Optimizing throughput, fairness and/or interference in cognitive radio networks

Due to the existence of limited resources, the cognitive radio networks should utilize the available spectrum in an efficient manner. This means that they should be able to achieve as high data rates as possible using a particular frequency band. From the other side, it is also important that all users within a network would have the same opportunities to access the spectrum. That way the distribution of resources among users would be fair. Another important issue that should be also taken into account is the amount of interference which is introduced to the licensed system. That amount should be less than a threshold and the probability of exceeding that threshold should be kept very small.

There are many approaches in the literature which propose a method to satisfy the above requirements. Most of them define an optimization problem with a utility function that depends on the spectral efficiency, fairness and/or interference to the licensed system. The parameters of the optimization problem could vary from one approach to the other. Examples of such parameters are power control, beamforming, channel allocation, interference cancellation and adaptive modulation. A description of each one of them is given in the following section.

A. Power control

The value of SINR at the receiver of a system is vital for a reliable communication to occur. In most cases it is required that this value should be above a certain threshold for the receiver to be able to decode the signal of the transmitter. Power control is a method that is used to increase the value of SINR if it is too low or decrease it if it is too high. This can be done by appropriate adjustment of transmission powers. That way the quality of weak links is greatly improved. In other words, the goal of power control is to minimize the overall power that is required to satisfy the SINR requirements of all links within a network.

An analytical description of the power control problem is given in [8]. In particular, a set of M transmitter–receiver pairs which share the same channel is considered. The link gain between transmitter i and receiver j is denoted by $G_{ij}$, and the ith transmitter power by $P_i$. For an isotropic antenna with unity gain in all directions, the signal power received at receiver i from transmitter j is $G_{ji}P_j$. It is assumed that transmitter i communicates with receiver i. Hence, the desired signal at receiver i is equal to $G_{ii}P_i$, while the interfering signal power from other transmitters to receiver i is:

$$I_i = \sum_{j \neq i} G_{ji}P_j$$

If we also consider thermal noise power, the SINR at the ith receiver is expressed as:

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{j \neq i} G_{ji}P_j + N_i}$$

(2)
The quality of a link is acceptable if $\Gamma_i$ is above a certain threshold $\gamma_0$, the minimum protection ratio, thus:

$$\Gamma_i > \gamma_0, 1 \leq i \leq M \text{ or in matrix form } [I - \gamma_0F]P \geq u \quad (3)$$

Where $P = [P_1, P_2, \ldots, P_M]^T$ is the power vector, $u$ is an element wise vector with elements $u_i = \gamma_0N_i/G_{ii}, 1 \leq i \leq M$ and $F$ is a nonnegative matrix defined as:

$$[F]_{ij} = 0 \text{ if } j=i \quad (4)$$

$$[F]_{ij} = G_{ji}/G_{ii} \text{ if } j \neq i$$

The power control problem can be now formally defined as follows:

$$\text{minimize } \sum_i P_i \quad (5)$$

subject to $[I - \gamma_0F]P \geq u$

It can be shown that, if the spectral radius of $F$ is less than $1/\gamma_0$, the matrix $I - \gamma_0F$ is invertible and positive. In this case, the power vector $P_{opt} = [I - \gamma_0F]^{-1} u$ solves the optimization problem. The solution to the power control problem can also be obtained by performing the following iterations:

$$P_i^{n+1} = \frac{\gamma_0}{G_{ii}} \left( \sum_{j \neq i} G_{ji} P_j^{n} + N_i \right) = \frac{\gamma_0}{G_{ii}} I_i \quad (6)$$

Where $P_i^n$ is the ith mobile power at the nth iteration step.

B. Beamforming

Consider the case that there are $M$ antennas at the receiver of a telecommunication system which create an antenna array. The outputs of the array elements are multiplied by a weight factor and are added together in order to construct the received signal. By varying the weight vectors of the array elements we can adjust the beam pattern of the receiver. That way we could place nulls at the directions of interfering sources and the main lobe at the direction of the signal of interest. The same method could also be applied at the transmitter. In this case the main lobe of the beam pattern is placed at the direction of the receiver and the nulls are placed at the directions of the receivers of other systems within the area.

