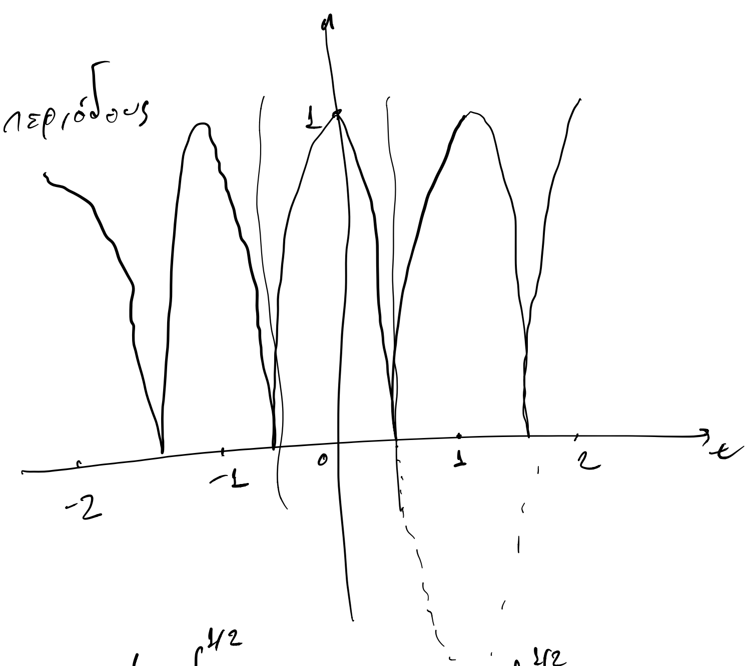
ΗΥ-215

Φροντιστήριο 4

Άσκηση 1 - (Ορισμός) (HW3-2018-19 Άσκηση 1)

$x(t) = |\cos(\pi t)|$

α.) T_0? Σχεδ. μερικές περιόδους



$T_0 = 1$

β) Εκθ. Σ.F.? $X_0 = \frac{1}{T_0}\int_{-1/2}^{1/2} x(t)\,dt = \int_{-1/2}^{1/2} |\cos(\pi t)|\,dt$

$$= 2\int_0^{1/2} \cos(\pi t)\,dt = \frac{2}{\pi}\Big[\sin(\pi t)\Big]_0^{1/2} = \frac{2}{\pi}$$

$$X_k = \frac{1}{T_0} \int_{-1/2}^{1/2} x(t) e^{-j2\pi k f_0 t} dt =$$

$$= \frac{1}{1} \int_{-1/2}^{1/2} \cos(\pi t) e^{-j2\pi k t} dt =$$

$$= \int_{-1/2}^{1/2} \left(\frac{1}{2} e^{j\pi t} + \frac{1}{2} e^{-j\pi t} \right) e^{-j2\pi k t} dt =$$

$$= \frac{1}{2} \int_{-1/2}^{1/2} e^{j\pi t} e^{-j2\pi k t} dt + \frac{1}{2} \int_{-1/2}^{1/2} e^{-j\pi t} e^{-j2\pi k t} dt$$

$$= \frac{1}{2} \int_{-1/2}^{1/2} e^{-j\pi(2k-1)t} dt + \frac{1}{2} \int_{-1/2}^{1/2} e^{-j\pi(2k+1)t} dt$$

$$= \frac{1}{2} \left(\frac{1}{-j\pi(2k-1)} e^{-j\pi(2k-1)t} \right]_{-1/2}^{1/2} + \frac{1}{2} \left[\frac{1}{-j\pi(2k+1)} e^{-j\pi(2k+1)t} \right]_{-1/2}^{1/2}$$

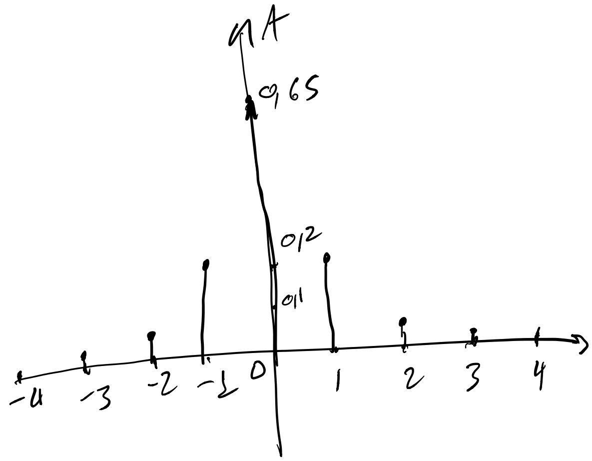
$$= \frac{-1}{j2\pi(2k-1)} \left(e^{-j\pi(2k-1)/2} - e^{j\pi(2k-1)/2} \right) -$$

