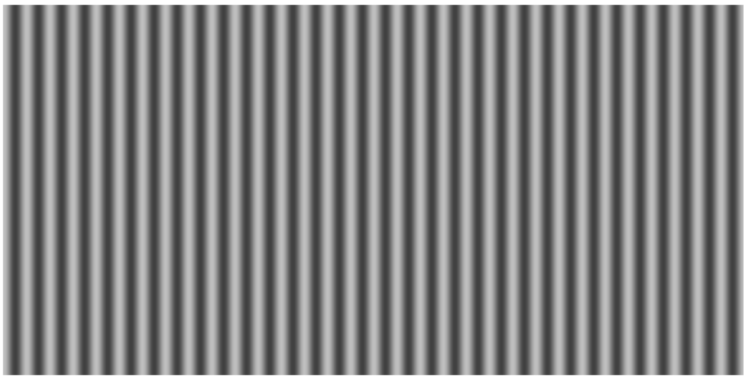
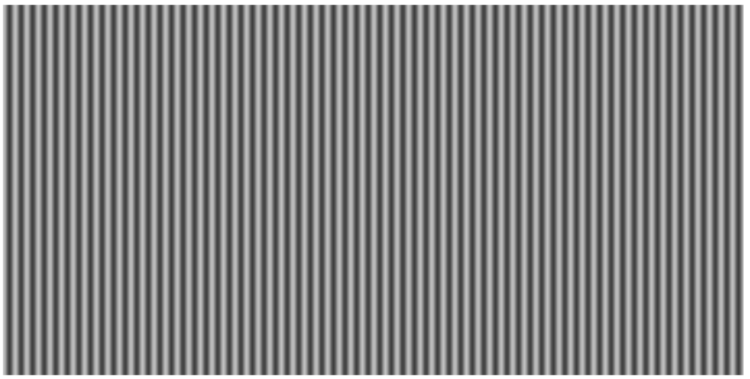
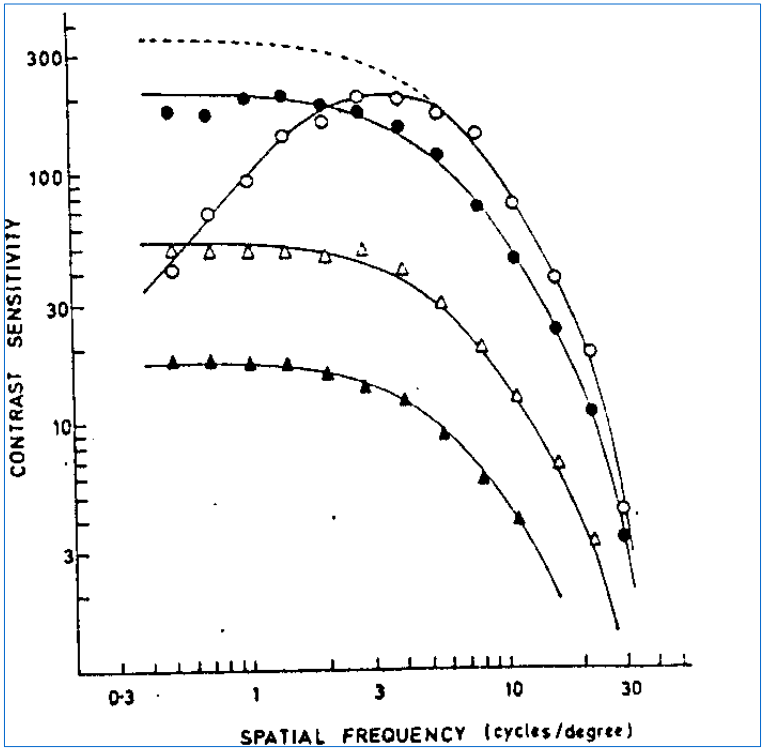
Μετασχηματισμός Fourier 2-Δ συνεχών σημάτων

Αναπαράσταση σε μια βάση σημάτων
Ορθοκανονική βάση
Περιοδικές συναρτήσεις
Αρμονικές συναρτήσεις

(1-Δ) αρμονικές συναρτήσεις







Ανάλυση σε αρμονικές συνιστώσες

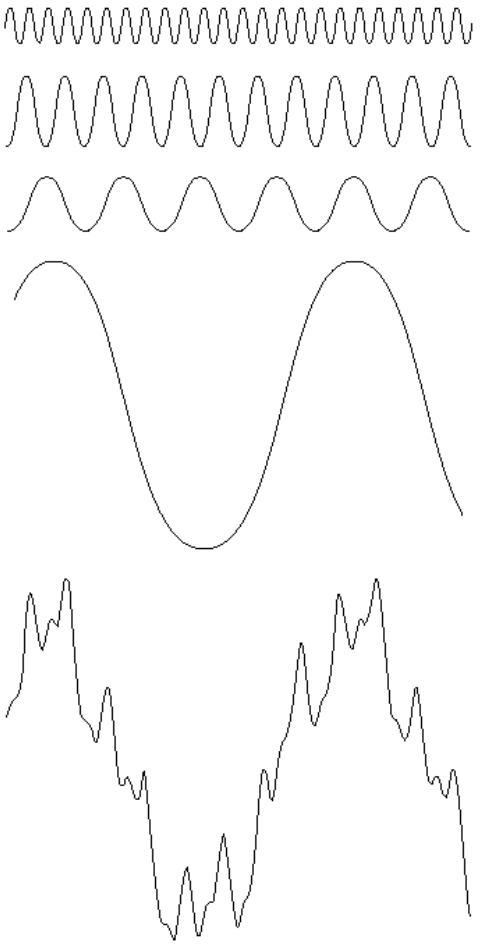


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

1-Δ Μετασχηματισμός Fourier

Ορθοκανονική βάση

$$\int_{-\infty}^{\infty} \varphi(x,u)\bar{\varphi}(x,v)dx = \delta(u-v)$$

Ευθύς μετασχηματισμός

$$F(u) = \int_{-\infty}^{\infty} f(x)\bar{\varphi}(x,u)dx$$

Αντίστροφος μετασχηματισμός

$$f(x) = \int_{-\infty}^{\infty} F(u)\varphi(x,u)du$$

$$\phi(x,u) = e^{j2\pi ux}, \quad u \in (-\infty,+\infty)$$

Ζεύγη μετασχηματισμού

$$f(x)=1 \quad \Leftrightarrow \quad F(u)=\delta(u)$$

$$f(x)=e^{j2\pi f_0 x} \quad \Leftrightarrow \quad F(u)=\delta(u-f_0)$$

$$f(x)=\cos(2\pi f_0 x) \quad \Leftrightarrow \quad F(u)=\frac{1}{2}\left(\delta(u-f_0)+\delta(u+f_0)\right)$$