An analytical formulation of the beamforming problem is also given in [8]. In particular, a cochannel set of $M$ transmitter and receiver pairs is assumed. Each receiver uses an antenna array with $K$ elements. Denote the array response of the ith antenna to the direction of arrival $\theta$ as $v_i(\theta)$ defined as $v_i(\theta) = [v_i^1(\theta) v_i^2(\theta) \ldots v_i^K(\theta)]$, where $v_i^k(\theta)$ is the response of the kth antenna element at the direction $\theta$. We consider multipath and slow fading channels with negligible delay spreads. That is the received vector at the ith antenna array can be written as:
\[
x_i(t) = \sum_{j=1}^{M} \sqrt{P_j} G_j \sum_{l=1}^{L} a_j^l \mathbf{v}_i(\theta_j) s_j(t - \tau_j) + \mathbf{n}_i(t)
\]  

where \( s_j(t) \) is the message signal transmitted from the \( j \)th user, \( \tau_j \) is the corresponding time delay, \( \mathbf{n}_i(t) \) is the thermal noise vector at the input of antenna array at the \( i \)th receiver, and \( P_j \) is the power of the \( j \)th transmitter. The attenuation due to shadowing in the \( l \)th path is denoted by \( \alpha_{ji} \).

We will now use the following definition:

\[
a_{ji} = \sum_{l=1}^{L} a_j^l \mathbf{v}_i(\theta_j)
\]  

The received signal at the \( i \)th receiver is now given by:

\[
x_i(t) = \sum_{j=1}^{M} \sqrt{P_j} G_j \sum_{l=1}^{L} a_{ji} s_j(t - \tau_j) + \mathbf{n}_i(t)
\]  

In order to minimize the interference, we minimize the variance or average power at the output of the beamformer subject to maintaining unity gain at the direction of the desired signal. We can write the output of the beamformer at the \( i \)th receiver as:

\[
e_i(n) = \mathbf{w}_i^H \mathbf{x}_i(nT)
\]  

where \( \mathbf{w}_i \) is a vector which contains the weight of each element of the \( i \)th antenna array and \( T \) is the symbol duration. The average output power is then given by:

\[
\varepsilon_i = \mathbb{E}\{ \mathbf{w}_i^H \mathbf{x}_i(nT) \mathbf{x}_i^H(nT) \mathbf{w}_i \} = \mathbf{w}_i^H \mathbb{E}\{ \mathbf{x}_i(nT) \mathbf{x}_i^H(nT) \} \mathbf{w}_i = \mathbf{w}_i \Phi_i \mathbf{w}_i
\]

where \( \Phi_i \) is the correlation matrix of the received vector \( \mathbf{x}_i(n) \). If the message signals are uncorrelated and zero mean, the correlation matrix is given by:

\[
\Phi_i = \sum_{j \neq i} P_j G_j a_j a_j^H + N_i \mathbf{I} + P_i G_i a_i a_i^H = \Phi_{ii} + P_i G_i a_i a_i^H
\]

where:

\[
\Phi_{ii} = \sum_{j \neq i} P_j G_j a_j a_j^H + N_i \mathbf{I}
\]

If we combine the previous result with equation (11) we get:

\[
\varepsilon_i = P_i G_i + \sum_{j \neq i} P_j G_j \mathbf{w}_i^H a_j a_j^H \mathbf{w}_i + N_i \mathbf{w}_i^H \mathbf{w}_i
\]
Here, we use the fact that the gain at the direction of interest is unity, i.e., \( w_i^H a_{ii} = 1 \). The goal of beamforming is to find a weight vector \( w_i \) that minimizes the average power \( \varepsilon_i \), subject to \( w_i^H a_{ii} = 1 \). It can be shown that the unique solution to this problem is given by:

\[
w_{\text{optm}} = \frac{\Phi_m^{-1} a_n}{a_n^H \Phi_m^{-1} a_{ii}}
\]  

(15)

C. Interference cancellation

Assume that at a particular region two systems are using the same channel in order to communicate. One of them is licensed for that channel (primary system) and the other is a pair of cognitive radios who use the channel in an opportunistic way (secondary system). This means that the cognitive radios should not introduce a significant amount of interference to the licensed system. From the other side it is widely known that the achievable rate of a system is strongly dependent on the value of SINR at the receiver. That way if the power of the primary system’s signal is high at the secondary receiver, the corresponding value of SINR would be small and as a result the achievable rate would also be limited.

