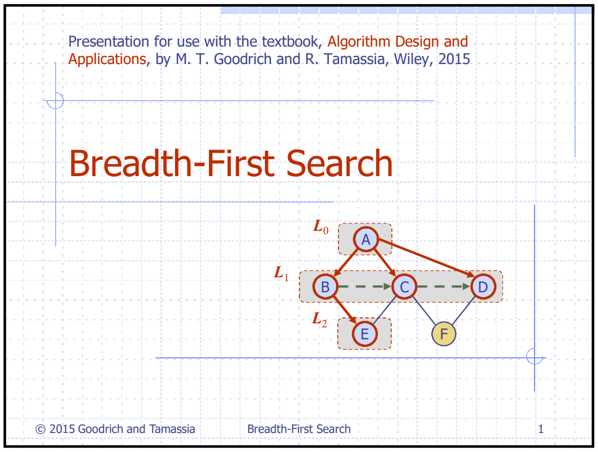
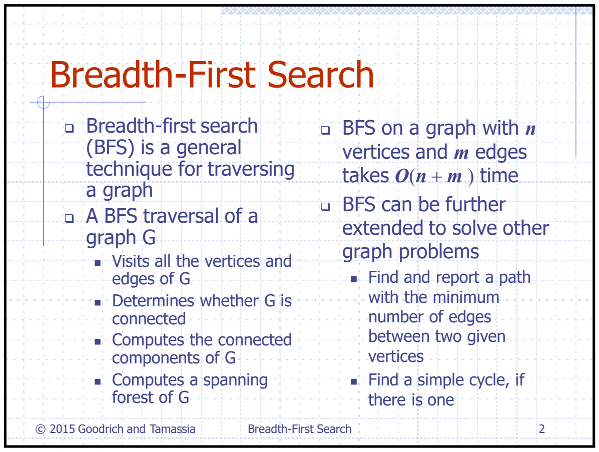


Presentation for use with the textbook, Algorithm Design and Applications, by M. T. Goodrich and R. Tamassia, Wiley, 2015

Breadth-First Search

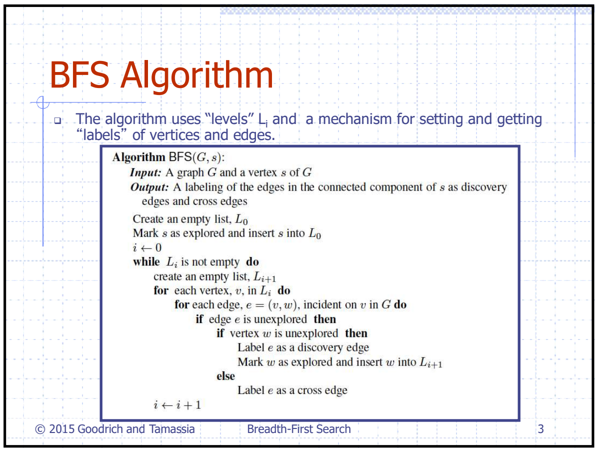
Breadth-First Search 1



Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph
- A BFS traversal of a graph G
  - Visits all the vertices and edges of G
  - Determines whether G is connected
  - Computes the connected components of G
  - Computes a spanning forest of G
- BFS on a graph with n vertices and m edges takes $O(n+m)$ time
- BFS can be further extended to solve other graph problems
  - Find and report a path with the minimum number of edges between two given vertices
  - Find a simple cycle, if there is one