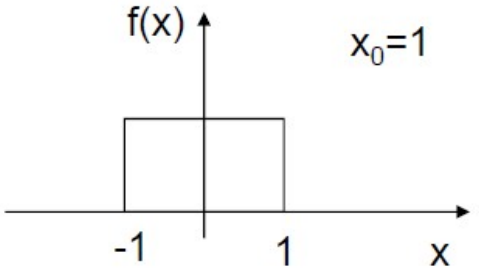
$$f(x)=\sin(2\pi f_0 x) \quad \Leftrightarrow \quad F(u)=\frac{1}{2j}\left(\delta(u-f_0)-\delta(u+f_0)\right)$$

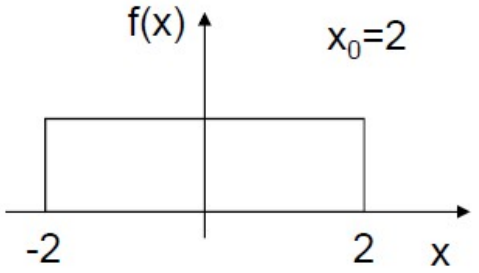
$$f(x)=\begin{cases} 1, & |x|<x_0 \\ 0, & otherwise \end{cases} \quad \Leftrightarrow \quad F(u)=\frac{\sin(2\pi x_0 u)}{\pi u}=2x_0\,\mathrm{sinc}(2x_0 u)$$

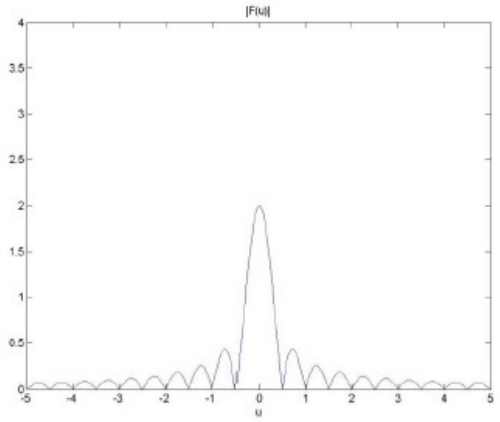
$$where,\ \mathrm{sinc}(t)=\frac{\sin(\pi t)}{\pi t}$$

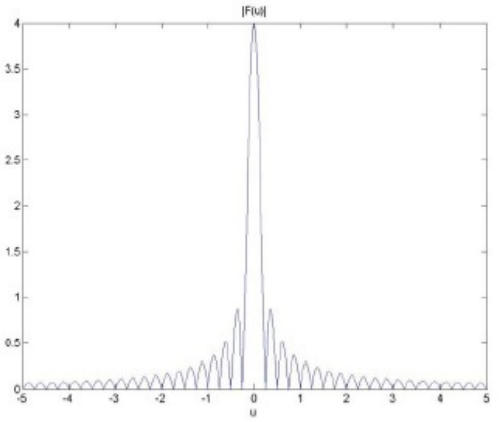
Μετασχηματισμός σταθερού παλμού

$$F(u) = \frac{\sin(2\pi x_0 u)}{\pi u} = 2x_0 \,\mathrm{sinc}(2x_0 u) \quad where, \mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$









Ιδιότητες μετασχηματισμού

Γραμμική $$F\{a_1 f_1(x) + a_2 f_2(x)\} = a_1 F\{f_1(x)\} + a_2 F\{f_2(x)\}$$

Δυϊκή $$f(t) \Leftrightarrow F(u)$$
$$F(t) \Leftrightarrow f(-u)$$

Μετατόπιση $$f(x - x_0) \Leftrightarrow F(u)e^{-j2\pi x_0 u}, \quad f(x)e^{j2\pi u_0 x} \Leftrightarrow F(u - u_0)$$

Συνέλιξη $$f(x) \otimes g(x) = \int f(x - \alpha) g(\alpha) d\alpha$$
$$f(x) \otimes g(x) \Leftrightarrow F(u)G(u)$$

2-Δ Μετασχηματισμός Fourier

Βάση

$$\phi(x,y;u,v) = e^{j(2\pi ux+2\pi vy)} = e^{j2\pi ux}e^{j2\pi vy}, \quad u,v \in (-\infty,+\infty).$$

