

HY-215

Φροντιστήριο 3

Άσκηση 1 (HW3-2016-17 Άσκ 2a)

$$\frac{d^2}{dt^2} y(t) + 7 \frac{d}{dt} y(t) + 10 y(t) = x(t) + \frac{d}{dt} x(t)$$

$$y(0^-) = 2 \quad y'(0^-) = -2 \quad \text{y}_{zi}(t)?$$

$$\text{Χαρακ. Πολυώνυμο: } \lambda^2 + 7\lambda + 10 \begin{cases} \lambda_1 = -2 \\ \lambda_2 = -5 \end{cases}$$

$$\leadsto y_{zi}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}, \quad t > 0$$

$$= C_1 e^{-2t} + C_2 e^{-5t}, \quad t > 0$$

$$\leadsto y(0^-) = C_1 + C_2 = 2$$

$$\leadsto \frac{d}{dt} y_{zi}(t) = -2C_1 e^{-2t} - 5C_2 e^{-5t} \quad \left. \begin{array}{l} C_1 = \frac{8}{3} \\ C_2 = -\frac{2}{3} \end{array} \right\}$$

$$\left. \frac{d}{dt} y_{zi}(t) \right|_{t=0} = -2C_1 - 5C_2 = -2$$

$$y_{zi}(t) = \frac{8}{3} e^{-2t} - \frac{2}{3} e^{-5t}, \quad t > 0 \quad \text{D(ευναρτῆς)}$$

Άσκηση 2 (HW3-2016-17 Άσκ 36)

$$\frac{d^2}{dt^2} y(t) - 3 \frac{d}{dt} y(t) + 2y(t) = x(t) - \underbrace{\frac{d}{dt} x(t)}_{\text{hca?}}$$

Έστω $h_0(t)$ η β.α.

$$\frac{d^2}{dt^2} h_0(t) - 3 \frac{d}{dt} h_0(t) + 2h_0(t) = \underline{\delta(t)}$$

Αρχικές Συνθήκες: $h_0(0^+) = 0$ $h_0'(0^+) = 1$

X.P. $\lambda^2 - 3\lambda + 2 \begin{cases} \nearrow \lambda_1 = 2 \\ \searrow \lambda_2 = 1 \end{cases}$

$$\leadsto h_0(t) = C_1 e^{2t} + C_2 e^t, \quad t > 0$$

$$\leadsto h(0^+) = \frac{C_1 + C_2 = 0}{}$$

$$\leadsto h'(0^+) = 2C_1 e^{2 \cdot 0} + C_2 e^{1 \cdot 0} = 1 \Rightarrow$$

$$\underline{2C_1 + C_2 = 1}$$

$$C_1 = -1$$

$$C_2 = 1$$

$$\text{Άρα } \underline{h_0(t) = (-e^t + e^{2t}) u(t)}$$

$$h_c(t) = h_0(t) - \frac{d}{dt} h_0(t) =$$

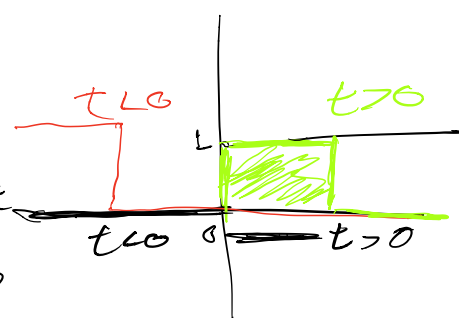
$$\begin{aligned}
& -e^t u(t) + e^{2t} u(t) + \frac{d}{dt} e^t u(t) - \frac{d}{dt} e^{2t} u(t) \\
& = -e^t u(t) + e^{2t} u(t) + e^t u(t) + e^t \delta(t) - 2e^{2t} u(t) - e^{2t} \delta(t) \\
& = e^{2t} u(t) + \delta(t) - 2e^{2t} u(t) - \delta(t) = \underline{\underline{-e^{2t} u(t)}} \\
& \qquad \qquad \qquad \downarrow \\
& \qquad \qquad \qquad \text{Αναδες}
\end{aligned}$$

