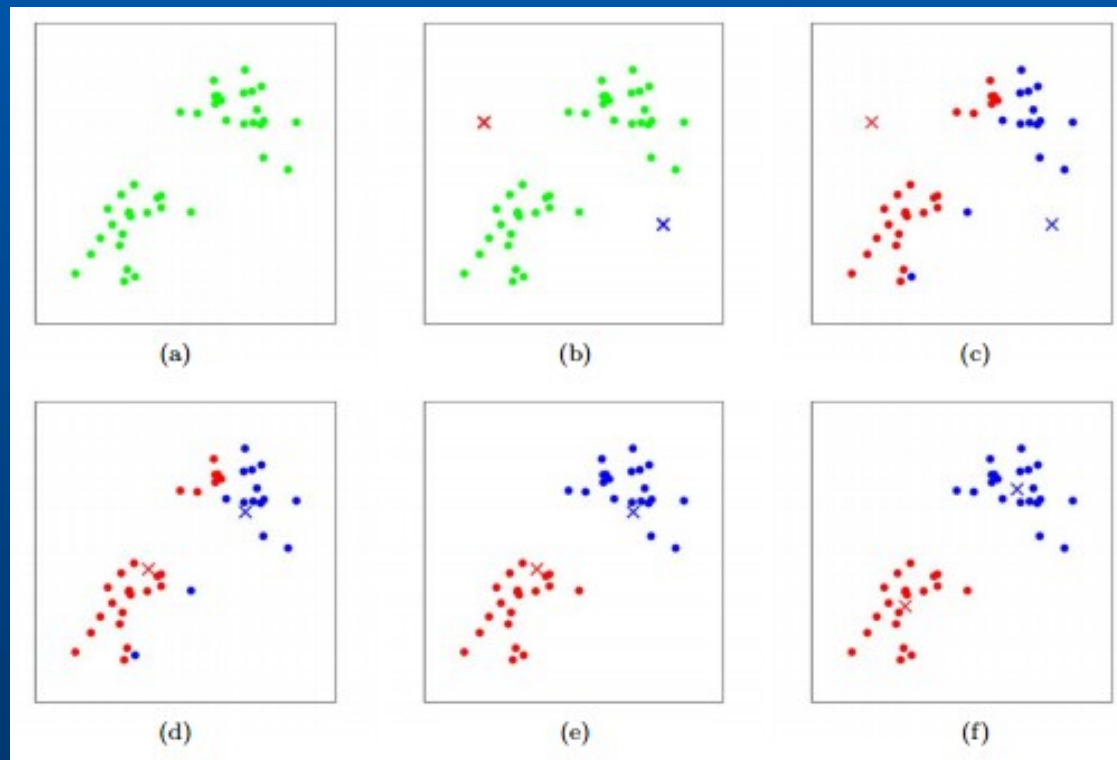
**k-means algorithm**

Necessary conditions :

