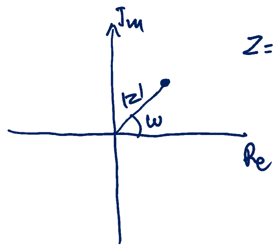
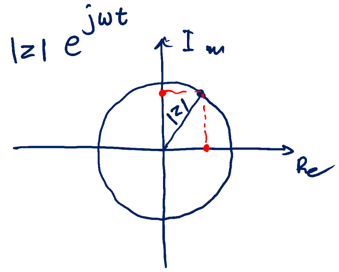
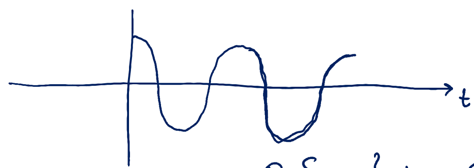
HY 215



$z = |z| \cdot e^{j\omega}$

$z(t) = |z|\, e^{j\omega t}$





$x(t) = \mathrm{Re}\{z(t)\} = |z| \cdot \mathrm{Re}\{e^{j\omega t}\} = |z|\, \mathrm{Re}\{\cos(\omega t) + j\sin(\omega t)\} =$

$= |z| \cos(\omega t)$

$x(t) = A\cos(\omega_0 t + \varphi)$ $\Rightarrow x(t) = \underbrace{\overbrace{\frac{A}{2} e^{j\varphi}}^{\text{Phasor}}}_{\text{Βάρη } X_{\omega_0}} \cdot \underbrace{e^{j\omega_0 t}}_{\text{συνάρτηση βάσης } \Psi_{\omega_0}} + \underbrace{\frac{A}{2} e^{-j\varphi}}_{X^*_{\omega_0}} \cdot \underbrace{e^{-j\omega_0 t}}_{\Psi^*_{\omega_0}} = X_{\omega_0} \cdot \Psi_{\omega_0} + X^*_{\omega_0} \Psi^*_{\omega_0}$

Euler

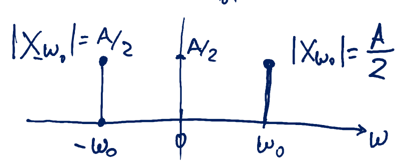
$\cos\theta = \frac{1}{2} e^{j\theta} + \frac{1}{2} e^{-j\theta}$

$|X_{\omega_0}| \quad \measuredangle X_{\omega_0}$

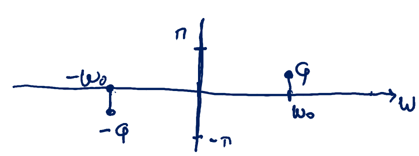
$= [X_{\omega_0} \quad X^*_{\omega_0}] \begin{bmatrix} \Psi_{\omega_0} \\ \Psi^*_{\omega_0} \end{bmatrix}$

Αναπαράσταση Συχνότητας:

A) Πλάτους $|X_{\omega_0}|$



B) Φάσης



Παράδειγμα:

$$x(t) = 3\cos t + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right) = \frac{3}{2}e^{jt} + \frac{3}{2}e^{-jt} + \frac{1}{2j}e^{-j\pi/6}e^{j5t} - \frac{1}{2j}e^{+j\pi/6}e^{-j5t} - \left(e^{-j\pi/3}e^{j8t} + e^{j\pi/3}\cdot e^{-j8t}\right)$$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

$$\frac{1}{2j} = \frac{j}{2j^2} = -\frac{j}{2} = \frac{1}{2}(-j) = \frac{1}{2}e^{-j\pi/2}$$

$$\frac{1}{2j}e^{-j\pi/6} = \frac{1}{2}e^{-j\pi/2}\cdot e^{-j\pi/6} = \frac{1}{2}e^{-j\left(\frac{3\pi}{6}+\frac{\pi}{6}\right)} = \frac{1}{2}e^{-j4\pi/6} = \frac{1}{2}e^{-j2\pi/3} \quad (1)$$

$$-e^{-j\pi/3} = (-1)e^{-j\pi/3} = e^{j\pi}e^{-j\pi/3} = e^{j(\pi-\pi/3)} = e^{j2\pi/3} \quad (2)$$

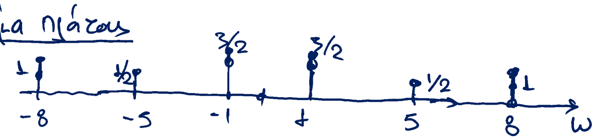
$$-e^{j\pi/3} = (-1)e^{j\pi/3} = e^{j\pi}\cdot e^{j\pi/3} = e^{j4\pi/3} = e^{-j2\pi/3} \quad (3) \qquad \frac{4\pi}{3} \bmod_{2\pi} = \frac{4\pi}{3} - 2\pi = \frac{4\pi}{3} - \frac{6\pi}{3} = -\frac{2\pi}{3}$$

βάρη

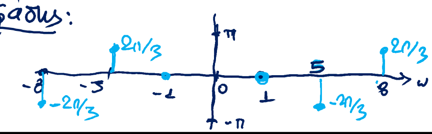
$$\text{Άρα: } x(t) = e^{-j2\pi/3}e^{-j8t} + \frac{1}{2}e^{j2\pi/3}e^{-j5t} + \frac{3}{2}e^{-jt} + \frac{3}{2}e^{jt} + \frac{1}{2}e^{-j2\pi/3}e^{j5t} + e^{j2\pi/3}e^{j8t}$$

συναρτήσεις βάσης

Φάσμα πλάτους



Φάσμα φάσης:



