Machine Learning in Telecommunications

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Roadmap

- Motivation
- Supervised Learning
- Algorithms
  - Artificial Neural Networks
  - Naïve Bayes Classifier
  - Decision Trees
- Application on VoIP in Wireless Networks
Landscape in Telecommunications

- Dramatic growth of mobile data, streaming services, telepresence
- By 2019 mobile data traffic over 24 exabytes/month worldwide
- Growth in video delivery segment
- Robust growth opportunity for networking, server, and specialized hardware providers, due to:
  - mobile device capacity growth
  - advances in networked home
  - cloud services
  - user-generated content
- Existing & emerging large access markets & services
Motivation

- Need to analyze a large amount of heterogeneous data
- Detect trends and patterns
- Characterize the Quality of Experience of various services
- Analyze the performance of various services, providers, and networks
Objectives: QoE Modeling & Analysis

How does the network performance affect the perceived quality of experience (QoE) of a user?

- To predict the QoE based on network performance, apply machine learning and data mining algorithms, such as:
  - Decision Trees, Support Vector Regression, Artificial Neural Networks, Gaussian Naïve Bayes
- Train the models based on network measurements and opinion scores collected in the context of a service
- Demonstrate this methodology for VoIP and video services
Machine Learning

- The study of algorithms and systems that improve their performance with experience (Mitchell book)
- Experience = data / measurements / observations
Where to Use Machine Learning

- You have past data, you want to predict the future
- You have data, you want to make sense out of them (find patterns)
- You have a problem which is hard to be modeled
  - Gather input-output pairs to learn the mapping
- Measurements + intelligent behavior usually lead to some form of Machine Learning
Supervised Learning

- Learn from examples
- Would like to be able to predict an outcome of interest $y$ for an object $x$
- Learn function $y = f(x)$
- For example, $x$ is a VoIP call, $y$ is an indicator of QoE
- Given data $\{<x_i, y_i> : i=1, ..., n\}$,
  - $x_i$ the representation of an object, i.e., predictors
  - $y_i$ the representation of a known outcome, i.e., class labels
- Learn the function $y = f(x)$ that generalizes from the data the “best” (has minimum average error)
Classification vs. Regression

- Classification
  - Constructs decision surfaces
  - Predicts **categorical** class **labels** (discrete or nominal)
  - Classifies (assigns a label) to new data

- Regression
  - Constructs a regression line
  - Predicts **continuous values** along the line
Algorithms:
Artificial Neural Networks
Binary Classification Example
Possible Decision Areas

Class area: Green triangles

Class area: red circles
Binary Classification Example

The simplest non-trivial decision function is the straight line

One decision surface

Decision surface partitions space into two subspaces

In the case of high dimensional space, a **hyperplane** is the decision function
Specifying a Line (1)

Line equation:
\[ w_2x_2 + w_1x_1 + w_0 = 0 \]

Classifier model:
If \( w_2x_2 + w_1x_1 + w_0 \geq 0 \)
- Output 1
Else
- Output -1
Specifying a Line (2)

Classifier becomes

\[ \text{sgn}(w_2 x_2 + w_1 x_1 + w_0) = \]
\[ \text{sgn}(w_2 x_2 + w_1 x_1 + w_0 x_0) , \]
set \( x_0 = 1 \) always

Let \( n \) be the number of predictors

\[ \text{sgn}(\sum_{i=0}^{n} w_i x_i) \text{, or} \]
\[ \text{sgn}(\vec{w} \cdot \vec{x}) \]
The simplest neural network: the Perceptron

The Perceptron is a 2-layer neural network

Weights

Example X
The simpler Neural: The Perceptron

\[ \sum_{i=0}^{n} w_i x_i = 3 \]

- \( w_4 = 3 \)
- \( w_2 = 0 \)
- \( w_2 = -2 \)
- \( w_1 = 4 \)
- \( w_0 = 2 \)