**Best cluster representative = cluster centroid**

**Best partition = minimization of distance to centroid**

# K-means algorithm



# Named colors (Web)

	<i>aliceblue</i>	#F0F8FF	240,248,255		<i>deeppink</i>	#FF1493	255,20,147		<i>lightslategrey</i>	#778899	119,136,153
	<i>antiquewhite</i>	#FAEBD7	250,235,215		<i>deepskyblue</i>	#00BFFF	0,191,255		<i>lightsteelblue</i>	#B0C4DE	176,196,222
	<i>aqua</i>	#00FFFF	0,255,255		<i>dimgray</i>	#696969	105,105,105		<i>lightyellow</i>	#FFFFE0	255,255,224
	<i>aquamarine</i>	#7FFFD4	127,255,212		<i>dimgrey</i>	#696969	105,105,105		<i>lime</i>	#00FF00	0,255,0
	<i>azure</i>	#F0FFFF	240,255,255		<i>dodgerblue</i>	#1E90FF	30,144,255		<i>limegreen</i>	#32CD32	50,205,50
	<i>beige</i>	#F5F5DC	245,245,220		<i>firebrick</i>	#B22222	178,34,34		<i>linen</i>	#FAF0E6	250,240,230
	<i>bisque</i>	#FFE4C4	255,228,196		<i>floralwhite</i>	#FFFAF0	255,250,240		<i>magenta</i>	#FF00FF	255,0,255
	<i>black</i>	#000000	0,0,0		<i>forestgreen</i>	#228B22	34,139,34		<i>maroon</i>	#800000	128,0,0
	<i>blanchedalmond</i>	#FFEB3D	255,235,205		<i>fuchsia</i>	#FF00FF	255,0,255		<i>mediumaquamarine</i>	#66CDAA	102,205,170
	<i>blue</i>	#0000FF	0,0,255		<i>gainsboro</i>	#DCDCDC	220,220,220		<i>mediumblue</i>	#0000CD	0,0,205
	<i>blueviolet</i>	#8A2BE2	138,43,226		<i>ghostwhite</i>	#F8F8FF	248,248,255		<i>mediumorchid</i>	#BA55D3	186,85,211
	<i>brown</i>	#A52A2A	165,42,42		<i>gold</i>	#FFD700	255,215,0		<i>mediumpurple</i>	#9370DB	147,112,219
	<i>burlywood</i>	#DEB887	222,184,135		<i>goldenrod</i>	#DAA520	218,165,32		<i>mediumseagreen</i>	#3CB371	60,179,113
	<i>cadetblue</i>	#5F9EA0	95,158,160		<i>gray</i>	#808080	128,128,128		<i>mediumslateblue</i>	#7B68EE	123,104,238
	<i>chartreuse</i>	#7FFF00	127,255,0		<i>green</i>	#008000	0,128,0		<i>mediumspringgreen</i>	#00FA9A	0,250,154
	<i>chocolate</i>	#D2691E	210,105,30		<i>greenyellow</i>	#ADFF2F	173,255,47		<i>mediumturquoise</i>	#48D1CC	72,209,204
	<i>coral</i>	#FF7F50	255,127,80		<i>grey</i>	#808080	128,128,128		<i>mediumvioletred</i>	#C71585	199,21,133
	<i>cornflowerblue</i>	#6495ED	100,149,237		<i>honeydew</i>	#F0FF00	240,255,240		<i>midnightblue</i>	#191970	25,25,112
	<i>cornsilk</i>	#FFF8DC	255,248,220		<i>hotpink</i>	#FF69B4	255,105,180		<i>mintcream</i>	#F5FFFA	245,255,250
	<i>crimson</i>	#DC143C	220,20,60		<i>indianred</i>	#CD5C5C	205,92,92		<i>mistyrose</i>	#FFE4E1	255,228,225
	<i>cyan</i>	#00FFFF	0,255,255		<i>indigo</i>	#4B0082	75,0,130		<i>moccasin</i>	#FFE4B5	255,228,181
	<i>darkblue</i>	#00008B	0,0,139		<i>ivory</i>	#FFFF00	255,255,240		<i>navajowhite</i>	#FFDEAD	255,222,173
	<i>darkcyan</i>	#008B8B	0,139,139		<i>khaki</i>	#F0E68C	240,230,140		<i>navy</i>	#000080	0,0,128
	<i>darkgoldenrod</i>	#B8860B	184,134,11		<i>lavender</i>	#E6E6FA	230,230,250		<i>oldlace</i>	#FDF5E6	253,245,230