Breadth-First Search 2

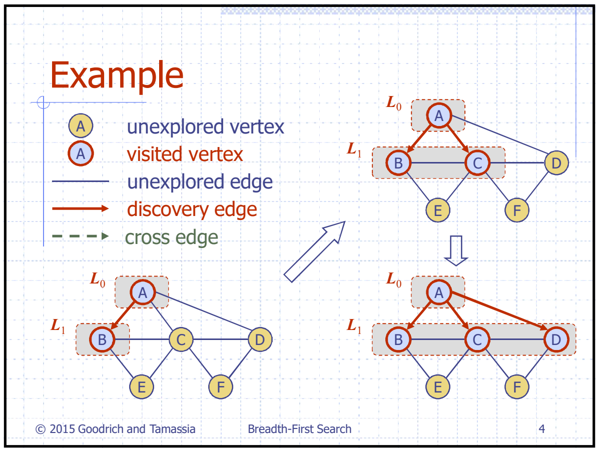


BFS Algorithm

- The algorithm uses “levels” L_i and a mechanism for setting and getting “labels” of vertices and edges.

```
Algorithm BFS($G, s$):
  Input: A graph $G$ and a vertex $s$ of $G$
  Output: A labeling of the edges in the connected component of $s$ as discovery
    edges and cross edges
  Create an empty list, $L_0$
  Mark $s$ as explored and insert $s$ into $L_0$
  $i \leftarrow 0$
  while $L_i$ is not empty do
      create an empty list, $L_{i+1}$
      for each vertex, $v$, in $L_i$ do
          for each edge, $e = (v, w)$, incident on $v$ in $G$ do
              if edge $e$ is unexplored then
                  if vertex $w$ is unexplored then
                      Label $e$ as a discovery edge
                      Mark $w$ as explored and insert $w$ into $L_{i+1}$
                  else
                      Label $e$ as a cross edge
      $i \leftarrow i + 1$
```

Breadth-First Search 3



Example

A unexplored vertex

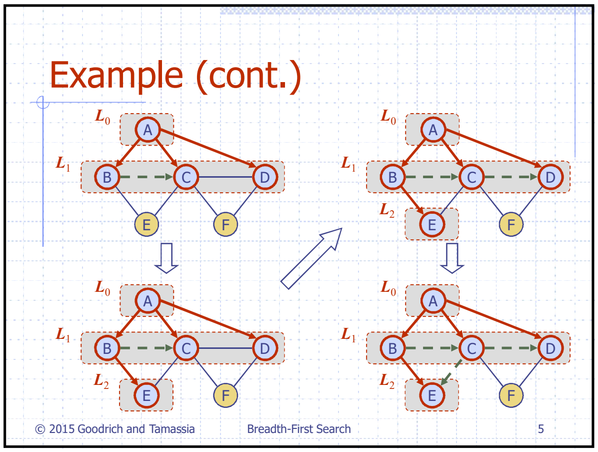
A visited vertex

unexplored edge

discovery edge

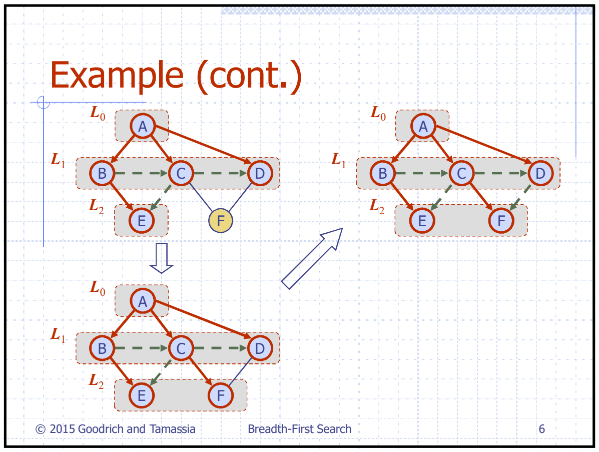
cross edge

Breadth-First Search 4



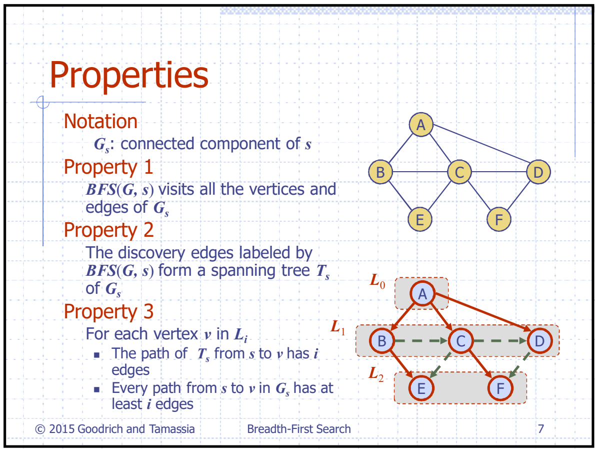
Example (cont.)

Breadth-First Search 5



Example (cont.)