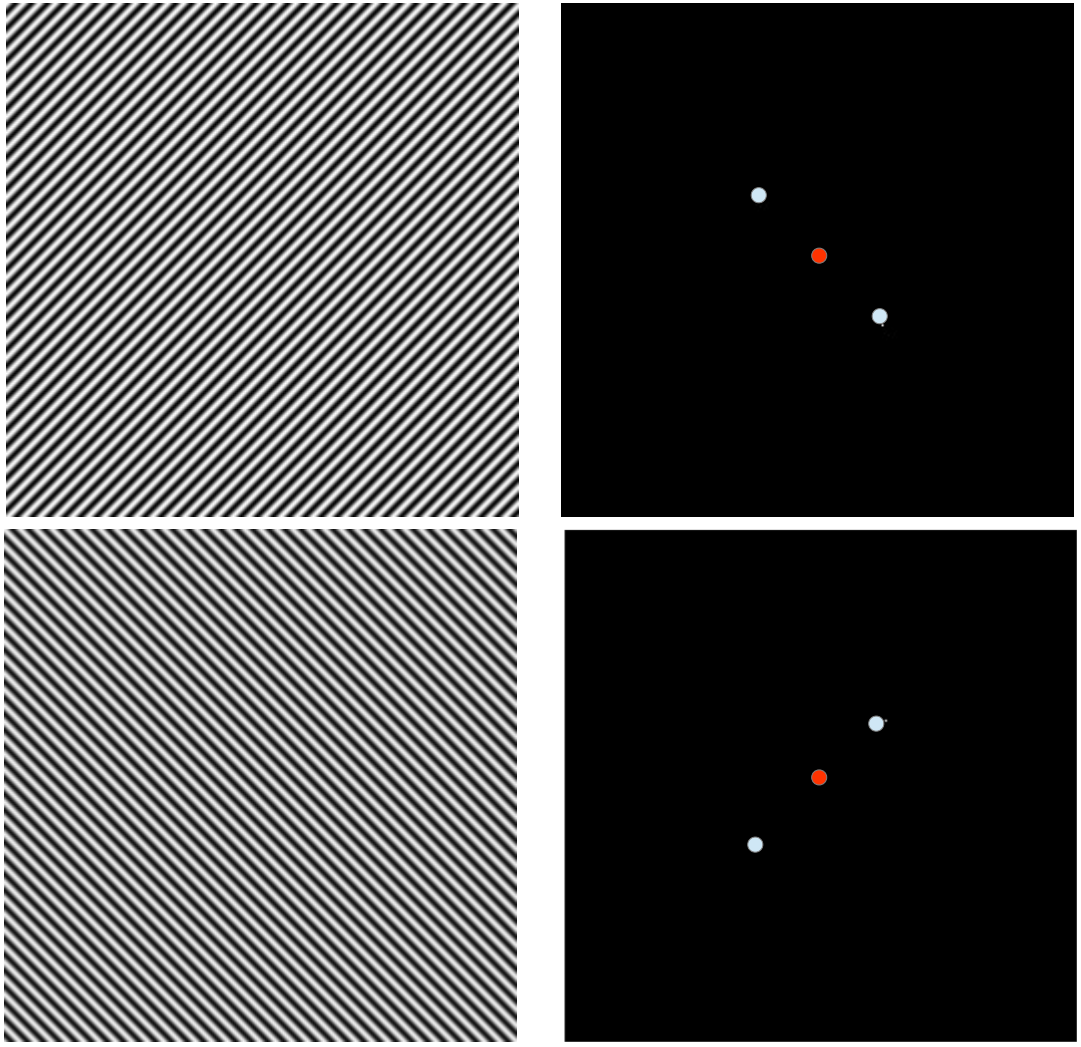
Ευθύς μετασχηματισμός

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\bar{\varphi}(x,y;u,v)\,dx\,dy$$

Αντίστροφος μετασχηματισμός

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\varphi(x,y;u,v)\,du\,dv$$

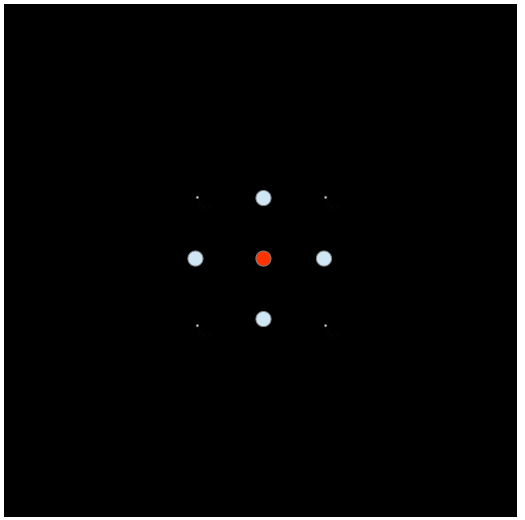
Μετασχηματισμός Fourier ημιτονοειδών
Περιστροφή



Σήμα Μετασχηματισμός

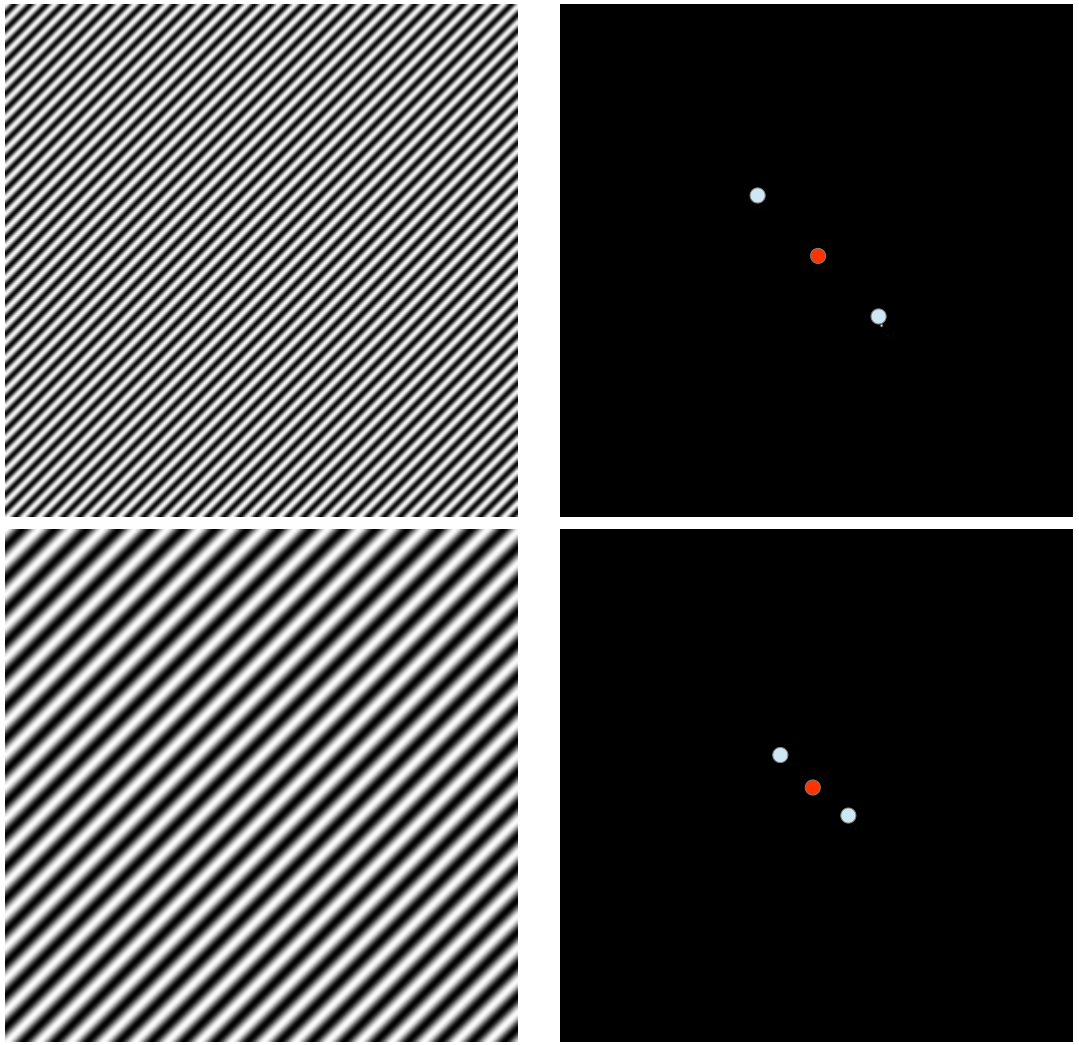
Μετασχηματισμός Fourier
Άθροισμα ημιτονοειδών