\( x_4 \ | \ 3 \)
\( x_3 \ | \ 3.1 \)
\( x_2 \ | \ 4 \)
\( x_1 \ | \ 0 \)
\( x_0 \ | \ 1 \)
The simpler Neural: The Perceptron

\[ \sum_{i=0}^{n} w_i x_i = 3 \]

\[ \text{sgn}(3) = 1 \]

\[ W_4 = 3 \]

\[ W_2 = 0 \]

\[ W_2 = -2 \]

\[ W_1 = 4 \]

\[ W_0 = 2 \]

\[ x_4 \]

\[ x_3 \]

\[ x_2 \]

\[ x_1 \]

\[ x_0 \]
Training Perceptrons

- Start with random weights
- Update in an intelligent way to improve them using the data
- Intuitively:
  - Decrease the weights that increase the sum
  - Increase the weights that decrease the sum
- Repeat for all training instances until convergence
Perceptron Training Rule

For each misclassified example $\bar{x}_d$ update weights:

$$\Delta w_i = \eta (t_d - o_d) x_{i,d}$$

$$w'_i \leftarrow w_i + \Delta w_i$$

In vector form:

$$\bar{w}' \leftarrow \bar{w} + \eta (t_d - o_d) \bar{x}_d$$

- $\eta$: arbitrary learning rate (e.g. 0.5)
- $t_d$: (true) label of the $d$-th example
- $o_d$: output of the perceptron on the $d$th example
- $x_{i,d}$: value of predictor variable $i$ of example $d$
- $t_d = o_d$: No change (for correctly classified examples)
Analysis of the Perceptron Training Rule

- Algorithm will always converge within finite number of iterations if the data are **linearly separable**
- Otherwise, it may oscillate (no convergence)
Gradient Descent

- A first-order optimization algorithm
- Finds a local minimum
- Steps proportional to the negative of the gradient of the function at the current point
Training by Gradient Descent

Idea:

◦ Define an error function
◦ Search for weights that minimize the error, i.e., find weights that zero the error gradient

Similar with the Perceptron training rule, but it the gradient descent:

◦ Always converges
◦ Generalizes to training networks of perceptrons (neural networks) and training networks for multicategory classification or regression
Setting Up the Gradient Descent

Squared Error: $t_d$ label of $d$th example, $o_d$ current output on $d$th example

$$E(\vec{w}) = \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

Minima exist where gradient is zero:

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

$$= \frac{1}{2} \sum_{d \in D} \frac{\partial}{\partial w_i} (t_d - o_d)^2$$

$$= \frac{1}{2} \sum_{d \in D} 2(t_d - o_d) \frac{\partial}{\partial w_i} (t_d - o_d)$$

$$= \sum_{d \in D} (t_d - o_d) \frac{\partial}{\partial w_i} (-o_d)$$
The Sign Function is not Differentiable

\[
\frac{\partial}{\partial w_i} (-o_d) = -\frac{\partial o_d}{\partial w_i} = 0, \text{ everywhere except } o_d = 0
\]
Use Differentiable Transfer Functions

- Replace with the sigmoid

\[ \text{sig}(\mathbf{w} \cdot \mathbf{x}_d) \]

\[ \text{sig}(y) = \frac{1}{1 + e^{-y}} \]

\[ \frac{d\text{sig}(y)}{dy} = \text{sig}(y)(1 - \text{sig}(y)) \]
Updating the Weights with Gradient Descent

\[ \tilde{w} \leftarrow \tilde{w} - \eta \nabla E(\tilde{w}) \]

\[ \tilde{w} \leftarrow \tilde{w} + \eta \sum_{d \in D} (t_d - o_d) \sigma(\tilde{w} \cdot \tilde{x}_d)(1 - \sigma(\tilde{w} \cdot \tilde{x}_d)) \cdot \tilde{x}_d \]

- Each weight update goes through all training instances
- Each weight update more expensive but more accurate
- Always converges to a local minimum regardless of the data
- When using the sigmoid: output is a real number between 0 & 1
- Thus, labels (desired outputs) have to be represented with numbers from 0 to 1
Feed-Forward Neural Networks

