CS240 — Data Structures

1st Series of Exercises

Submission Instructions

Exercises can be submitted to the course assistants on **Monday, October 16, 2023, from 11:00 AM to 12:00 PM** at the TAs' office (**B.208** / **B.210**). Exercises submitted after 12:00 PM on Monday, 16/10/2023, will have a penalty. **Late submissions are accepted in electronic format**, and they must be submitted using the `turnin` program. For more information visit the course website.

Semester:	Spring (2023-24)
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Exercise 1 **[20 points]**

Let $T_1(n)$ be the total number of times the `x = x + 3` command is executed by procedure `Puzzle1()`, and $\mathrm{T}_2(n)$ be the total number of times the `x = x + 3` command is executed by procedure `Puzzle2()`. Calculate $T_1(n)$ and $\mathrm{T}_2(n)$. Analyze the order (denoted as O notation) of $T_1(n)$ and $\mathrm{T}_2(n)$.

```
procedure Puzzle1(integer n) {
  for i = 1 to sqrt(n) do
    for j = i to n do
      for k = 1 to 2n do
        x = x+3;
}
```

```
procedure Puzzle2(integer n)
  for i = 0 to 2n do
    if i is even then {
      for j = i to n do x = x+3;
    }
  }
```

Exercise 2 **[25 points]**

a. Which of the following is true and why?

- $n^2 - 2n - 7 = \Theta(n^2)$ [5P]
- $n \times \sum_{i=1}^{n}(3 \times (i+1)) = O(n^4)$ [5P]
- $n^2 \log n^2 - n \log n - n = \Theta(n^2 \log n)$ [5P]

b. Study the asymptotic complexity (using the Θ notation) of the function: [10P]

$$f(n) = 10n^3 \log n^6 + n^3\sqrt{n} - n \log n - 5$$

Exercise 3 **[20 points]**

Solve the following recursive relations using iterative substitution.

a. [5M]
$$T(n) := \begin{cases} 1, & \text{for } n \in \{0,1\} \\ 3T\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 3n, & \text{otherwise} \end{cases}$$

b. [5M]
$$T(n) := \begin{cases} 1, & \text{for } n \leq 0 \\ T(n-c) + c \cdot n, & \text{otherwise, where } c \text{ is an integer constant} \end{cases}$$

For the recursive relation of question a, the following are requested:

c. Draw the recursion tree. [5P]

d. Verify, using mathematical induction, that the solution you found with iterative substitution is correct. [5P]

Exercise 4 **[15 points]**

Consider the following function, which performs Egyptian multiplication:

$$m(x,y) := \begin{cases} 0, & \text{if } y = 0 \\ m\big(x+x, \frac{y}{2}\big), & \text{if } y \text{ even, and } y \neq 0 \\ x + m(x, y-1), & \text{if } y \text{ add, and } y \neq 0 \end{cases}$$

a. Present pseudocode for a recursive function [5P]

```
int EgyptianMultiplication(int x, int y)
```

which will perform Egyptian multiplication according to the above recursive formula.

b. Trace the execution of `EgyptianMultiplication(5,20)` (i.e., provide a detailed explanation of its execution, as well as the activation records in memory, when called with `x = 5` and `y = 20`). [10P]

Exercise 5 **[20 points]**

`InsertionSort()` can be expressed as a recursive procedure in the following way. To sort the array `A[1..n]`, we recursively sort the subarray `A[1..n-1]` and then we insert the element `A[n]` into the sorted subarray A[1..n-1].

a. Present pseudocode for this recursive version of `InsertionSort()`.

b. Formulate a recursive relation for the runtime of this recursive version of `InsertionSort()`.

c. Solve the recursive relation you proposed in question b. and study (based on the O and Ω notations) the order of complexity of the recursive version of `InsertionSort()`.

d. What is the space complexity of the algorithm (i.e., how much memory is required to run the algorithm)?