	<i>darkgray</i>	#A9A9A9	169,169,169		<i>lavenderblush</i>	#FFF0F5	255,240,245		<i>olive</i>	#808000	128,128,0
	<i>darkgreen</i>	#006400	0,100,0		<i>lawngreen</i>	#7CFC00	124,252,0		<i>olivedrab</i>	#6B8E23	107,142,35
	<i>darkgrey</i>	#A9A9A9	169,169,169		<i>lemonchiffon</i>	#FFFACD	255,250,205		<i>orange</i>	#FFA500	255,165,0
	<i>darkkhaki</i>	#BDB76B	189,183,107		<i>lightblue</i>	#ADD8E6	173,216,230		<i>orangered</i>	#FF4500	255,69,0
	<i>darkmagenta</i>	#8B008B	139,0,139		<i>lightcoral</i>	#F08080	240,128,128		<i>orchid</i>	#DA70D6	218,112,214
	<i>darkolivegreen</i>	#556B2F	85,107,47		<i>lightcyan</i>	#E0FFFF	224,255,255		<i>palegoldenrod</i>	#EEE8AA	238,232,170
	<i>darkorange</i>	#FF8C00	255,140,0		<i>lightgoldenrodyellow</i>	#FAFAD2	250,250,210		<i>palegreen</i>	#98FB98	152,251,152
	<i>darkorchid</i>	#9932CC	153,50,204		<i>lightgray</i>	#D3D3D3	211,211,211		<i>paleturquoise</i>	#AFEEEE	175,238,238
	<i>darkred</i>	#8B0000	139,0,0		<i>lightgreen</i>	#90EE90	144,238,144		<i>palevioletred</i>	#DB7093	219,112,147
	<i>darksalmon</i>	#E9967A	233,150,122		<i>lightgrey</i>	#D3D3D3	211,211,211		<i>papayawhip</i>	#FFEDF5	255,239,213
	<i>darkseagreen</i>	#8FBC8F	143,188,143		<i>lightpink</i>	#FFB6C1	255,182,193		<i>peachpuff</i>	#FFDAB9	255,218,185
	<i>darkslateblue</i>	#483D8B	72,61,139		<i>lightsalmon</i>	#FFA07A	255,160,122		<i>peru</i>	#CD853F	205,133,63
	<i>darkslategrey</i>	#2F4F4F	47,79,79		<i>lightseagreen</i>	#20B2AA	32,178,170		<i>pink</i>	#FFC0CB	255,192,203
	<i>darkslategrey</i>	#2F4F4F	47,79,79		<i>lightskyblue</i>	#87CEFA	135,206,250		<i>plum</i>	#DDA0DD	221,160,221
	<i>darkturquoise</i>	#00CED1	0,206,209		<i>lightslategray</i>	#778899	119,136,153		<i>powderblue</i>	#B0E0E6	176,224,230
	<i>darkviolet</i>	#9400D3	148,0,211						<i>purple</i>	#800080	128,0,128
					<i>red</i>	#FF0000	255,0,0		<i>rosybrown</i>	#BC8F8F	188,143,143
					<i>royalblue</i>	#4169E1	65,105,225		<i>saddlebrown</i>	#8B4513	139,69,19
					<i>salmon</i>	#FA8072	250,128,114		<i>sandybrown</i>	#F4A460	244,164,96
					<i>seagreen</i>	#2E8B57	46,139,87		<i>seashell</i>	#FFF5EE	255,245,238
					<i>sienna</i>	#A0522D	160,82,45		<i>silver</i>	#C0C0C0	192,192,192
					<i>skyblue</i>	#87CEEB	135,206,235		<i>slateblue</i>	#6A5ACD	106,90,205
					<i>slategray</i>	#708090	112,128,144		<i>slategrey</i>	#708090	112,128,144
					<i>snow</i>	#FFFAFA	255,250,250		<i>springgreen</i>	#00FF7F	0,255,127
					<i>steelblue</i>	#4682B4	70,130,180		<i>tan</i>	#D2B48C	210,180,140
					<i>teal</i>	#008080	0,128,128		<i>thistle</i>	#D8BFD8	216,191,216
					<i>tomato</i>	#FF6347	255,99,71		<i>turquoise</i>	#40E0D0	64,224,208
					<i>violet</i>	#EE82EE	238,130,238		<i>wheat</i>	#F5DEB3	245,222,179
					<i>white</i>	#FFFFFF	255,255,255		<i>whitesmoke</i>	#F5F5F5	245,245,245
					<i>yellow</i>	#FFFF00	255,255,0		<i>yellowgreen</i>	#9ACD32	154,205,50