Σήμα



Μετασχηματισμός

Μετασχηματισμός Fourier ημιτονοειδών
Αλλαγή συχνότητας



Σήμα Μετασχηματισμός

Ιδιότητες 2-Δ μετασχηματισμού

Γραμμική $$F\{a_1 f_1(x,y) + a_2 f_2(x,y)\} = a_1 F\{f_1(x,y)\} + a_2 F\{f_2(x,y)\}$$

Μετατόπιση $$f(x-x_0, y-y_0) \quad \Leftrightarrow \quad F(u,v)e^{-j2\pi(x_0 u + y_0 v)},$$
$$f(x,y)e^{j2\pi(u_0 x + v_0 y)} \quad \Leftrightarrow \quad F(u-u_0, v-v_0)$$

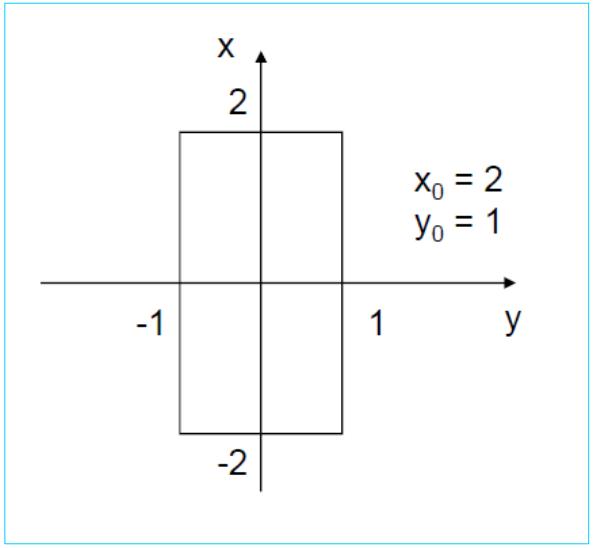
Συμμετρία $$f(x,y)\ is\ real \quad \Leftrightarrow \quad |F(u,v)| = |F(-u,-v)|$$

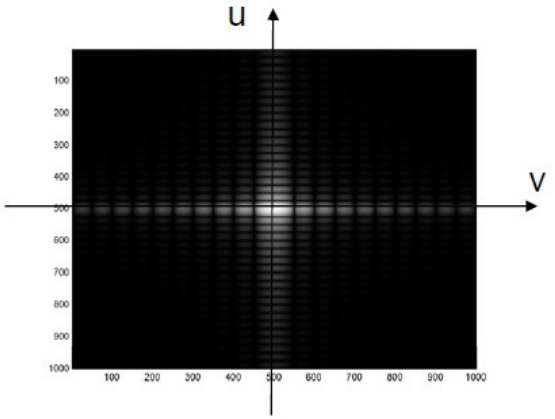
Συνέλιξη $$f(x,y) \otimes g(x,y) = \iint f(x-\alpha, y-\beta) g(\alpha,\beta) d\alpha d\beta$$
$$f(x,y) \otimes g(x,y) \quad \Leftrightarrow \quad F(u,v)G(u,v)$$

Διαχωρισιμότητα $f(x,y) = f_x(x) f_y(y)$
$F(u,v) = F_x(u) F_y(v)$

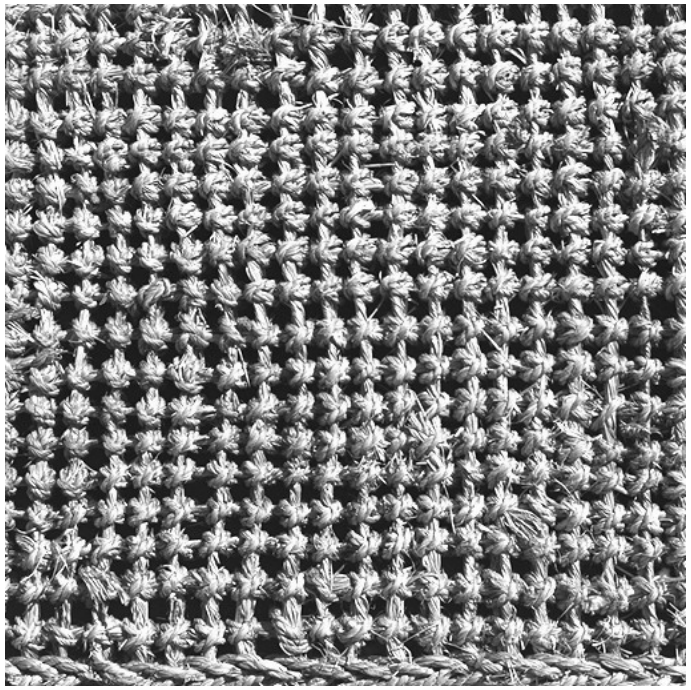
Αλλαγή κλίμακας $$\mathcal{F}[f(ax,by)] = \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right)$$