Output Layer

Hidden Layer 2

Hidden Layer 1

Input Layer

\( x_0 \)

\( x_1 \)

\( x_2 \)

\( x_3 \)

\( x_4 \)
Increased Expressiveness Example: Exclusive OR

No line (no set of three weights) can separate the training examples (learn the true function).
From the Viewpoint of the Output Layer

- Each hidden layer maps to a new feature space
- Each hidden node is a new constructed feature
- Original Problem may become separable (or easier)
How to Train Multi-Layered Networks

- Select a network structure (number of hidden layers, hidden nodes, and connectivity)
- Select transfer functions that are differentiable
- Define a (differentiable) error function
- Search for weights that minimize the error function, using gradient descent or other optimization method
- Backpropagation
Back-Propagating the Error

Output unit(s)  \( o \)

Hidden unit(s)

Input unit(s)  \( x \)
Back-Propagating the Error

\[ \delta_o = o (1-o) (t-o) \]

\[ \delta_i = z_i (1-z_i) \sum_{o \in \text{Outputs}} w_i \delta_o \]

\[ \delta_j = z_j (1-z_j) \sum_{i \in \text{NextLayer}} w_j \delta_i \]
Back-Propagation

For a given input vector \( \bar{x} \)

Notation:
- \( o_u \) output of every unit \( u \) in network
- \( t \) desired output
- \( w_{i \rightarrow j} \) weight from unit \( i \) to unit \( j \)
- \( x_{i \rightarrow j} \) input from unit \( i \) going to unit \( j \)

Define:
- \( \delta_k = o_k (1 - o_k)(t - o_k) \), when \( k \) is the output unit
- \( \delta_k = o_k (1 - o_k) \sum_{u \in \text{Output of unit } k} w_{k \rightarrow u} \delta_u \)

Update weights rule:
- \( w'_{i \rightarrow j} = w_{i \rightarrow j} + \eta \delta_j x_{i \rightarrow j} \)
Back-Propagation Algorithm

- Propagate the input forward through the network
- Calculate the outputs of all nodes (hidden and output)
- Propagate the error backward
- Update the weights:

\[
\begin{align*}
  w_{r \rightarrow t} &\leftarrow w_{r \rightarrow t} - \eta \frac{\partial E( w_{r \rightarrow t} )}{\partial w_{r \rightarrow t}} \\
  w_{r \rightarrow t} &\leftarrow w_{r \rightarrow t} + \eta \cdot \delta_t \cdot x_r
\end{align*}
\]
Training with Back-Propagation

- Go once through all training examples & update the weights (1 epoch)
- Iterate until a stopping criterion is satisfied
- The hidden layers learn new features and map to new spaces
- Training reaches a local minimum of the error surface
Overfitting with Neural Networks

- If number of hidden units (and weights) is large, it is easy to “memorize” the training set (or parts of it) and not generalize.

- Typically, the optimal number of hidden units is much smaller than the input units.

- Each hidden layer maps to a space of smaller dimension.
Representational Power

- **Perceptron**: Can learn only linearly separable functions
- **Functions learnable by a neural network**
  - **Boolean Functions**: one hidden layer
  - **Continuous Functions**: one hidden layer and sigmoid units
  - **Arbitrary Functions**: two hidden layers and sigmoid units
- **Number of hidden units in all cases unknown**
ANN in Matlab

- Create an ANN
  ```matlab
  net = feedforwardnet(hiddenSizes)
  ```
- `[net] = train(net,X,T)` takes a `network` net, input data `X` and target data `T` and returns the `network` after training it.
- `sim(net, X)` takes a `network` net and inputs `X` and returns the estimated outputs `Y` generated by the `network`.