This problem could be solved if the secondary receiver would be able to decode the signal of the primary system. In that case it is required that the secondary receiver would know the modulation scheme used by the primary transmitter and would have the proper hardware to perform the demodulation procedure. That way when the primary signal would be decoded, it could be subtracted from the overall signal and as a result a significant amount of interference would be cancelled. This means that the signal of the secondary system could now be decoded under a higher value of SINR.

Let us now denote as \( \gamma_p \), the ratio of the primary signal’s power to the noise power at the secondary receiver and as \( \beta_p \), the minimum value of SINR required for a successful decoding of the primary signal. Similarly, by \( \gamma_s \) we could denote the ratio of the secondary’s signal power to the noise power. We also assume that the channel width that is used is \( B \). Based on the above definitions we can conclude that the value of SINR for the primary signal at the secondary receiver is:

\[
\text{SINR} = \frac{\gamma_p}{1 + \gamma_s}
\]  

(16)

If \( \gamma_p < \beta_p \), the primary signal cannot be decoded by the secondary receiver and as a result the interference cancellation method which was described earlier cannot be applied. This means that the achievable rate for the secondary system is:

\[
B \log_2(1 + \frac{\gamma_s}{1 + \gamma_p})
\]

If \( \gamma_p > \beta_p \), we can distinguish two different cases:
1. $\text{SINR} \geq \beta_p \Rightarrow \gamma_s \leq \frac{\gamma_p}{\beta_p} - 1$

In this case the primary signal can be decoded and the achievable rate for the secondary system is $\log_2(1+\gamma_s)$.

2. $\text{SINR} < \beta_p \Rightarrow \gamma_s > \frac{\gamma_p}{\beta_p} - 1$

Now the signal of the primary system cannot be immediately decoded because the condition for SINR does not apply. In [10], a method called superposition coding is proposed, that achieves any pair of achievable rates for the multiple access channel. In particular the secondary transmitter could send two different streams of information denoted by $x_1$ and $x_2$. The first stream uses a portion $\alpha$ of the total transmission power and the remaining power is used for the modulation of the second stream. Let us now define as $\beta_{s1}(\beta_{s2})$ the minimum value of SINR that is required for a successful decoding of the signal $x_1(x_2)$. If:

$$\frac{\alpha \gamma_s}{1 + \gamma_p + (1-\alpha)\gamma_s} \geq \beta_{s1}$$

Then the signal of the first stream can be decoded and subtracted from the overall signal. That way only the signal of the second stream, the primary signal and the noise will remain. As a result, if:

$$\frac{\gamma_p}{1+(1-\alpha)\gamma_s} \geq \beta_p$$

Then the primary signal can be decoded and subtracted from the overall signal. That way the second stream can be decoded under the SINR $(1-\alpha)\gamma_s$. The total achievable rate for the secondary system in this case is:

$$B \log_2(1+\frac{\alpha \gamma_s}{1 + \gamma_p + (1-\alpha)\gamma_s}) + B \log_2(1+(1-\alpha)\gamma_s)$$

D. Adaptive modulation

Consider a telecommunication system that consists of a single transmitter-receiver pair. The power of the transmitted signal within that system is assumed to be constant and equal to $P$. According to the theorem of Shannon, the capacity of the channel that is used by the communicating pair is given by the following equation:

$$C = B \log_2(1+\gamma) \quad (17)$$

Where $B$ is the Bandwidth of the channel and $\gamma$ is the corresponding value of SINR. If we now assume that the channel is affected by phenomena like multipath fading
and shadowing then the value of $\gamma$ will vary according to a probability density function denoted by $p(\gamma)$. For example in the case that shadowing is the dominant fading phenomenon, $p(\gamma)$ would be lognormal and in the case that multipath fading is dominant, $p(\gamma)$ would be exponential (Rayleigh fading). We could now define the mean capacity of the channel which is given by the following integral:

$$C_m = \int_0^\infty B \log_2(1 + \gamma) p(\gamma) d\gamma$$

(18)

The above value of mean capacity is a theoretical result based on the theorem of Shannon which does not provide a practical method to achieve this maximum rate in real networks.

Let us now consider the case that the transmitter adapts its rate based on an estimation of the channel gain that is provided by the receiver. This estimation becomes available to the transmitter through a feedback channel which is assumed to be perfect. In other words the delay that is introduced by the feedback channel is negligible. Based on this information, the transmitter can calculate an estimation of the value of SINR at the receiver. According to this value, the transmitter adapts its modulation and coding scheme in a way that maximizes the achievable throughput of the communicating pair.