Άσκηση 3 (HW3 - 2016-17 Άσκ. 4b)

$$x(t) = 6e^{-t} u(t) \quad C_{xh} ? \quad (\text{Ευρίδιο } J_h)$$

$$h(t) = 2u(t)$$

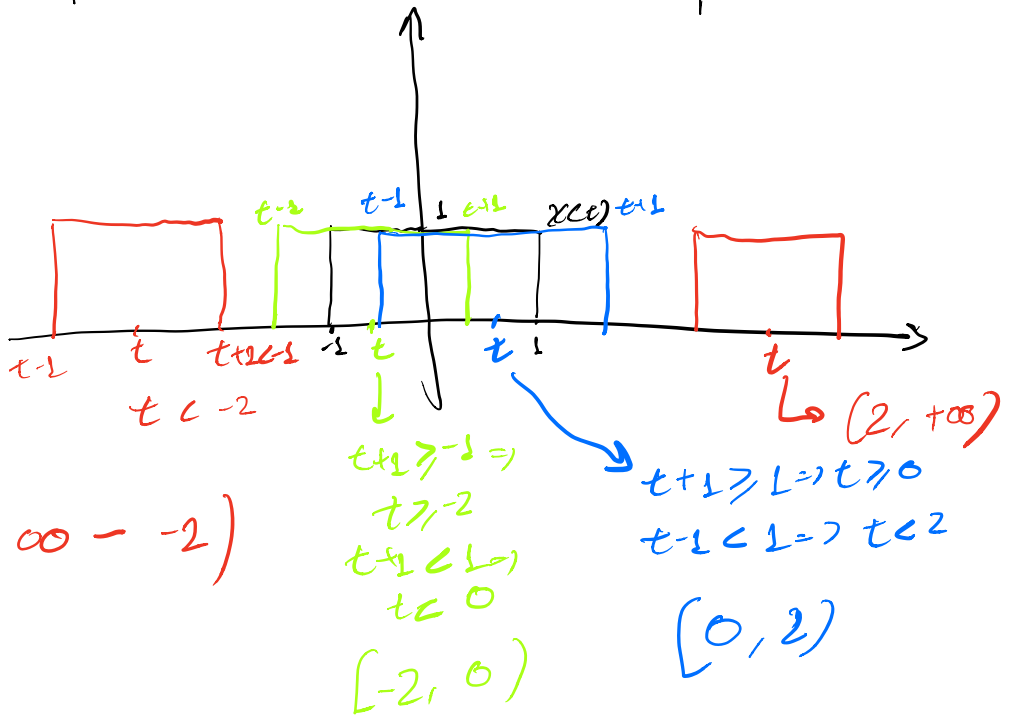
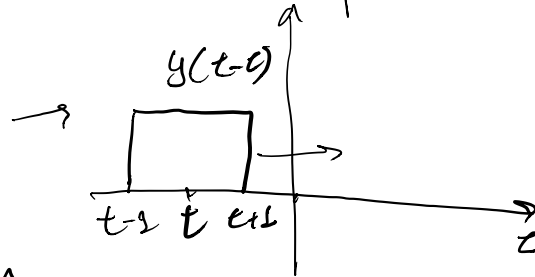
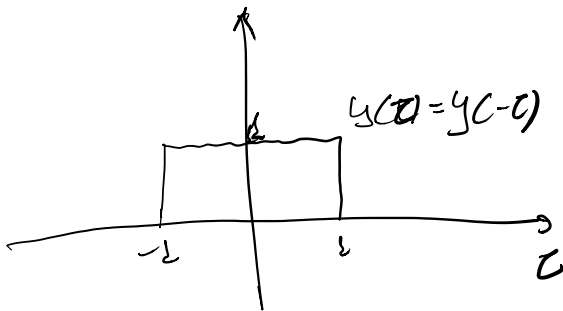
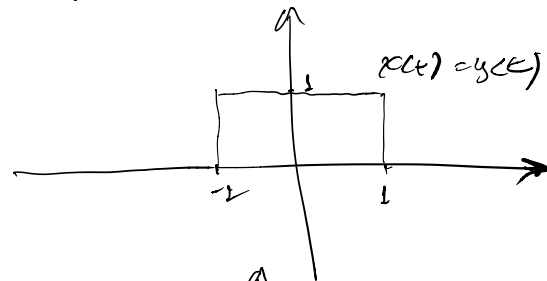
$$\begin{aligned}
y(t) &= h(t) * x(t) = \int_{-\infty}^{+\infty} 6e^{-\tau} u(\tau) \cdot 2u(t-\tau) d\tau \\
&= 12 \int_{-\infty}^{+\infty} e^{-\tau} \underbrace{u(\tau)u(t-\tau)} d\tau \\
&= 12 \int_0^t e^{-\tau} d\tau = 12[-e^{-\tau}]_0^t \\
&= \underline{\underline{12(1 - e^{-t})}}, \quad t > 0
\end{aligned}$$