- Example
```matlab
layers = [2 4];  % 2 hidden layers with size 2 and 4, respectively
net = feedforwardnet([2 4]);
net = init(net);  % initialize
% traindata is a struct that contains the training set
net = train(net, traindata.examples, traindata.labels);
```

% testdata is a struct that contains the testing set
```matlab
predictions = sim(net, testdata.examples);
```
Conclusions

- Can deal with both real and discrete domains
- Can also perform density or probability estimation
- Very fast classification time
- Relatively slow training time (does not easily scale to thousands of inputs)
- One of the most successful classifiers yet
- Successful design choices still a black art
- Easy to overfit or underfit if care is not applied
Algorithms:
Naïve Bayes Classifier
Bayes Rule

\[ P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} \]

Which is shorthand for:

\[ (\forall i, j) P(Y = y_i|X = x_j) = \frac{P(X = x_j|Y = y_i)P(Y = y_i)}{P(X = x_j)} \]

- Random Variable
- It’s ith possible value
Bayes Rule

\[
P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}
\]

Which is shorthand for:

\[
(\forall i, j) P(Y = y_i | X = x_j) = \frac{P(X = x_j | Y = y_i)P(Y = y_i)}{P(X = x_j)}
\]

Equivalently:

\[
(\forall i, j) P(Y = y_i | X = x_j) = \frac{P(X = x_j | Y = y_i)P(Y = y_i)}{\sum_k P(X = x_j | Y = y_k)P(Y = y_k)}
\]
Bayes Rule

\[ P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} \]

Which is shorthand for:

\[ (\forall i, j) P(Y = y_i|X = x_j) = \frac{P(X = x_j|Y = y_i)P(Y = y_i)}{P(X = x_j)} \]

Common abbreviation:

\[ (\forall i, j) P(y_i|x_j) = \frac{P(x_j|y_i)P(y_i)}{P(x_j)} \]
Bayes Classifier

- **Training data:**

<table>
<thead>
<tr>
<th>Sky</th>
<th>Temp</th>
<th>Humid</th>
<th>Wind</th>
<th>Water</th>
<th>Enjoy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunny</td>
<td>Warm</td>
<td>Normal</td>
<td>Strong</td>
<td>Warm</td>
<td>Yes</td>
</tr>
<tr>
<td>Sunny</td>
<td>Warm</td>
<td>High</td>
<td>Strong</td>
<td>Warm</td>
<td>Yes</td>
</tr>
<tr>
<td>Rain</td>
<td>Cold</td>
<td>High</td>
<td>Strong</td>
<td>Warm</td>
<td>No</td>
</tr>
<tr>
<td>Sunny</td>
<td>Warm</td>
<td>High</td>
<td>Strong</td>
<td>Cool</td>
<td>Yes</td>
</tr>
</tbody>
</table>

- **Learning** = estimating \( P(X|Y), P(Y) \)
- **Classification** = using Bayes rule to calculate \( P(Y | X^{new}) \)
- \( X^{new} \) is a new example
Naïve Bayes Assumption

\[ X = \langle X_1, \ldots, X_n \rangle, \ n: \text{dimensions of } X \]
\[ Y \text{ discrete-valued} \]

\[ X_i \text{ and } X_j \text{ are conditionally independent given } Y, \text{ for all } i \neq j \]

\[ P(X_1 \ldots X_n | Y) = \prod_i P(X_i | Y) \]
Naïve Bayes classification

- Bayes rule:
  \[
P(Y = y_k | X_1 \ldots X_n) = \frac{P(Y = y_k)P(X_1 \ldots X_n | Y = y_k)}{\sum_j P(Y = y_j)P(X_1 \ldots X_n | Y = y_j)}
  \]

- Assuming conditional independence:
  \[
P(Y = y_k | X_1 \ldots X_n) = \frac{P(Y = y_k) \prod_i P(X_i | Y = y_k)}{\sum_j P(Y = y_j) \prod_i P(X_i | Y = y_j)}
  \]