Γενικότερα:

$$x(t) = \sum_{k=0}^{L} A_k\cos(\omega_k t + \varphi_k) = \sum_{k=-L}^{L} \frac{A_k}{2}\cdot e^{j\varphi_k}\cdot e^{j\omega_k t} = (1)$$

$$\boxed{A_k = A_{-k} \quad \omega_k = -\omega_{-k} \quad \varphi_k = -\varphi_{-k}}$$

Euler: $\cos\theta = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}$

παράδειγμα $k=3$:

$$A_3\cos(\omega_3 t + \varphi_3) = \underbrace{\frac{A_3}{2}\cdot e^{-j\varphi_3}\cdot e^{-j\omega_3 t}} + \frac{A_3}{2}e^{j\varphi_3}\cdot e^{j\omega_3 t}$$

$$= \frac{A_{-3}}{2}e^{j\varphi_{-3}}\cdot e^{j\omega_{-3}t} + \frac{A_3}{2}e^{j\varphi_3}\cdot e^{j\omega_3 t}$$

$$(1)\quad x(t) = \underbrace{\frac{A_{-L}}{2}e^{j\varphi_{-L}}}_{X_{-L}}\underbrace{e^{j\omega_{-L}t}}_{\psi_{-L}(t)} + \dots + \underbrace{\frac{A_{-1}}{2}e^{j\varphi_{-1}}}_{X_{-1}}\underbrace{e^{j\omega_{-1}t}}_{\psi_{-1}(t)} + \underbrace{\frac{A_0}{2}e^{j\varphi_0}}_{X_0}\underbrace{e^{j\omega_0 t}}_{\psi_0(t)} + \underbrace{\frac{A_1}{2}e^{j\varphi_1}}_{X_1}\underbrace{e^{j\omega_1 t}}_{\psi_1(t)} + \dots \underbrace{\frac{A_L}{2}e^{j\varphi_L}}_{X_L}\underbrace{e^{j\omega_L t}}_{\psi_L(t)} \Rightarrow$$

$$\varphi_0 = 0,\ \omega_0 = 0 \quad \Big| \quad \psi_0(t) = 1,\ e^{j\varphi_0} = 1$$

$$\Rightarrow x(t) = X_{-L}\,\psi_{-L}(t) + \dots + X_{-1}\,\psi_{-1}(t) + X_0\,\cancel{\psi_0(t)} + X_1\,\psi_1(t) + \dots + X_L\,\psi_L(t)$$

Ορισμός εσωτερικού γινομένου:

$$\psi_k(t) = e^{j\omega_k t} \qquad \omega_k = k\omega_0 \qquad \Big| \Rightarrow \psi_k(t) = e^{jk\omega_0 t}$$

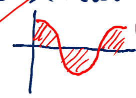
$\psi_k(t) = \psi_k(t+T_0)$ όπου $T_0 = \frac{2\pi}{\omega_0}$

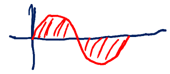
$$\langle \psi_k(t), \psi_\ell(t) \rangle_{(t_1,t_2)} = \int_{t_1}^{t_2} \psi_k(t)\cdot\psi_\ell^*(t)\,dt$$

όταν $\psi_k(t) = e^{jk\omega_0 t}$ $\qquad k,\ell \in \mathbb{Z}$, $\ k-\ell = m \in \mathbb{Z}$

$$\langle \psi_k(t), \psi_\ell(t) \rangle_{(0,T_0)} = \int_0^{T_0} \psi_k(t)\cdot\psi_\ell^*(t)\,dt = \int_0^{T_0} e^{jk\omega_0 t}\cdot e^{-j\ell\omega_0 t}\,dt = \int_0^{T_0} e^{j(k-\ell)\omega_0 t}\,dt =$$

$$k\neq\ell: \quad \int_0^{T_0} e^{jm\omega_0 t}\,dt = \int_0^{T_0} \cos(m\omega_0 t)\,dt + j\int_0^{T_0}\sin(m\omega_0 t)\,dt = 0$$





$$k=\ell: \quad \int_0^{T_0} e^{j0t}\,dt = \int_0^{T_0} dt = T_0$$

$$\Rightarrow \boxed{\int_0^{T_0} \psi_k(t)\cdot\psi_\ell^*(t)\,dt = \begin{cases} 0, & k\neq\ell \\ T_0, & k=\ell \end{cases}}$$

$\langle \psi_k(t), \psi_\ell(t) \rangle$

$$x(t) = X_{-L}\psi_{-L}(t) + X_{-(L-1)}\psi_{-(L-1)}(t) + \dots + X_{-1}\psi_{-1}(t) + X_0 + X_1\psi_1(t) + \dots X_{L-1}\psi_{L-1}(t) + X_L\cdot\psi_L(t)$$

$x(t) = x(t+T_0)$

$\psi_k(t) = e^{jk\omega_0 t}$

$$\langle x(t), \psi_1(t) \rangle_{(0,T_0)} = \int_0^{T_0} x(t)\cdot\psi_1^*(t)\,dt = \int_0^{T_0} x(t)\cdot e^{-j\omega_0 t}\,dt$$

$$\Big\langle \sum_{k=-L}^{L} X_k\,\psi_k(t), \psi_1(t) \Big\rangle = \sum_{k=-L}^{L} X_k \underbrace{\langle \psi_k(t), \psi_1(t) \rangle}_{\begin{cases} 0 & k\neq 1 \\ T_0 & k=1\end{cases}} = X_1\cdot T_0 \Rightarrow$$

$$\Rightarrow X_1 = \frac{1}{T_0}\langle x(t), \psi_1(t) \rangle_{(0,T_0)}$$ και γενικότερα:

$$\boxed{X_k = \frac{1}{T_0}\langle x(t), \psi_k(t) \rangle_{(0,T_0)}}$$