If the transmitter was able to change its rate in a continuous manner then the achievable throughput of the pair could be very close to the channel capacity. In most cases though, the transmitter would have to choose among a limited number of modulation schemes. This means that the transmission rate could also take a finite number of different values. Let us now assume that the number of the available modulation schemes for the transmitter is $N$. Then we could partition the space of the possible values of SINR into $N+1$ non overlapping regions, as shown in figure 1.

Figure 1: Partition of the space of possible values of SINR into $N+1$ non overlapping region.

<table>
<thead>
<tr>
<th>Transmission</th>
<th>MS1</th>
<th>MS2</th>
<th>...</th>
<th>MSN</th>
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<td>0</td>
<td>$\gamma_1$</td>
<td>$\gamma_2$</td>
<td>$\gamma_3$</td>
<td>$\gamma_N$</td>
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</table>

If the value of SINR is less than $\gamma_1$, then the channel condition is very poor and thus no transmission is performed. From the other side if SINR lies between $\gamma_1$ and $\gamma_2$ the first available modulation scheme is used. Similarly between $\gamma_2$ and $\gamma_3$ the second scheme is used and finally in the last region where $\gamma > \gamma_N$ the $N$th scheme is used. The goal of adaptive modulation is to determine the values of $\gamma_1$, $\gamma_2$, ..., $\gamma_N$ such that the achievable throughput for the communicating pair is maximized. Because the number of available modulation schemes is finite, the achievable throughput will be less than
the capacity limit expressed by equation (18). On the other hand by increasing the number of modulation schemes we can achieve a better approximation of the capacity.

E. Channel allocation

Consider that at a particular location a set of M transmitter and receiver pairs compete for access in the spectrum. If only one band [F_{low}, F_{high}] is available for transmissions, then a protocol should be determined to define the way that the spectrum would be shared among the M systems. In other words, for each pair, a decision should be made about the best way to distribute its transmission power in the range of frequencies [F_{low}, F_{high}]. If we now assume that the M pairs use a selfish strategy regarding the utilization of spectrum i.e. they try to maximize only their potential gains then they would spread their power in the whole interval [F_{low}, F_{high}]. This strategy is known in the literature as the water filling strategy.

In many cases the overall utilization of spectrum that is accomplished if all systems spread their power in the entire available band is suboptimal. This means that a certain level of cooperation between the communicating pairs could achieve a better performance. Assume for example, that c_{ij} denotes the channel gain between the transmitter of the ith system and the receiver of the jth system. In [12] it is stated that if c_{ij}c_{ji}/(c_{ii}c_{jj}) > 1, then the optimal power allocation strategy for systems i and j is orthogonal, which means that p_i(f)p_j(f) = 0, for f ∈[F_{low}, F_{high}]. In other words, it is better for systems i and j to function at different portions of the spectrum. In general in [12] it is also stated that every point in the achievable rate region can be obtained with power allocations p_i(f), i=1…M which are piece wise constant in the intervals [F_{low}, w_1), [w_1, w_2), ..., [w_N, F_{high}], where w_i < w_{i+1} for all i=1,...,N. The previous intervals can be considered as frequency bins and we say that a bin is allocated at a particular pair i if p_i(f) > 0 within the frequency range of that bin.

The problem of channel allocation tries to determine the best mapping of frequency bins to the communicating pairs such that a global utility function is maximized. This function could be dependent on the global spectrum utilization, fairness and/or the interference that is introduced to other systems. In this case the pairs do not simply try to maximize their own potential gains but they exchange information and cooperate in order to achieve a better performance for the whole system.

The channel allocation problem is also affected by a number of implementation issues regarding the hardware and software of the devices that access the spectrum. Examples of such issues are the allocation of contiguous or non contiguous bins to a particular device, the use of a distributed or centralized algorithm to perform the channel allocation and the a priori, on demand or proactive allocation of bins to different pairs. Each one of these issues is investigated in the following sections.