Assignment 4 (lec 06 2021 slide 17)

$x(t) = \text{rect}(\frac{t}{2})$ $C_{xy}?$

$y(t) = \text{rect}(\frac{t}{2}) = x(t)$

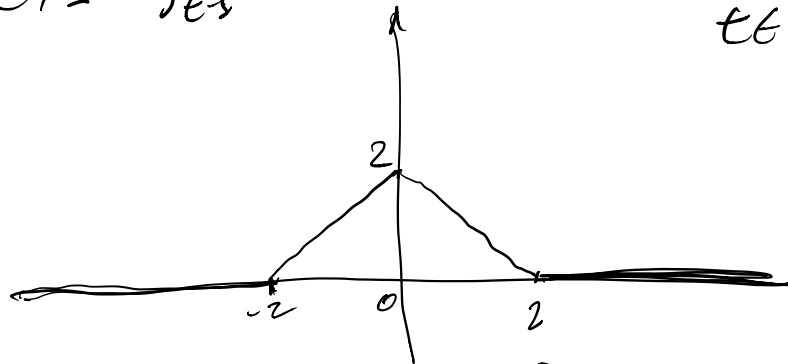


$C_{xy}(\tau) = 0$, $\forall \tau$ $t < -2, t > 2$

$C_{xy} = \int_{-1}^{t+1} 1 \cdot 1 \cdot d\tau = \tau \Big|_{-1}^{t+1} = t+1 - (-1) = t+2$
 $t \in [-2, 0)$

$$\bullet C(x) = \int_{e_1}^1 t \cdot s = z \Big|_{e_1}^1 = 1 - (t-1) = t+2$$

$$t \in [0, 2]$$



Άσκηση 5 (lec 07 2021 sl. 7)

$$x(t) = 3 - 2 \cos\left(2\pi \cdot 10t + \frac{\pi}{3}\right) + \sin\left(2\pi \cdot 15t - \frac{\pi}{6}\right)$$

✓ η ε periodic? Αν ναι, η περίοδος είναι?

$$f_0 = \text{MkL} \{10, 15\} = 5 \Rightarrow T_0 = \frac{1}{5} = 0,2 \text{ sec}$$

$$x(t) = 3 - 2 \left(\frac{1}{2} e^{j\frac{\pi}{3}} e^{j2\pi 10t} + \frac{1}{2} e^{-j\frac{\pi}{3}} e^{-j2\pi 10t} \right) +$$

$$\left(\frac{1}{2j} e^{-j\frac{\pi}{6}} e^{j2\pi 15t} - \frac{1}{2j} e^{j\frac{\pi}{6}} e^{-j2\pi 15t} \right) \Rightarrow$$

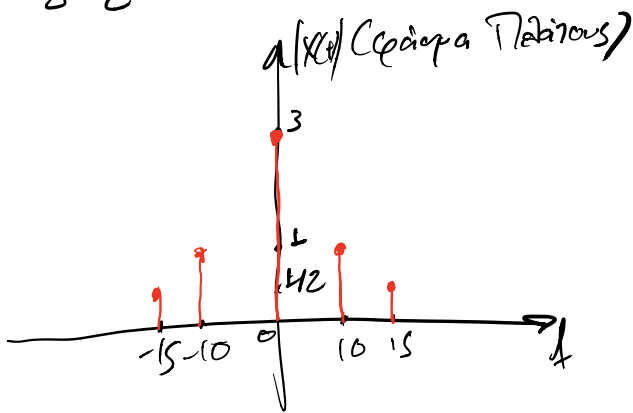
$$j^2 = -1 \Rightarrow \bar{j} = \frac{+1}{j} = e^{j\frac{\pi}{2}}$$

$$x(t) = 3 - e^{j\frac{\pi}{3}} e^{j2\pi 10t} - e^{-j\frac{\pi}{3}} e^{-j2\pi 10t} - \frac{1}{2} j e^{-j\frac{\pi}{6}} e^{j2\pi 15t} +$$