- So, the classification rule for a new example \(X_{\text{new}} = \langle X_i, \ldots, X_n \rangle\)
  \[
  Y_{\text{new}} \leftarrow \arg \max_{y_k} P(Y = y_k) \prod_i P(X_i | Y = y_k)
  \]
Naïve Bayes Algorithm

- Train Naïve Bayes (examples)
  for each label $y_k$
    estimate $P(Y = y_k)$
    for each value $x_{ij}$ of each predictor $X_i$
      estimate $P(X_i = x_{ij} | Y = y_k)$
    end
  end
end

$$Y^{\text{new}} \leftarrow \arg \max_{y_k} P(Y = y_k) \prod_i P(X_i | Y = y_k)$$

- Classify a new example $X^{\text{new}}$
Estimating Parameters: $Y, X_i$ discrete-valued

- Parameter estimation:

$$\hat{\pi}_k = \hat{P}(Y = y_k) = \frac{\#D\{Y = y_k\}}{|D|}$$

$$\hat{\theta}_{ijk} = \hat{P}(X_i = x_{ij} | Y = y_k) = \frac{\#D\{X_i = x_{ij} \land Y = y_k\}}{\#D\{Y = y_k\}}$$
What if we have continuous $X_i$?

- Gaussian Naïve Bayes (GNB) assume

$$P(X_i = x \mid Y = y_k) = \frac{1}{\sigma_{ik} \sqrt{2\pi}} e^{-\frac{(x-\mu_{ik})^2}{2\sigma_{ik}^2}}$$

Sometimes assume variance
- is independent of $Y$ (i.e., $\sigma_i$),
- or independent of $X_i$ (i.e., $\sigma_k$)
- or both (i.e., $\sigma$)
Estimating Parameters: $Y$ discrete, $X_i$ continuous

- Maximum likelihood estimates:

\[
\hat{\mu}_{ik} = \frac{1}{\sum_j \delta(Y^j = y_k)} \sum_j X^j_i \delta(Y^j = y_k)
\]

\[
\hat{\sigma}^2_{ik} = \frac{1}{\sum_j \delta(Y^j = y_k)} \sum_j (X^j_i - \hat{\mu}_{ik})^2 \delta(Y^j = y_k)
\]

$\delta(x) = 1$ if $x$ true, else 0

jth training example
Naïve Bayes in Matlab

• Create a new Naïve object:
  
  \[ \text{nb} = \text{NaiveBayes.fit}(X, Y), \text{X is a matrix of predictor values, } Y \text{ is a vector of n class labels} \]

• post = posterior(nb, test) returns the posterior probability of the observations in test

• Predict a value
  
  \[ \text{predictedValue} = \text{predict}(nb, \text{test}) \]
Algorithms:
Decision Trees
A small dataset: Miles Per Gallon

- Suppose we want to predict MPG

<table>
<thead>
<tr>
<th>mpg</th>
<th>cylinders</th>
<th>displacement</th>
<th>horsepower</th>
<th>weight</th>
<th>acceleration</th>
<th>modelyear</th>
<th>maker</th>
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<td>75to78</td>
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<td>bad</td>
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<td>europe</td>
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</table>

- From the UCI repository
A Decision Stump

mpg values: bad good

root

22 18

pchance = 0.001

cylinders = 3

0 0

Predict bad

cylinders = 4

4 17

Predict good

cylinders = 5

1 0

Predict bad

cylinders = 6

8 0

Predict bad

cylinders = 8

9 1

Predict bad
Recursion Step

Take the Original Dataset.

And partition it according to the value of the attribute we split on.

Build Tree from these Records.
Recursively build a tree from the seven records in which there are four cylinders and the maker was based in Asia

(Similar recursion in the other cases)
The final tree
Classification of a new example

- Classifying a test example
- Traverse tree
- Report leaf label
Learning decision trees is hard!!!