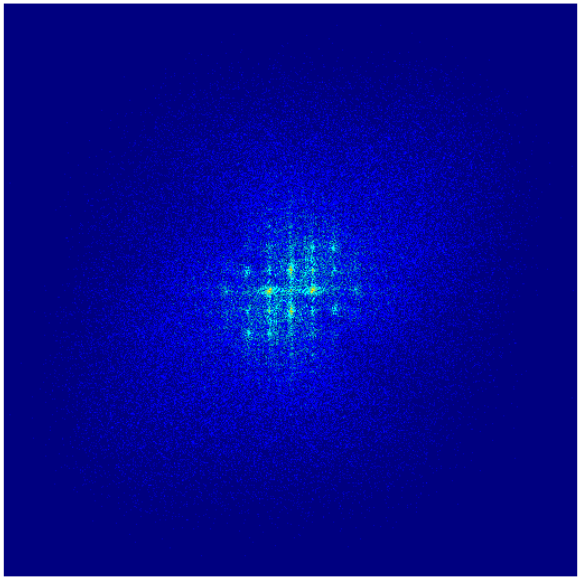
Μετασχηματισμός Fourier



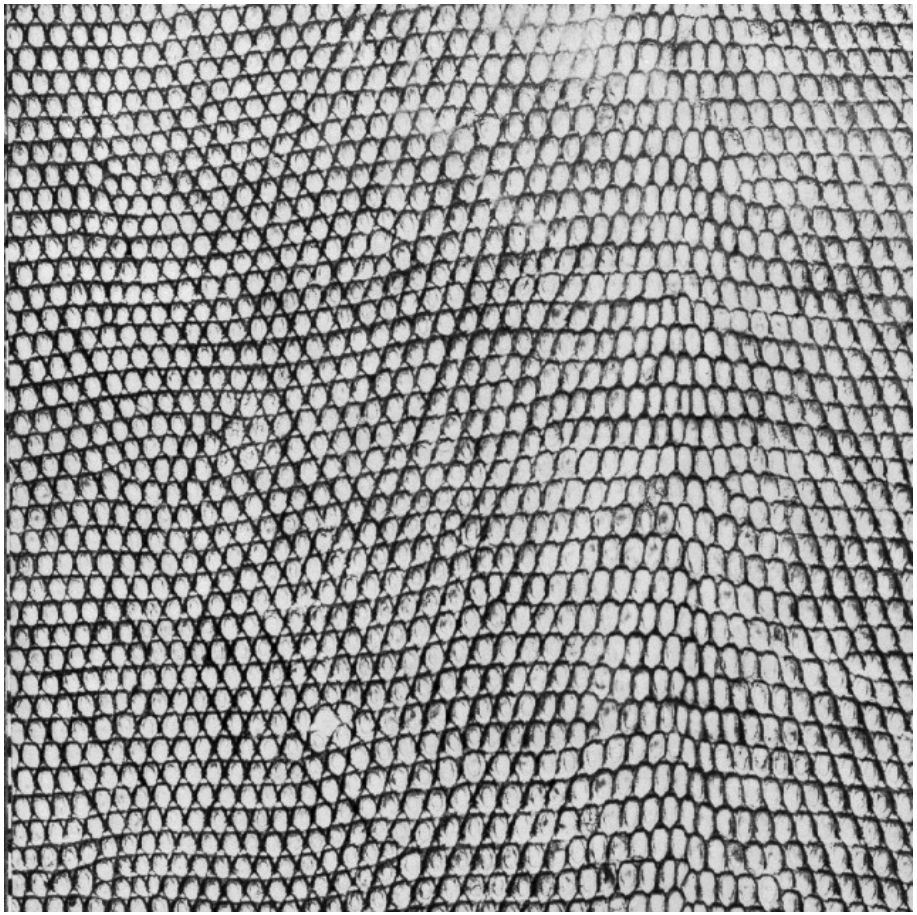


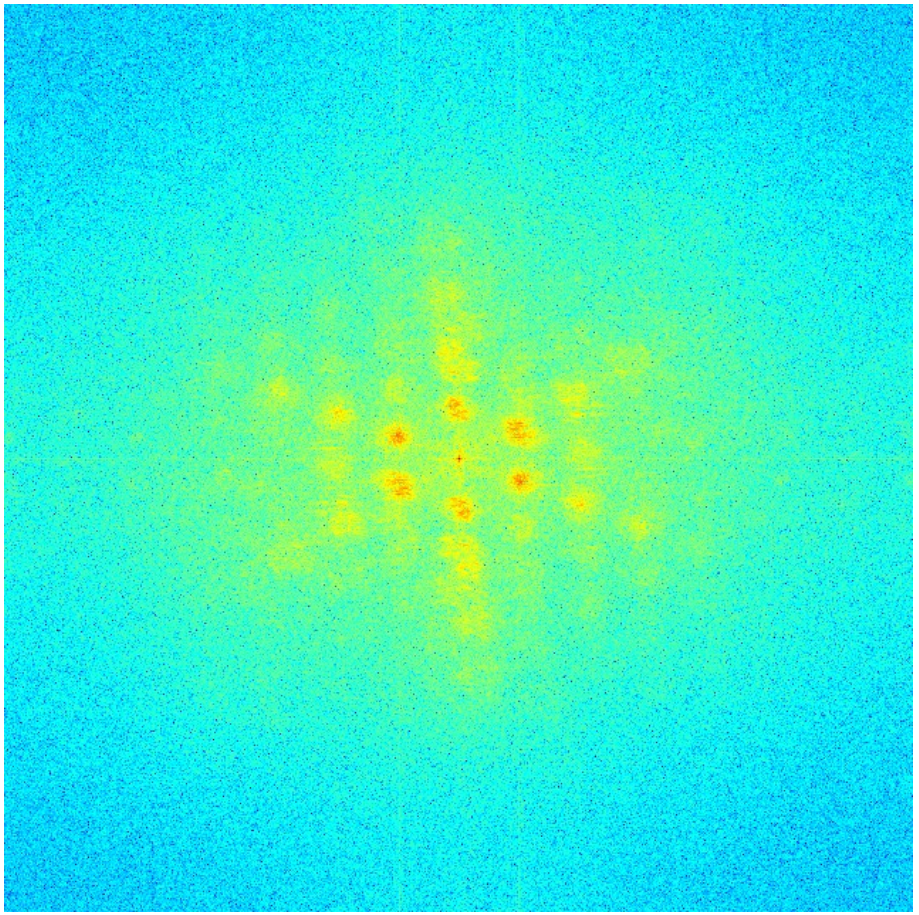
