HY-215

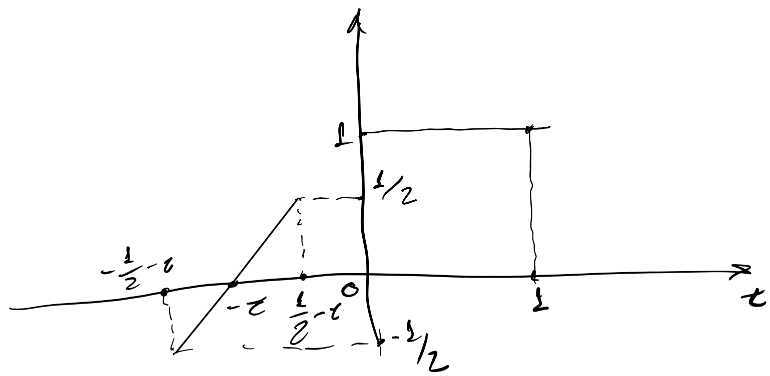
Φροντιστήριο 7

Άσκηση 1 (HW6-2015-16 Άσκηση 1)

$x(t) = rect\left(\frac{t-\frac{1}{2}}{1}\right)$

φ_{xy}? φ_{yx}?

$\Rightarrow y(t) = \begin{cases} t, & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0, & \text{αλλού} \end{cases}$



▷ $\frac{1}{2} - \tau < 0 \Rightarrow \tau > \frac{1}{2}$, $\varphi_{xy}(t) = 0$

▷ $-\frac{1}{2} - \tau > 1 \Rightarrow \tau \le \frac{-3}{2}$, $\varphi_{xy}(t) = 0$

▷ Για $\frac{-1}{2} < \tau < \frac{1}{2}$: $\varphi_{xy}(t) = \int_{-\infty}^{+\infty} x(t)\, y(t+\tau)\, dt \Rightarrow$

$\Rightarrow \int_0^{1/2-\tau} 1\cdot(t+\tau)\, dt = \left[\frac{t^2}{2} + t\cdot\tau\right]_0^{\frac{1}{2}-\tau} = \frac{1}{2}\left(\frac{1}{2}-\tau\right)^2 + \left(\frac{1}{2}-\tau\right)\tau$

$= \frac{1}{2}\left(\frac{1}{4} - \tau + \tau^2\right) + \frac{1}{2}\tau - \tau^2 = \frac{1}{8} - \frac{1}{2}\tau + \frac{1}{2}\tau^2 + \frac{1}{2}\tau - \tau^2$

$= \frac{1}{8} - \frac{1}{2}\tau^2$

• $\frac{-3}{2} < \tau < \frac{-1}{2}$: $\varphi_{xy} = \int_{-\frac{1}{2}-\tau}^{1} 1(t+\tau)\,dt = \dots = \frac{3}{8} + \tau + \frac{1}{2}\tau^2$

$\varphi_{yx}(\tau) = \varphi_{xy}(-\tau)$ . . .

Άσκηση 2 (Hw6-2016-17 Άσκηση 3)

$x(t) = e^{-2t}u(t)$

$y(t) = e^{t}u(-t)$

a) φ_{xx}?

$\varphi_{xx}(\tau) = \int_{-\infty}^{+\infty} e^{-2t}u(t)\, e^{-2(t+\tau)} u(t+\tau)\,dt =$

$= e^{-2\tau} \int_{-\infty}^{+\infty} e^{-2t} e^{-2t} u(t)u(t+\tau)\,dt = e^{-2\tau} \int_{-\infty}^{+\infty} e^{-4t} \underbrace{u(t)u(t+\tau)}\,dt$

$u(t)\,u(t+\tau) = \begin{cases} 1, & t>0,\ t>-\tau \\ 0, & \text{αλλού} \end{cases}$

• $-\tau < 0 \Rightarrow \tau > 0$: $\varphi_{xx}(\tau) = e^{-2\tau} \int_{0}^{+\infty} e^{-4t}\,dt = \dots = \frac{1}{4} e^{-2\tau}, \tau > 0$

• $-\tau > 0 \Rightarrow \tau < 0$: $\varphi_{xx}(\tau) = e^{-2\tau} \int_{-\tau}^{+\infty} e^{-4t}\,dt = \dots = \frac{1}{4} e^{2\tau}, \tau < 0$

Συνολικά: $\varphi_{xx}(\tau) = \frac{1}{4}\left(e^{-2\tau}u(\tau) + e^{2\tau}u(-\tau)\right)$

b) $\Phi_{yy}(f)$?

$\Phi_{yy}(f) = |Y(f)|^2 = \left|\frac{1}{-1+j2\pi f}\right|^2 = \frac{1}{1+4\pi^2 f^2}$

γ.) $\varphi_{xy}(\tau)$?