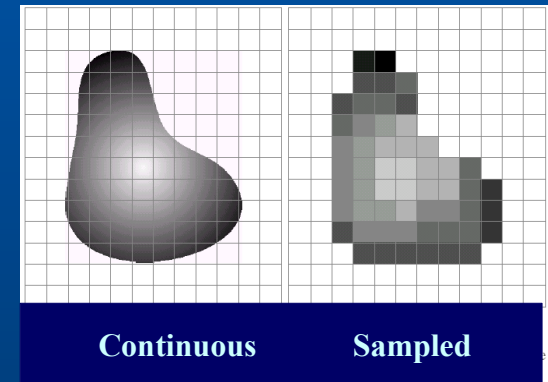
# Image digitalization

- Sampling

periodic, rectangular (pixel = picture element)  
samples' density related to image details (ppi)

- Quantization

discrete values, uniform quantization  
(256 values or 8 bits)



# Sampling (1/3)

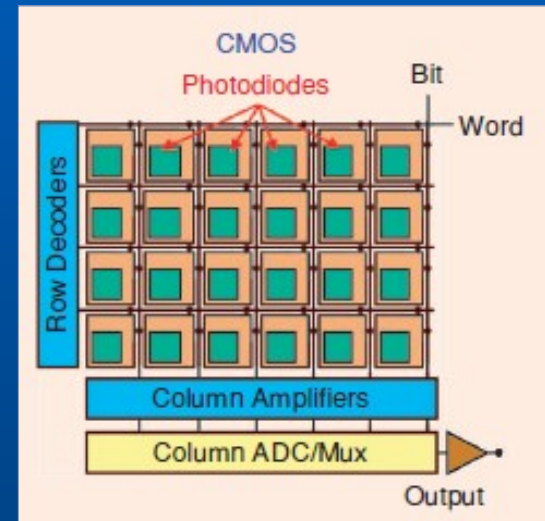
- sampling
- scanning
- sensors

electrical signal proportional to the luminance

CCD (charge-coupled device)

CMOS (complementary metal-oxide semiconductor)

- Pixel size
- Digital image size



# Sampling (2/3)

Resolution depends on image signal content :  
more details in the image, more dense are  
the samples.

Otherwise, sampling results in distortion.

If the maximum resolution is not needed,  
details are filtered and sampling density  
is adapted to the desired resolution.