Μετασχηματισμός Fourier



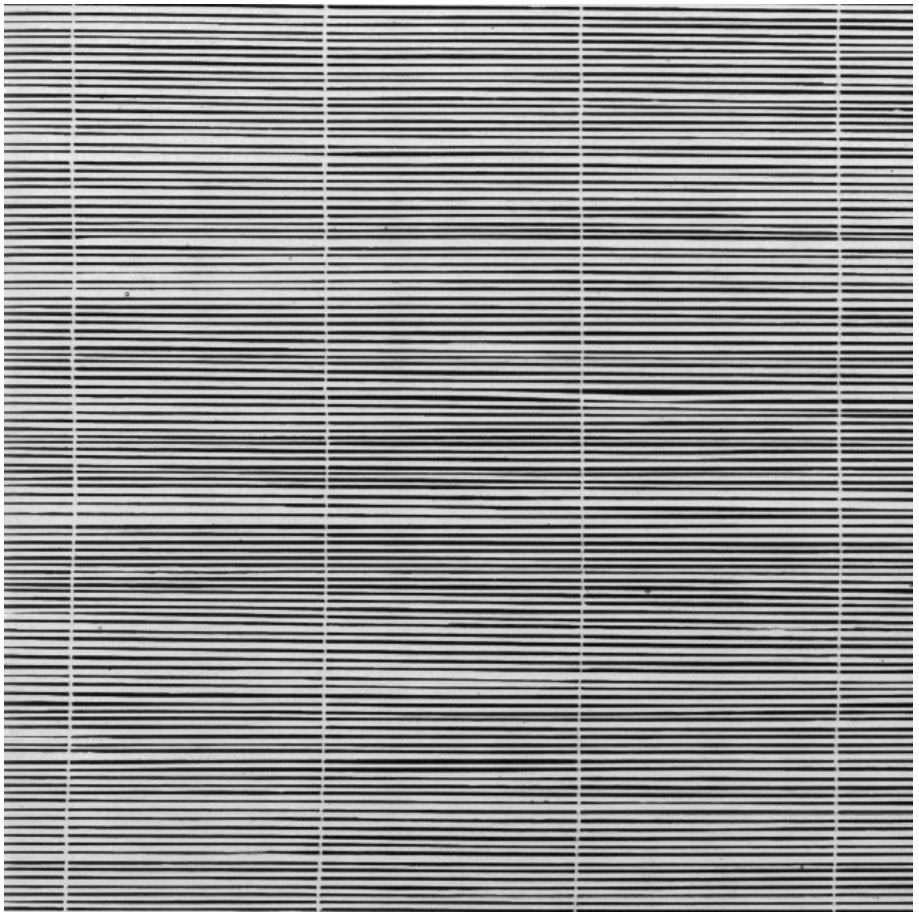


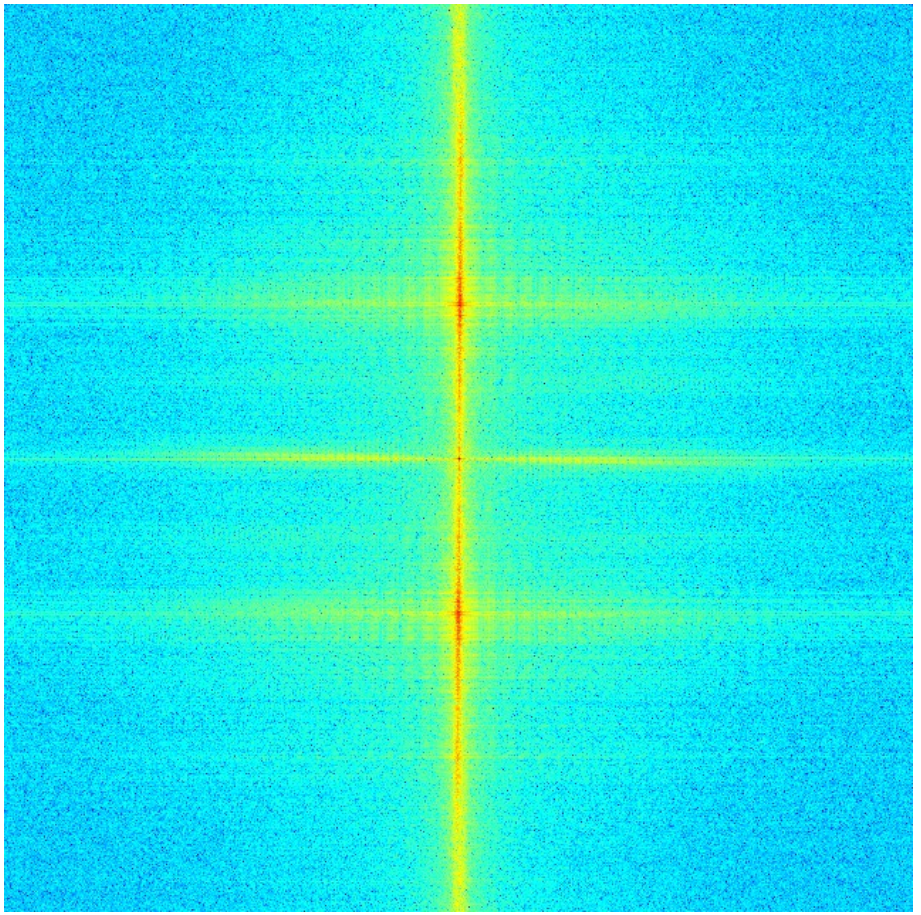
Μετασχηματισμός Fourier



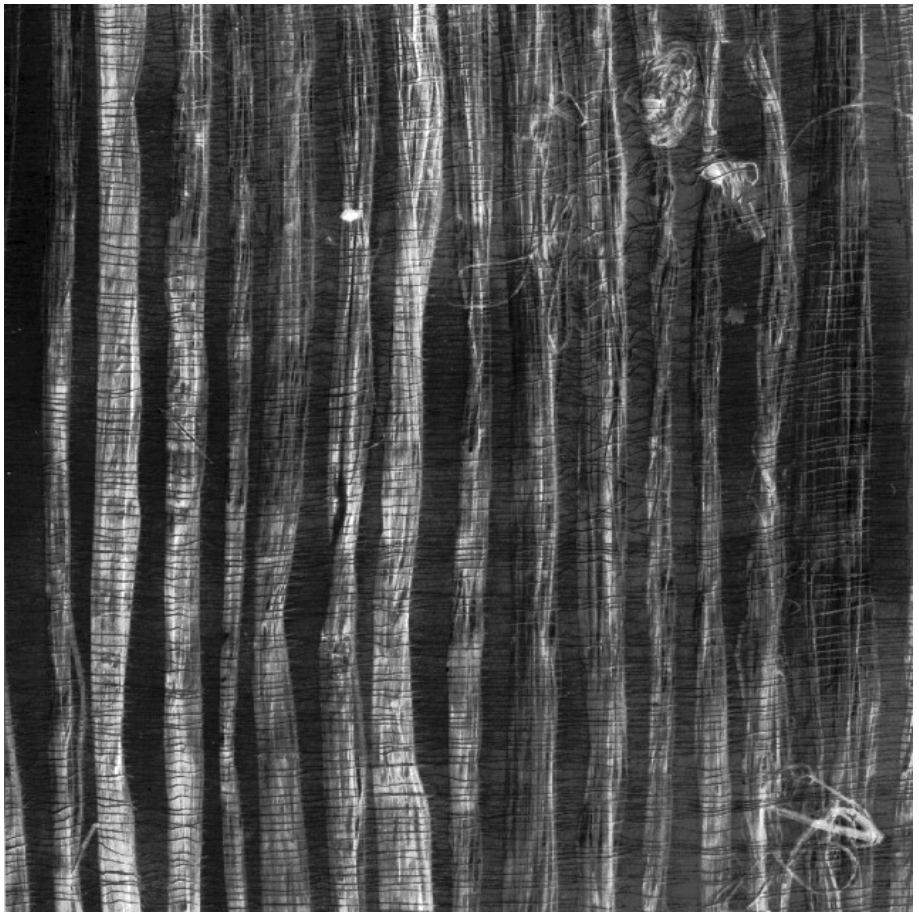