$\Phi_{xy}(f)$?

$$\varphi_{xy}(\tau) = \int_{-\infty}^{+\infty} e^{-2t} u(t) e^{(t+\tau)} u(-t-\tau)\, dt$$

$$\underline{u(t)u(-t-\tau)} = \begin{cases} 1, & 0<t<-\tau \\ 0, & \text{αλλού} \end{cases}$$

- $-\tau < 0 \Rightarrow \tau > 0$: $\varphi_{xy} = 0$
- $-\tau > 0 \Rightarrow \tau < 0$: $\varphi_{xy}(\tau) = \int_0^{-\tau} e^{-t} dt = (e^{\tau} - e^{-2\tau})$

$$\varphi_{xy} = (e^{\tau} - e^{-2\tau})\, u(-\tau)$$

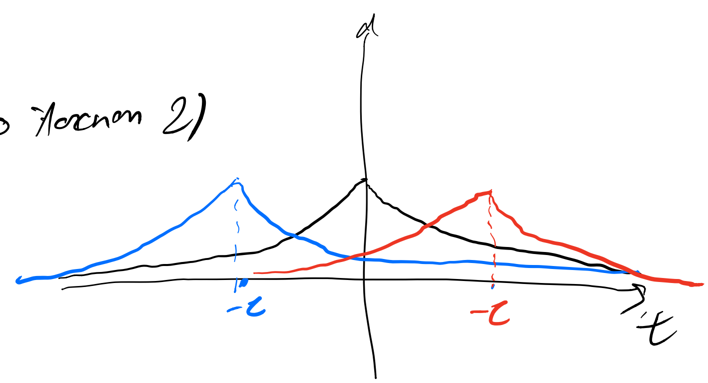
$$\Phi_{xy}(f) = X^*(f)\, Y(f) = \left(\frac{1}{2+j2\pi f}\right)^* \frac{1}{1-j2\pi f} = \dots$$

$$\frac{1}{(2-j2\pi f)(1-j2\pi f)}$$

Άσκηση 3 (HW 6-2019-20 Άσκηση 2)

$x(t) = e^{-a|t|}$, $a>0$

φ_{xx}?



$-\tau < 0 \Rightarrow \tau > 0$

$$\varphi_{xx}(\tau) = \int_{-\infty}^{-\tau} e^{at+a\tau} e^{at}\, dt + \int_{-\tau}^{0} e^{-at-a\tau} e^{at}\, dt +$$

$$+ \int_{0}^{+\infty} e^{-at-a\tau} e^{-at}\, dt = \ldots = \underline{\frac{1}{a} e^{-a\tau} + \tau e^{-a\tau}}$$

$-\tau > 0 \Rightarrow \tau < 0$:

$$\varphi_{xx}(\tau) = \int_{-\infty}^{0} e^{at+a\tau} e^{at}\, dt + \int_{0}^{-\tau} e^{at-a\tau} e^{a\tau}\, dt$$

$$+ \int_{-\tau}^{+\infty} e^{-at-a\tau} e^{-a\tau}\, dt = \ldots = \varphi_{xx}(\tau) = \frac{1}{a} e^{a\tau} - \tau e^{a\tau}$$

Συνολικά: $\underline{\varphi_{xx}(\tau) = \left(\frac{1}{a} + |\tau|\right) e^{-a|\tau|}}$

<u>Άσκηση 4</u> (HW6-2019-20 Άσκηση 3)

$x_1(t) = \cos(12\pi t) \longleftrightarrow \cos(12\pi \frac{n}{16}) = \cos\left(\frac{12}{16}\pi n\right) = \underline{\cos\left(\frac{3}{4}\pi n\right)}$

$x_2(t) = \cos(20\pi t) \longleftrightarrow \cos(20\pi \frac{n}{16}) = \cos\left(5\pi \frac{n}{4}\right) =$

$x_3(t) = \cos(44\pi t) \longleftrightarrow \cos\left(\frac{44}{16}\pi n\right)$ $= \cos\left(\frac{8\pi}{4}n - \frac{3\pi n}{4}\right) =$

$= \cos\left(\frac{11}{4}\pi n\right) = \cos\left(\frac{8\pi n}{4} + \frac{3\pi n}{4}\right)$ $\cos\left(-\frac{3}{4}\pi n\right) = \underline{\cos\left(\frac{3}{4}\pi n\right)}$

$f_s = 16\,Hz$ Ν.δ.ο. $= \underline{\cos\left(\frac{3}{4}\pi n\right)}$

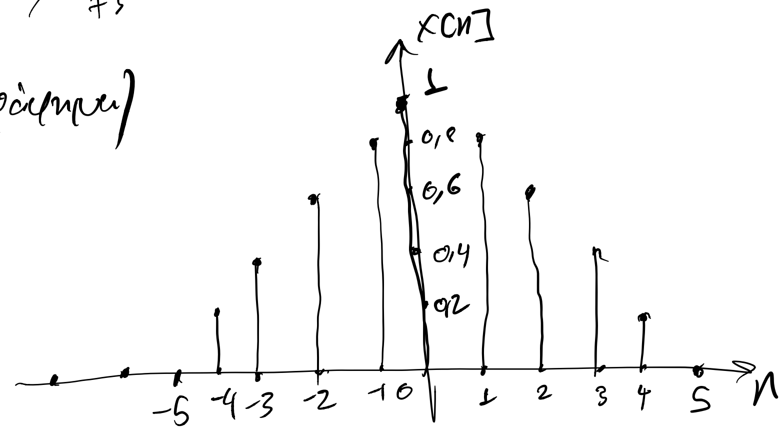
$x_1[n] = x_2[n] = x_3[n]$ ✓

Άσκηση 5 (HW6 - 2019-20 Άσκηση 4)

$x(t) = A\,tri\left(\frac{t}{T}\right) \quad X(f) = AT\,sinc^2(fT)$

Έστω $A=1$, $T=1$, $f_s = 5\,Hz$

α.) $x[n]$? (γράφημα)



β.) $X_s(f)$? $\quad X_s(f) = \frac{1}{T_s}\sum_{k=-\infty}^{+\infty} X(f - k\cdot f_s)$

$$X_s(f) = 5\sum_{k=-\infty}^{+\infty} X(f-5k) = 5\sum_{k=-\infty}^{+\infty} sinc^2(f-5k),\ k\in \mathbb{Z}$$

γ.) Γράφημα X_s (-10...10 Hz)