a b c

**FIGURE 2.22** (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

# Sampling (3/3)

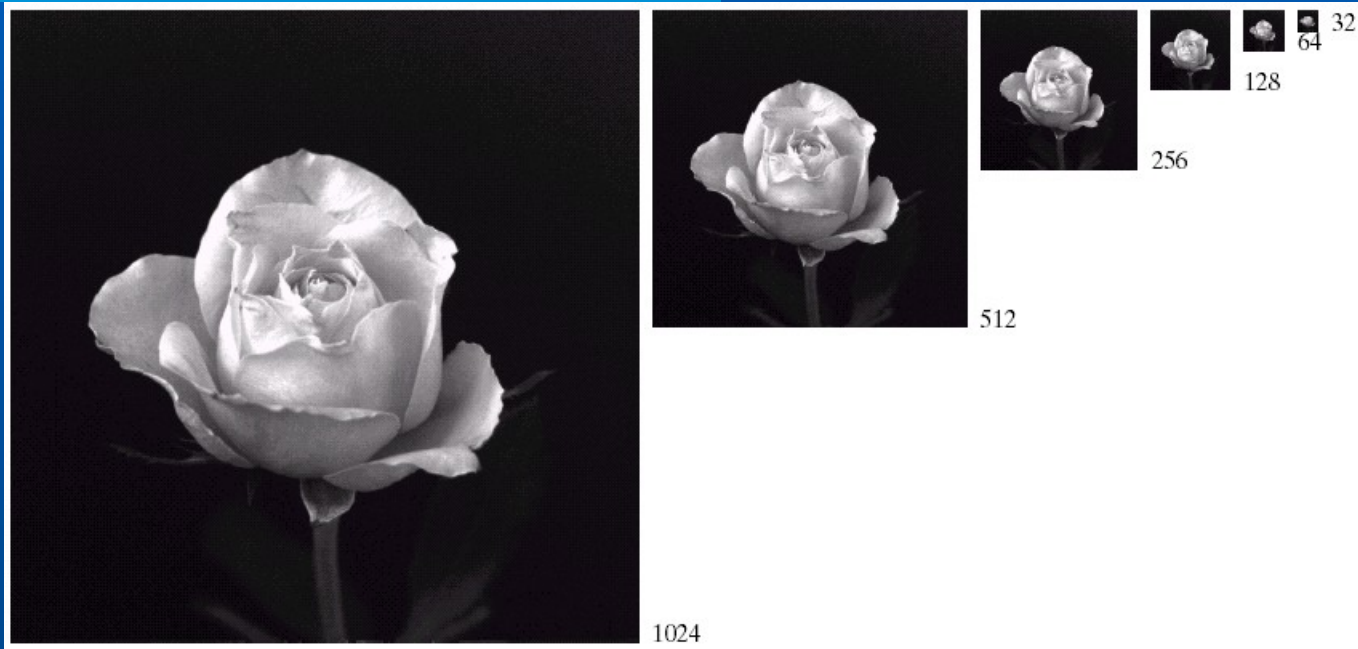


72 ppi



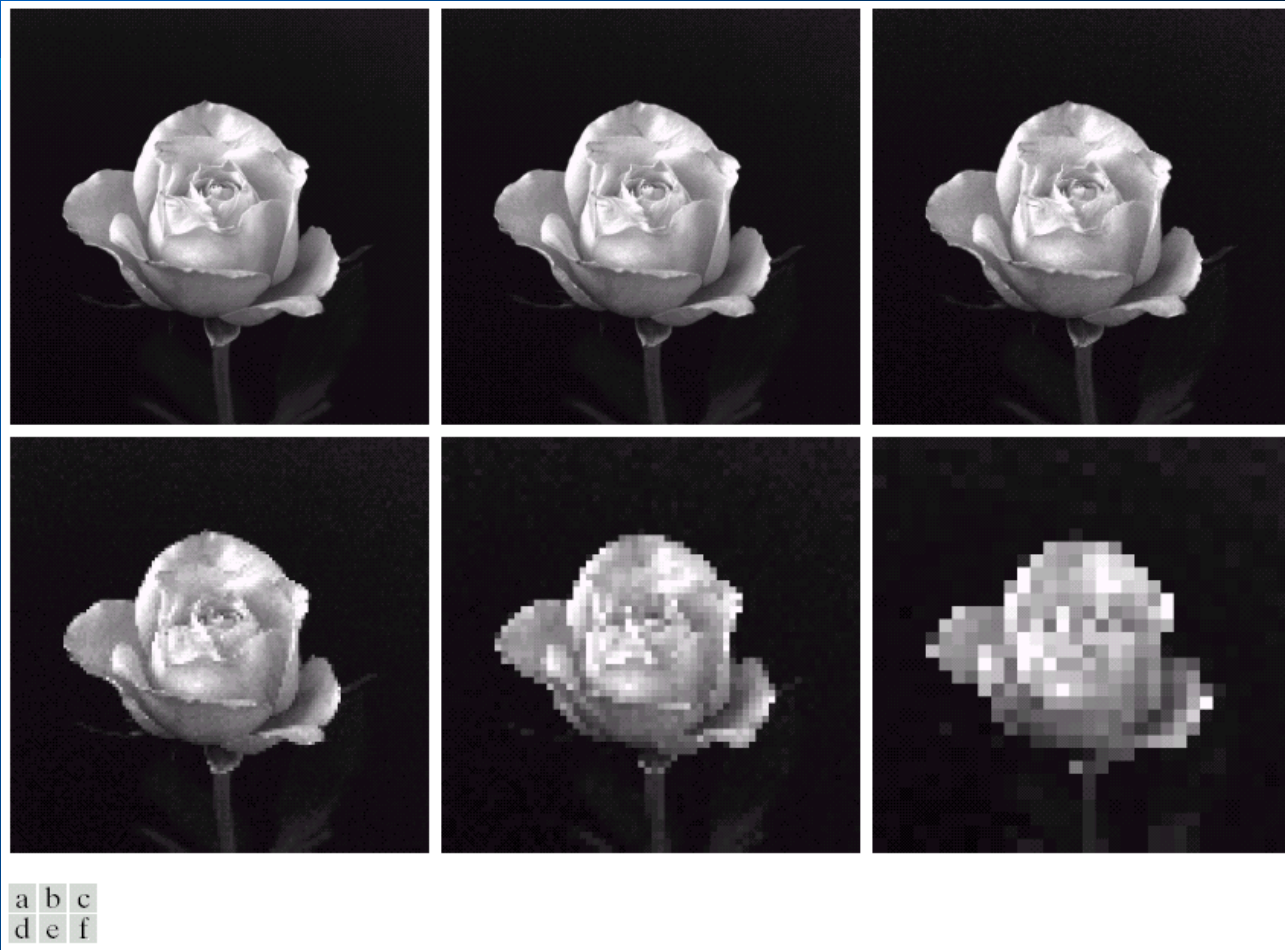
300 ppi

# Subsampling / resize





# Subsampling / reconstruction



# Subsampling / reconstruction



Nearest

Bilinear  
interpolation

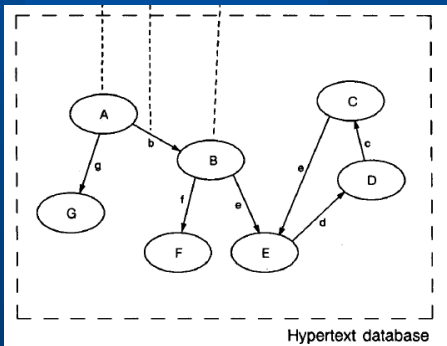
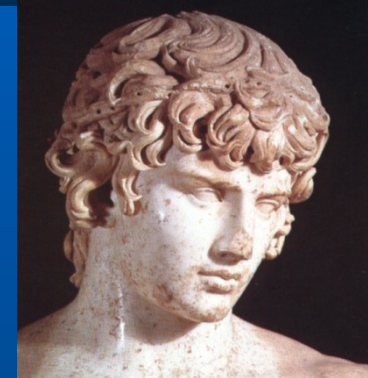
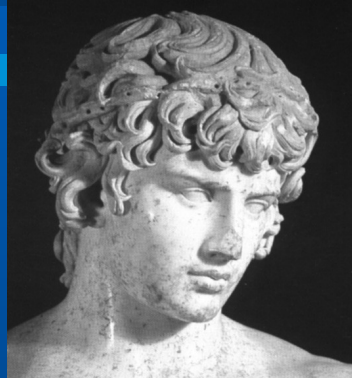
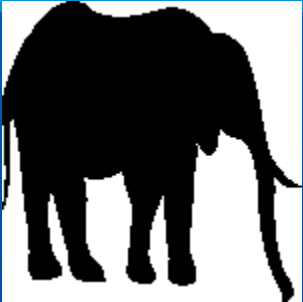
128

64

32

a	b	c
d	e	f

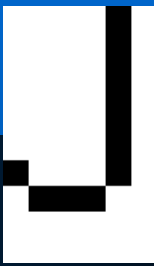
# Data types



Binary images

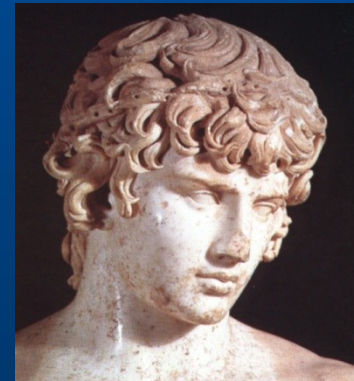