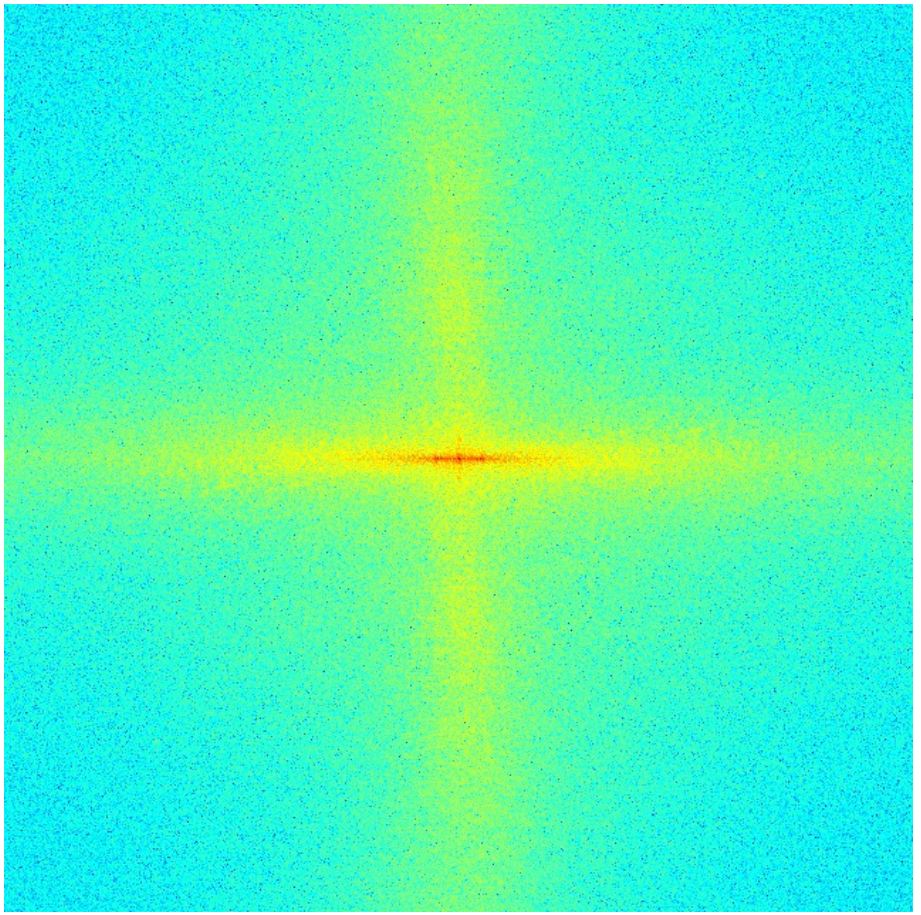
Μετασχηματισμός Fourier



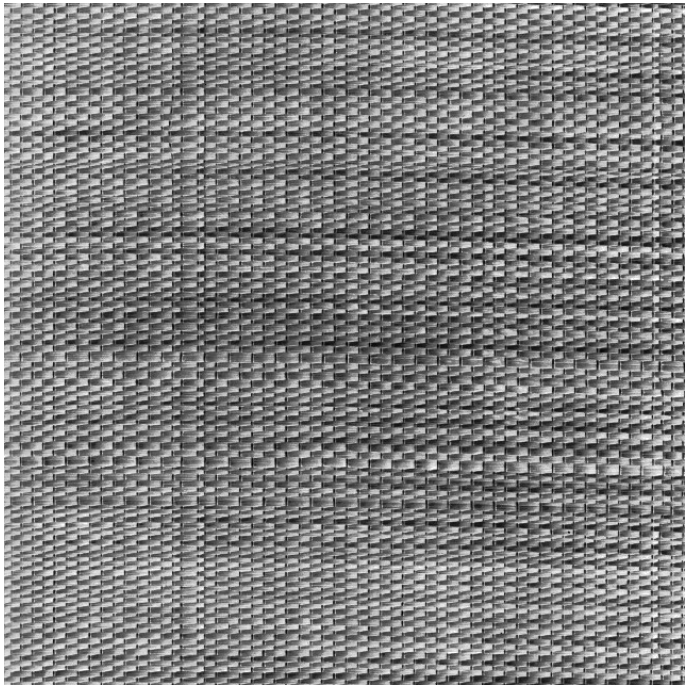


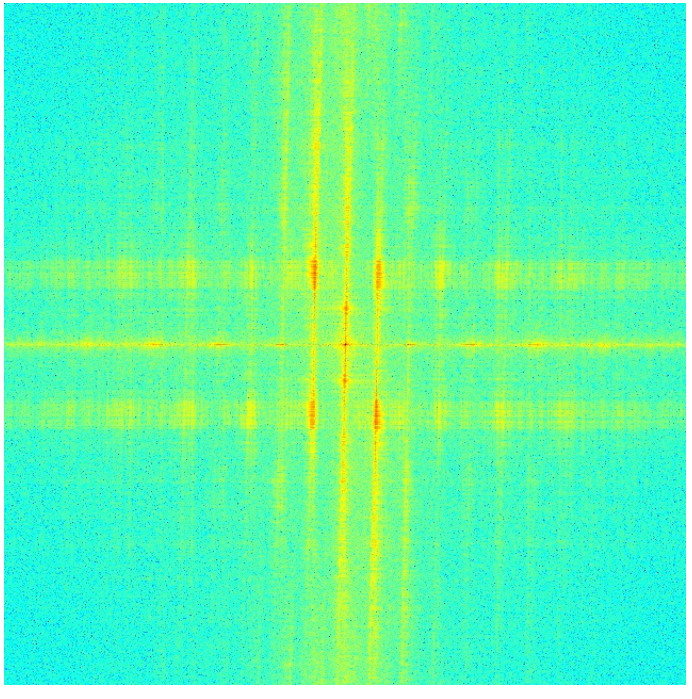