Use of contiguous or non contiguous bins

One fundamental problem regarding the spectrum availability in cognitive radio networks is that in many cases the unoccupied bands are very short and not consecutive. This means that sometimes a device has to function in frequency bins
that are not contiguous in order to cover its communicational needs. One question that naturally arises is if a proper modulation scheme exists for such a case. The answer is NC-OFDM (non contiguous orthogonal frequency division multiplexing). This scheme is exactly the same as conventional OFDM except that the subcarriers which are not allocated to a particular device are deactivated. The structures of an NC-OFDM transmitter and receiver are shown in pictures 2 and 3 respectively.

Picture 2: The NC-OFDM transmitter

Picture 3: The NC-OFDM receiver

As it is shown in picture 3, the transmitter is periodically informed about the output of the channel allocation algorithm. That way the subcarriers which are allocated to other devices are deactivated. The stream of bits that is intended to the receiver is then split into a number of parallel subflows. Each one of them corresponds to a subcarrier that is still active. Then the bits of each subflow are modulated using a MQAM or MPSK schema. This procedure produces a time series for each subcarrier which is then fed as input to a module that performs IFFT. The resulting signal is then transformed from digital to analog brought to the RF frequencies and fed to the antenna of the transmitter.
At the receiver, first the signal is brought down to baseband using a low pass filter. Then analog to digital conversion is applied resulting in a signal that is fed to a module which performs FFT. Finally the demodulation procedure takes place for all active subcarriers and the initial bit stream is reclaimed through parallel to serial conversion. The information regarding the output of the channel allocation algorithm is sent to the receiver from the transmitter through a control channel.

The basic problem that arises through the use of the NC-OFDM schema is the introduction of significant amount of interference in neighboring bands. This is because NRZ pulses are used for the modulation of the baseband signal. As it is known, the Fourier transformation of an NRZ pulse is analog to the function $\text{sync}(x)$. This means that sidelobes of significant power could appear in neighboring bands.

One way to deal with this problem is by windowing the transmitted signal in the time domain. In other words, we could use raised cosine pulses instead of NRZ pulses for the modulation of the baseband signal. That way the interference power will be reduced. This is because the Fourier transformation of the raised cosine pulse is characterized by lower levels of sidelobes compared to the NRZ pulses. The shape of a raised cosine pulse in time domain is shown in figure 4.

![Figure 4: A raised cosine pulse](image)

From this figure we can see that two consecutive raised cosine pulses are overlapping at an interval denoted by $\beta T_s$, where $T_s$ is the total duration of a single pulse. By increasing the value of $\beta$ we can achieve lower sidelobe levels as it is shown in figure 5. This is because the use of raised cosine pulses smoothes the transition between symbols and reduces the out-of-band radiation in the frequency domain. This is done at the cost of increasing the symbol interval and computational complexity. The method of windowing the signal in time domain can achieve the reduction of interference power in neighboring bands. This can be clearly evaluated from figure 5. In most cases though the amount of reduction that is achieved is not enough which means that additional methods should also be applied.
Another way to reduce interference is to deactivate a number of subcarriers at the edges of the bands that are allocated to a particular device. That way the sidelobe power at neighboring bands will be very limited. From the other side though, a large portion of the available spectrum should remain unutilized. Also this portion will be even larger as the number of non consecutive zones increase. This effect is shown if figure 6.

From the above figure we can clearly see that the method of subcarrier deactivation is more suitable for the case of conventional OFDM. That is because in this case only one consecutive zone of frequencies will be allocated to the corresponding device. Thus the number of the subcarriers that should be deactivated is limited.

According to the results of recent research, two new methods for interference reduction have been proposed [16]. The first one is called CE (constellation
expansion) and is based on the mapping of the signal constellation to a new larger one. In particular, each point in the initial constellation is mapped to \( n \) (usually 2) points of the new constellation. An example of such a mapping is shown in figure 7.

Figure 7: Mapping of a QPSK to an 8-PSK constellation. Point a of the QPSK constellation is mapped to points \( a_1 \) and \( a_2 \) of the 8-PSK constellation etc.

In the expanded constellation, every symbol can be represented by \( n \) different points, each one of which corresponds to a different phase. This means that if we take a series of \( M \) symbols, we can use \( n^M \) different ways to represent it. Thus, we can choose among them the one which introduces the least amount of interference to the neighboring bands.