- Learning the simplest (smallest) decision tree is an NP-complete problem [Hyafil & Rivest ’76]

- Resort to a greedy heuristic:
  - Start from empty decision tree
  - Split on next best attribute (feature)
  - Recurse

- How to choose the best attribute and the value for a split?
Entropy

- Entropy characterizes our uncertainty about our source of information

**More uncertainty, more entropy!**

- Information Theory interpretation: $H(Y)$ is the expected number of bits needed to encode a randomly drawn value of $Y$ (under most efficient code)
Information gain

- Advantage of attribute – decrease in uncertainty
  - Entropy of Y before you split
    \[
    H(Y) = - \sum_{i=1}^{k} P(Y = y_i) \log_2 P(Y = y_i)
    \]
  - Entropy after split
    - Weight by probability of following each branch, i.e., normalized number of records
    \[
    H(Y \mid X) = - \sum_{j=1}^{v} P(X = x_j) \sum_{i=1}^{k} P(Y = y_i \mid X = x_j) \log_2 P(Y = y_i \mid X = x_j)
    \]
  - Information gain is difference
    \[
    IG(X) = H(Y) - H(Y \mid X)
    \]
Learning decision trees

- Start from empty decision tree
- Split on **next best attribute** (feature)
  - Use, for example, information gain to select attribute
  - Split on \( \arg \max_i IG(X_i) = \arg \max_i H(Y) - H(Y \mid X_i) \)
- Recurse
A Decision Stump

mpg values: bad good

root

22 18

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4 17

Predict good

cylinders = 5

1 0

Predict bad

cylinders = 6

8 0

Predict bad

cylinders = 8

9 1

Predict bad
Base Cases

- Base Case One: If all records in current data subset have the same output then **don’t recurse**
- Base Case Two: If all records have exactly the same set of input attributes then **don’t recurse**
Don’t split a node if all matching records have the same output value.
Don’t split a node if all records have exactly the same set of input attributes.
Basic Decision Tree Building Summarized

- **BuildTree**(DataSet, Output)
- If all output values are the same in DataSet, return a leaf node that says “predict this unique output”
- If all input values are the same, return a leaf node that says “predict the majority output”
- Else find attribute X with highest Info Gain
- Suppose X has $n_X$ distinct values (i.e. X has arity $n_X$).
  - Create and return a non-leaf node with $n_X$ children.
  - The i’th child should be built by calling BuildTree(DSi, Output)
    Where DSi built consists of all those records in DataSet for which $X = ith$ distinct value of X.
Decision trees will overfit

- Standard decision trees are have no learning biased
  - Training set error is always zero!
    - (If there is no label noise)
  - Lots of variance
  - Will definitely overfit!!!
  - Must bias towards simpler trees

- Many strategies for picking simpler trees:
  - Fixed depth
  - Fixed number of leaves
  - Or something smarter…
Consider this split
A statistical test

Suppose that mpg was completely uncorrelated with maker.
What is the chance we’d have seen data of at least this apparent level of association anyway?
Using to avoid overfitting

- Build the full decision tree as before
- But when you can grow it no more, start to prune:
  - Beginning at the bottom of the tree, delete splits in which have extreme low chance to appear \( \text{p\_chance} > \text{MaxPchance} \)
  - Continue working your way up until there are no more prunable nodes
What you need to know about decision trees

- Decision trees are one of the most popular data mining tools
  - Easy to understand
  - Easy to implement
  - Easy to use
  - Computationally cheap (to solve heuristically)
- Information gain to select attributes (ID3, C4.5, …)
- Presented for classification, can be used for regression and density estimation too
- Decision trees will overfit!!!
  - Zero bias classifier! Lots of variance
  - Must use tricks to find “simple trees”, e.g.,
    - Fixed depth/Early stopping
    - Pruning
Decision trees in Matlab

- Use `classregtree` class
- Create a new tree:
  \[ t = \text{classregtree}(X,Y), \text{ } X \text{ is a matrix of predictor values, } y \text{ is a vector of } n \text{ response values} \]
- Prune the tree:
  \[ tt = \text{prune}(t, \alpha, \text{pChance}) \text{ } \alpha \text{ defines the level of the pruning} \]
- Predict a value
  \[ y = \text{eval}(tt, X) \]
Performance Estimation