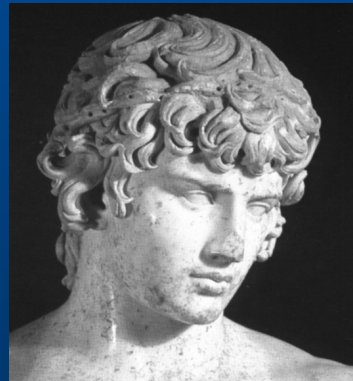
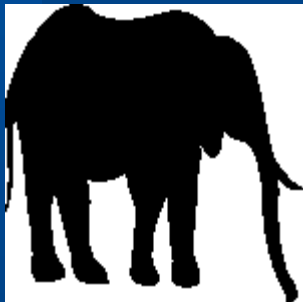
Gray scale images

Color images



# Portable PBM / PGM / PPM

PBM	binary images	1
PGM	Gray scale images	8, 16, 1-16
PPM	Color images	1-16 / component



# Graphics Interchange Format (GIF)

Images up to 8 bits/pixel

Compression algorithm Lempel-Ziv-Welch

Transparency is possible

Animation is possible



# Portable Network Graphics (PNG)

- Grayscale images 1, 2, 4, 8, 16 bits/pixel
- Grayscale images with transparency 16, 32 bits/pixel
- Color images with palette 1, 2, 4, 8 bits/pixel
- True color images 24, 48 bits/pixel
- True color images with transparency 32, 64 bits/pixel