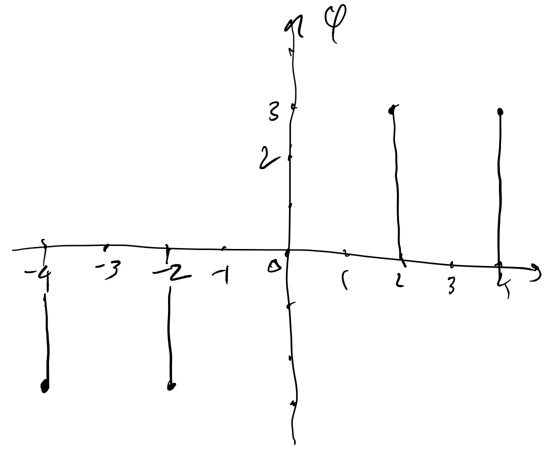
$$\frac{1}{j2\pi(2k+1)} \left(e^{-j\pi(2k+1)/2} - e^{j\pi(2k+1)/2} \right) =$$

$$= \frac{-1}{j2\pi(2k-1)}\left(-2j\sin(\pi(2k-1)/2)\right) - \frac{1}{j2\pi(2k+1)}\left(-2j\sin\left(\pi\frac{(2k+1)}{2}\right)\right)$$

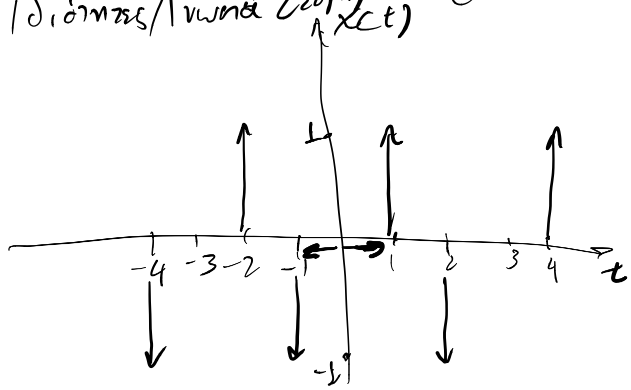
$$= \frac{\sin(\pi(2k-1)/2)}{\pi(2k-1)} + \frac{\sin(\pi(2k+1)/2)}{\pi(2k+1)}$$

γ.) Φ.Π., Φ.Φ. για $k = \pm 4, \pm 3, \pm 2, \pm 1, 0$





Άσκηση 2 (Ιδιότητες/Γνωστά Ζεύγη) (HW3-2018-19 Άσκηση 2)



α.) $T_0 = 3$

b) X_k ?

$$x(t) = \begin{cases} x_1(t) = \sum_{k=-\infty}^{+\infty} \delta(t-3k-\underbrace{1}) \\ + \\ x_2(t) = -\sum_{k=-\infty}^{+\infty} \delta(t-3k+\underbrace{1}) \end{cases}$$

Έστω $x_0(t) = \sum_{k=-\infty}^{+\infty} \delta(t-kT_0) \longleftrightarrow X_k = \frac{1}{T_0}$

$$\left[x(t-t_0) \longleftrightarrow X_k e^{-j2\pi k f_0 t_0} \right]$$

$X_{1k} = \frac{1}{3} \cdot e^{-j2\pi k \cdot \frac{1}{3}}$ $\quad$ $X_{2k} = -\frac{1}{3} e^{j2\pi k \cdot \frac{1}{3}}$

$$X_k = X_{1k} + X_{2k} = \frac{1}{3} e^{-j2\pi k \frac{1}{3}} - \frac{1}{3} e^{j2\pi k \frac{1}{3}} =$$

$$= \frac{-2j}{3} \sin\left(\frac{2\pi k}{3}\right) = \frac{2}{3} e^{-j\frac{\pi}{2}} \sin\left(\frac{2\pi k}{3}\right)$$

Άσκηση 3 (HW3-2017-18 Άσκηση 4)

Έστω $x(t)$ περιοδικό $\quad$? x_1, x_2 που ικανοποιούν

- $x(t) \in \mathbb{R}$, $x(t)$ περιττό ①
- $T_0 = 2$, X_k? ②
- $X_k = 0$, $|k| > 1$ ③
- $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1$ ④

Με βάση το (3), $x(t) = X_1^* e^{-j2\pi\frac{1}{2}t} + X_0 + X_1 e^{j2\pi\frac{1}{2}t}$

αφού $X_k = 0,\ k>1,\ k<-1$

Από (1), $x(t)$ περιττό άρα $\underline{X_k = -X_{-k}}$ (ιδιότητες)

και X_k φανταστικά

$\Rightarrow X_0 = 0$

Από (4) και Θ. Parseval, $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 1 \Rightarrow$

$\sum_{k=-1}^{1} |X_k|^2 = 1 \Rightarrow 2|X_1|^2 + |X_0|^2 = 1 \Rightarrow 2|X_1|^2 = 1$

$\Rightarrow X_1 = \pm j\frac{1}{\sqrt{2}} \Rightarrow X_{-1} = \mp j\frac{1}{\sqrt{2}}$

Αν $X_1 = j\frac{1}{\sqrt{2}},\ X_{-1} = -j\frac{1}{\sqrt{2}}$

Αν $X_1 = -j\frac{1}{\sqrt{2}},\ X_{-1} = j\frac{1}{\sqrt{2}}$

$j = -\frac{1}{j}$

Άρα $\underline{x_1(t)} = j\cdot\frac{1}{\sqrt{2}}e^{j\pi t} - j\frac{1}{\sqrt{2}}e^{-j\pi t} = \sqrt{2}\sin(\pi t)$

$\underline{x_2(t)} = -\frac{j}{\sqrt{2}}e^{j\pi t} + \frac{j}{\sqrt{2}}e^{-j\pi t} = -\sqrt{2}\sin(\pi t)$