Μετασχηματισμός Fourier

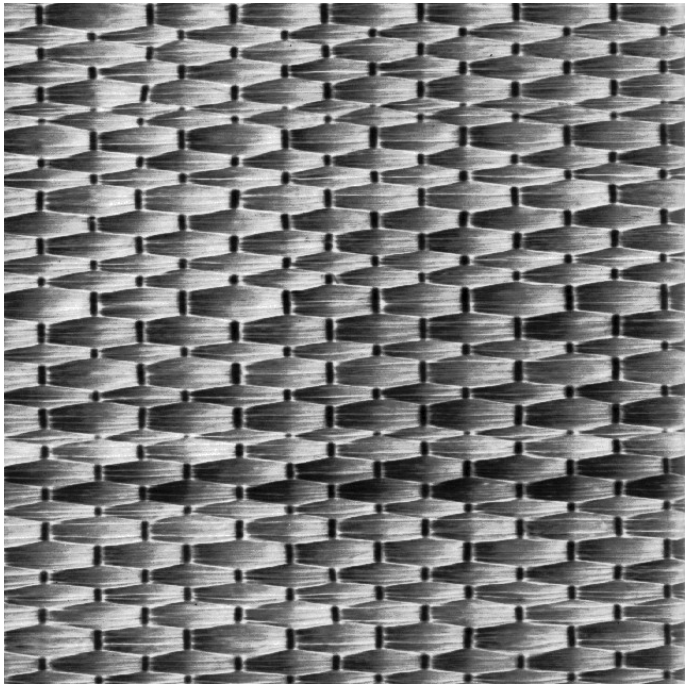


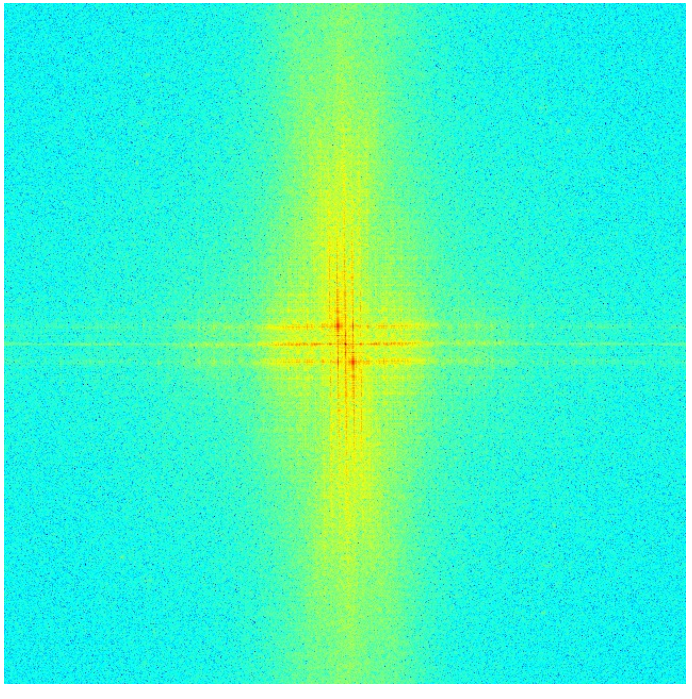


Μετασχηματισμός Fourier









Μετασχηματισμός Fourier