$$+ \frac{1}{2} j e^{j\frac{\pi}{6}} e^{-j2\pi 15t}$$

$$\begin{aligned}
 + x(t) &= 3 - e^{j\frac{\pi}{3}} e^{j2\pi 10t} - e^{-j\frac{\pi}{3}} e^{-j2\pi 10t} + \frac{1}{2} e^{-j\frac{\pi}{2}} e^{-j\frac{\pi}{6}} e^{j2\pi 15t} \\
 &= 3 + e^{-j\pi} \cdot e^{j\frac{\pi}{3}} e^{j2\pi 10t} + e^{j\pi} e^{-j\frac{\pi}{3}} e^{-j2\pi 10t} + \frac{1}{2} e^{j\frac{\pi}{2}} e^{j\frac{\pi}{6}} e^{-j2\pi 15t} \\
 &\quad + \frac{1}{2} e^{j\frac{2\pi}{3}} e^{-j2\pi 15t}
 \end{aligned}$$

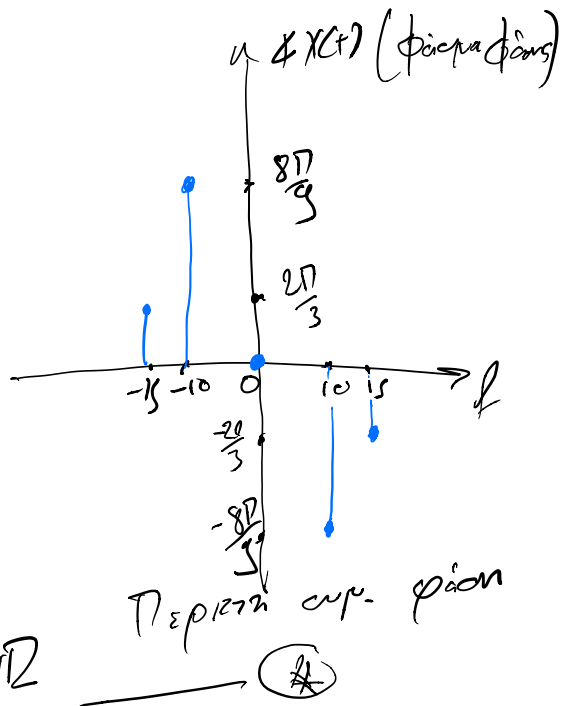
$$\begin{aligned}
 &= \textcircled{3} + \textcircled{1} e^{-j\frac{8\pi}{9}} e^{j2\pi 10t} + \textcircled{1} e^{j\frac{8\pi}{9}} e^{-j2\pi 10t} + \textcircled{\frac{1}{2}} e^{-j\frac{2\pi}{3}} e^{j2\pi 15t} \\
 &\quad + \textcircled{\frac{1}{2}} e^{j\frac{2\pi}{3}} e^{-j2\pi 15t} \\
 &\downarrow \\
 &3 \cdot e^{j2\pi \cdot 0t}
 \end{aligned}$$



Άρτια υπ. η)ατος



$x(t) \in \mathbb{R}$



Περίσπυ υπ. φάσπυ



Aufgaben 6 (HW4-2016-17 Aox. 2)

φ.Π. / b.φ. $x(t) = -2 + 2\cos(2\pi 100t + \frac{\pi}{3}) - \sin(2\pi 250t + \frac{\pi}{4})$

$$= -2 + 2 \left(\frac{1}{2} e^{j2\pi 100t} e^{j\frac{\pi}{3}} + \frac{1}{2} e^{-j2\pi 100t} e^{-j\frac{\pi}{3}} \right) +$$

$$\left(-\frac{1}{2j} e^{j2\pi 250t} e^{j\frac{\pi}{4}} + \frac{1}{2j} e^{-j2\pi 250t} e^{-j\frac{\pi}{4}} \right)$$

(Handwritten annotations: red arrows pointing to $2 \cdot e^{j\pi}$, $e^{j\frac{\pi}{2}}$, and $e^{-j\frac{\pi}{2}}$)

$$= 2 \cdot e^{j\pi} + 1 e^{j2\pi 100t} e^{j\frac{\pi}{3}} + 1 e^{-j2\pi 100t} e^{-j\frac{\pi}{3}} + \frac{1}{2} e^{j2\pi 250t} e^{j\frac{\pi}{4}} + \frac{1}{2} e^{-j2\pi 250t} e^{-j\frac{\pi}{4}}$$

(Handwritten annotations: blue circles around coefficients, green underlines on exponents)