Breadth-First Search 6



Properties

Notation

G_s: connected component of s

Property 1

$BFS(G, s)$ visits all the vertices and edges of G_s

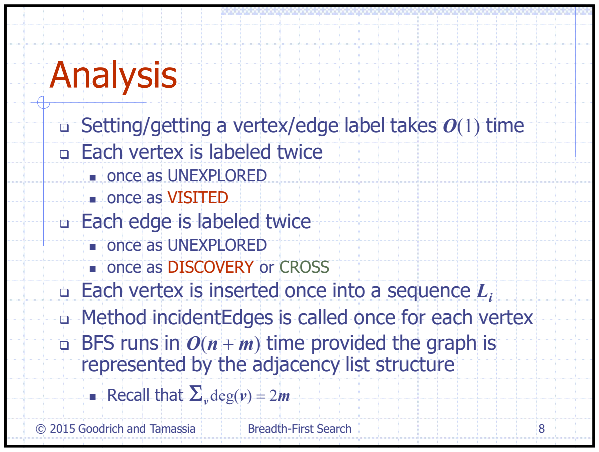
Property 2

The discovery edges labeled by $BFS(G, s)$ form a spanning tree T_s of G_s

Property 3

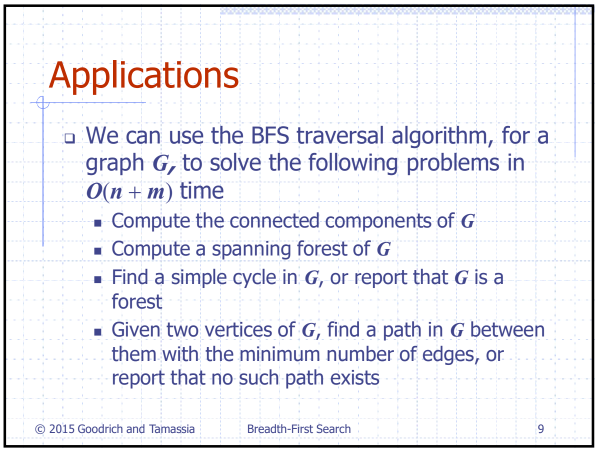
For each vertex v in L_i

- The path of T_s from s to v has i edges
- Every path from s to v in G_s has at least i edges



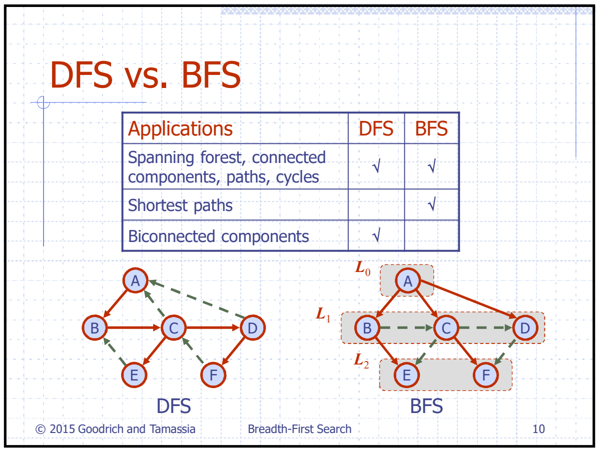
Analysis

- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into a sequence L_i
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$



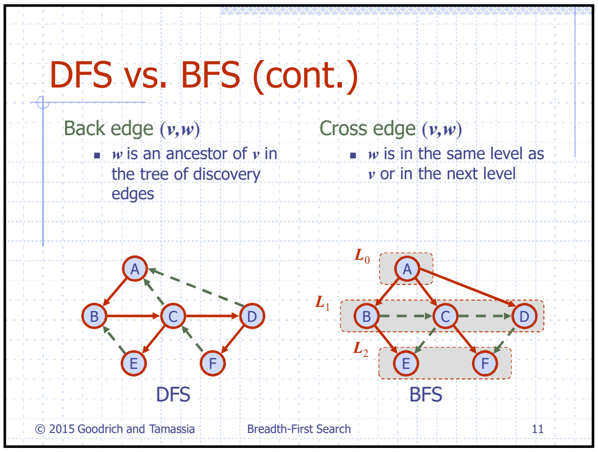
Applications

- We can use the BFS traversal algorithm, for a graph G, to solve the following problems in $O(n + m)$ time
  - Compute the connected components of G
  - Compute a spanning forest of G
  - Find a simple cycle in G, or report that G is a forest
  - Given two vertices of G, find a path in G between them with the minimum number of edges, or report that no such path exists



DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	√	√
Shortest paths		√
Biconnected components	√	



DFS vs. BFS (cont.)

Back edge (v,w)

- w is an ancestor of v in the tree of discovery edges

Cross edge (v,w)

- w is in the same level as v or in the next level