The second method that is proposed is based on the use of cancellation subcarriers or CCs. In particular, the subcarriers that are located at the edges of bands which are used by a device are modulated in such a way that the interference in neighboring bands is cancelled. The disadvantage of this method is that again a portion of the spectrum remains unutilized. This portion though is smaller compared to the case that the subcarrier deactivation method is used. From the other side, a part of the available transmission power is used for the modulation of CCs. That way, the value of SINR at the receiver is reduced, which means that the achievable rate of the communicating pair is degraded. The results of the combined use of the CE and CCs methods are shown in figure 8.

Figure 8: Combined use of the CE and CCs methods.
Use of a distributed or centralized channel allocation algorithm

An important issue regarding the channel allocation algorithm that is used in a cognitive radio network is if it is centralized or distributed. In centralized approaches, usually one entity is responsible for the division of the available channels to the nodes of the network. This entity should be periodically informed about various parameters such as the traffic demand of nodes, possible changes in the topology (departures or arrivals of new nodes), the quality of links and the conditions which correspond to the network in general. The amount of information that the centralized entity should analyze gets larger as the network grows. This means that it is difficult to ensure the scalability of the network.

From the other side in distributed approaches, each node should be kept informed about the conditions which correspond to its neighborhood. For example if two nodes decide to use a certain channel in order to communicate they should first inform their neighbors for this action. That way they can ensure that no other node will interfere with their communication. This means that each node should be able to store a certain amount of information in its memory and that a large number of messages should be exchanged for the distributed algorithm to function. From the other side though, distributed approaches can ensure the scalability of the network better than centralized approaches.

In general centralized approaches are the most appropriate choice for infrastructure networks. The topology of such networks does not change very often and usually there is an entity which can maintain all the required information in order to administrate the network. Distributed approaches on the other hand are more suitable for ad hoc networks. These networks are usually formed by nodes with limited computational resources and scale in an unpredicted way. Thus, it is difficult for a centralized entity to administrate the whole network.

Proactive or on demand channel allocation

The procedure of spectrum sensing periodically detects which frequency bands are not occupied by a licensed system. These bands are available for use by a cognitive radio network and are administrated by the channel allocation algorithm. This algorithm could assign the available bands to users periodically or it could allocate an amount of bandwidth to a certain user only when it needs it. In the first case we say that the channel allocation algorithm is proactive and in the second case on demand.

A priori channel allocation (Use of primary and secondary channels)

In many telecommunication systems the available bandwidth is divided into a set of orthogonal channels. These channels are allocated to the nodes of a wireless network in such a way that each node can get an equal share of the bandwidth. Even though the above mechanism is fair, it is not optimal in terms of throughput and cannot guarantee the quality of service. This is due to the high variation of load throughout the network. For example, in a cellular network the number of clients in a particular cell could be very large and at the same time the load of a neighboring cell could be almost zero. In such a case some of the clients of the congested cell would not be able
to communicate even though there are unused channels in the neighborhood. One way to solve the above mentioned problem is to let each node to be able to borrow channels from its neighbors at times of severe congestion. In that case each node could potentially use every channel in the available bandwidth. The channels that are allocated to a particular node are called primary and the remaining channels are called secondary. That way whenever a node needs to acquire a channel, it first checks to see if there is an available primary channel. If so it acquires that channel otherwise it searches for an available secondary channel. This is done by sending appropriate request messages to each neighbor. Upon receiving such a message, a node checks to see if it is possible to lend one of its primary channels to the transceiver of the message. If so it responds to the transceiver with an appropriate reply message. After collecting a number of replies from the neighborhood a node can chooses one of the available secondary channels and use it for its transmissions.

**Definition of the optimization Problem**

As we mentioned above, many approaches have been proposed in the literature which try to maximize the spectrum utilization, fairness and/or interference in cognitive radio networks. Each one of them defines an optimization problem with deferent degrees of freedom. Figure 9 contains a tree which shows the structure of such an optimization problem. In particular, all possible degrees of freedom and utility function parameters are listed. Also, tables 1 presents the parameters of the optimization problem that is defined by various articles in the literature.

Figure 9: Throughput, fairness and/or interference optimization in cognitive radio networks.
Table 1: Parameters of the optimization problem that is defined by various articles

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Channel allocation methods

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References:
[14] A. Shukla, B. Willamson, J. Burns, E. Burbidge, A. Taylor, and D. Robinson, “A study for the provision of aggregation of frequency to provide wider bandwidth services”, August 2006