- Need to produce a single, final model
- But also estimate its performance
- Why estimate performance
  - Know what to expect out of a model / system
  - Select the best model out of all possible models
  - Compare different learning algorithms
- Probably the most underestimated problem in machine learning, data mining, pattern recognition
Ideal Performance Estimation

1. Learn a model from samples in S (train-set)
2. Install the model in its intended operational environment
3. Observe its operation for some time, for new cases S’
4. Label with a gold-standard the cases in S’ (test-set)
5. Estimate the performance of the model on S’
Ideal Performance Estimation

Golden Rule:
Simulate: learn from S, make operational, test on new samples S’
Simulating the Ideal

- Randomly split original data
- Learn on Train
- Test on Test
- Called hold-out estimation
- Can it go wrong?
K-Fold Cross-Validation

- Split to K-folds
- Cross-Validation(Data D, number K)
  - Randomly split D to K folds
  - Returned Model: f(D)
Nested Cross Validation

- Different combinations of model parameters
- \( n = 5 \) folds

Input: Dataset \( S \), \( n \), \( k \), \( \gamma \), \( \varepsilon \), \( c \)

Output: \( \text{Mean}(\text{Error}_i) \)
Application on VoIP in Wireless Networks
Motivation

- Wide use of wireless services for communication
- Quality of Service (QoS):
  - Objective network-based metrics (e.g., delay, packet loss)
- Quality of Experience (QoE):
  - Objective and subjective performance metric (e.g., E-model, PESQ)
  - Objective factors: network, application related
  - Subjective factors: users expectation (MOS)
Problem Definition

- Users are not likely to provide QoE feedback
  - unless bad QoE is witnessed
- Estimation of QoE
  - difficult because of the many contributing factors using Opinion Models
- Use of machine learning algorithms for the estimation of the QoE
  - based on QoS metrics
Proposed Method

- Nested Cross Validation training of
  - ANN Models
  - GNB Models
  - Decision Trees models

- Preprocessing of data: normalization
Dataset

- 25 users
- 18 samples (segments of VoIP calls)
- Each user evaluated all the segments with QoE score
- 10 attributes as predictors
Dataset

- Predictors
  - average delay, packet loss, average jitter, burst ratio, average burst interarrival, average burst size, burst size variance, delay variance, jitter variance, burst interarrival variance
- QoE score
Experiments and Results

- For ANN we tested different values of nodes at the first and the second hidden layer, with and no normalization of the data.
- In this table we can see some statistics from the error which appears from the difference between the estimated QoE and the real QoE.

<table>
<thead>
<tr>
<th></th>
<th>ANN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean error</td>
<td>0.9018</td>
</tr>
<tr>
<td>Median error</td>
<td>0.6181</td>
</tr>
<tr>
<td>Std error</td>
<td>1.0525</td>
</tr>
</tbody>
</table>
Experiments and Results

- In order to train the GNB models we use the data with normalization or not.
- Statistics from the error of this model:

<table>
<thead>
<tr>
<th></th>
<th>GNB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean error</td>
<td>0.9018</td>
</tr>
<tr>
<td>Median error</td>
<td>0.6181</td>
</tr>
<tr>
<td>Std error</td>
<td>1.0525</td>
</tr>
</tbody>
</table>
Experiments and Results

• For the Decision Trees we used different values of alpha (a) parameter which defines the pruning level of the tree.

• Statistics:

<table>
<thead>
<tr>
<th></th>
<th>Decision Trees</th>
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</thead>
<tbody>
<tr>
<td>Mean error</td>
<td>0.5475</td>
</tr>
<tr>
<td>Median error</td>
<td>0.3636</td>
</tr>
<tr>
<td>Std error</td>
<td>0.5395</td>
</tr>
</tbody>
</table>
Material

Sources:

- Lectures from Machine Learning course CS577