Compression algorithm Lempel-Ziv (1977) and Huffman

Animation is possible (MNG)

# Windows bitmap (BMP)

- Grayscale images 1, 4, 8 bits/pixel
- Color images with palette 1, 4, 8 bits/pixel
- True color images 16 (=5+6+5), 24 bits/pixel
- True color images with transparence 32 bits/pixel

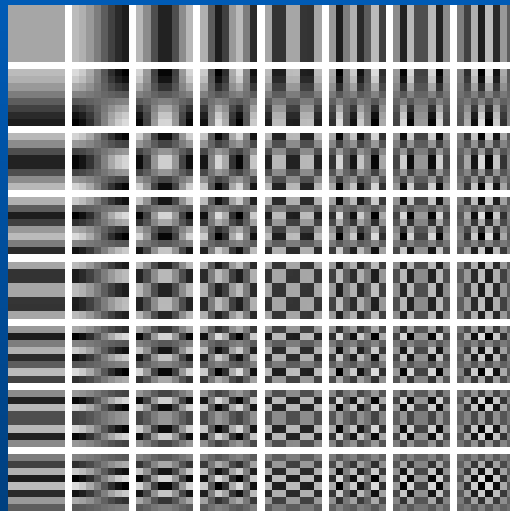
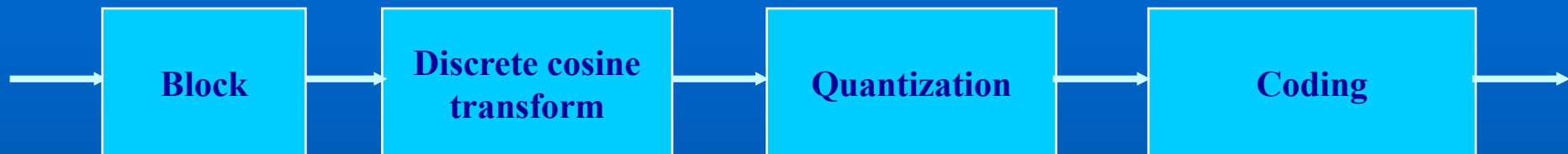
Compression option (RLE)

# Joint Photographic Experts Group (JPEG)

Lossless or lossy compression

Chromatic system YCbCr

Subsampling of chromatic components



Quantization  
according to  
human visual system



# Scanner



**Moving optical array CCD**  
**Tri-chromatic (filters RGB)**

**Optical resolution (ppi)**

**Scaling**

**Color depth**

**Optical character recognition**

<b>Resolution</b>	<b>Usage</b>
<b>75 ppi</b>	<b>Display, Web</b>
<b>100 ppi</b>	<b>Printer 300 dpi</b>
<b>150 ppi</b>	<b>Printer 600 dpi</b>
<b>300 ppi</b>	<b>Printer 1200 dpi</b>

# Digital camera

CCD or CMOS sensors (RGB)

Typical resolution

1280 x 1024

2048 x 1536

3072 x 2048

3520 x 2344

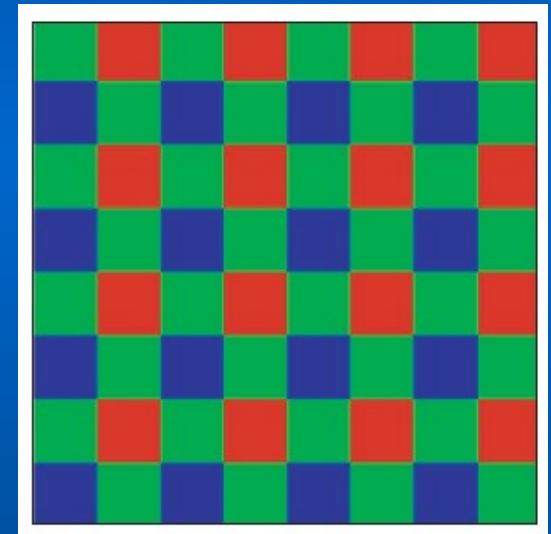
4256 x 2832

4992 x 3328

Pixel aspect

4:3 ή 3:2

Anti-aliasing filter



**Compression JPEG**