# Smoothing 2-D or 3-D Images Using Local Classification

K. HARIS, G. TZIRITAS, and S. ORPHANOUDAKIS

*Institute of Computer Science, Foundation for Research and Technology-Hellas, and*
*Department of Computer Science, University of Crete*
*P.O. Box 1470, Heraklion, Crete, Greece*

**Abstract.** We propose a new method for smoothing 2-D (or 3-D) images which preserves edge elements. Pixel data contained within a moving square (or cube), centered at each point under consideration, is tested to determine if it is homogeneous, or if a region boundary is present. If the area is homogeneous, the mean value of the whole area gives the smoothed value for the central point. If a region boundary is detected, then the data are classified into two clusters, and the smoothed value of the central point results from the mean value of the cluster including this point, if a context-based consistency criterion is respected. Two cases are considered: Gaussian noise and noise whose distribution is unknown. Results are given on a synthetic image corrupted by additive Gaussian noise, illustrating the efficiency of our method in relation with the signal to noise ratio. Results on a 2-D MR image are also shown.

## 1 Introduction

Almost all natural images are corrupted by random noise and this makes image analysis tasks, such as segmentation and edge detection, very difficult. Moreover, boundary localization accuracy is low for noisy images, and improves with noise reduction. A better edge detection and localization may be obtained by using nonlinear and/or space-variant edge-preserving smoothing. Among different approaches, a very promising one is that of adaptive smoothing. Methods of anisotropic diffusion are proposed in [4] and [5], where the image is iteratively convolved by a nonlinear space-variant filter with only a few coefficients ($3 \times 3$), which are determined based on image gradients. This method has two effects: sharpening of discontinuities and region smoothing. The convergence to smooth regions is slow requiring many iterations (about 200).

Other methods are based on Markov random field models using a line process representing edge elements [3]. The weak membrane model is also suitable for obtaining a smoothed image. The minimization of the resulting cost function may be obtained either by the graduated non-convexity algorithm [1], or mean field annealing [2].

The method proposed in this paper is based on local statistical hypothesis testing. In a fixed relatively small area around each point, it is assumed that there exist only two possibilities: either it is homogeneous, or two different regions are present. Thus, the image intensity is assumed piecewise constant corrupted by white noise. Two cases are considered: Gaussian noise and noise of unknown distribution. For both cases a statistical test is used to decide if the data are homogeneous or not. In the case of a homogeneous probability distribution of pixel values, and in accordance with the hypothesis of piecewise constant image intensity, the value of the smoothed signal is the mean value of the whole area. If the distribution of pixel values within a given area is decided to be a mixture of two distributions, the parameters of the two distributions are estimated and a threshold is obtained which permits the classification of the intensity value of the point under consideration to one of the two classes. If the last decision violates a context-based consistency criterion, it is relaxed.

In Section 2 we describe the proposed method in greater detail, for the case of Gaussian noise, and in Section 3 we give distribution free decision tests to carry out the above classification. In Section 4, we give results on a synthetic image corrupted by a simulated additive Gaussian noise and on a 2-D MR image.

## 2 Gaussian noise

Let us consider an area with $N = (2n+1)^2$ (or $N = (2n+1)^3$ for the 3-D images) points, and let us, for simplicity, denote the set of points as follows

$$\Omega = \{(-n,-n), \ldots, (0,0), \ldots, (n,n)\}$$

We limit ourselves to the 2-D case, the extension to the 3-D case being straightforward.

If the area is homogeneous, it is assumed that the data are distributed according to a Gaussian random variable, with mean $\mu$ and known variance $\sigma^2$. In the case of a mixture of two distributions, the mean of a set $\mathcal{C}_0$ of points is $\mu_0$, and for the complement set $\mathcal{C}_1$ ($\mathcal{C}_0 \cup \mathcal{C}_1 = \Omega$, $\mathcal{C}_0 \cap \mathcal{C}_1 = \emptyset$) the mean is $\mu_1$, while both have the same known variance $\sigma^2$. The maximum likelihood ratio gives the following test:

$$\text{An area is homogeneous, if} \quad \hat{S}^2 \leq (1+\alpha)\sigma^2 \qquad (1)$$

where $\hat{S}^2$ is the variance of the data

$$\hat{S}^2 = \frac{1}{N} \sum_{(i,j)\in\Omega} \left( x(i,j) - \frac{1}{N} \sum_{(i,j)\in\Omega} x(i,j) \right)^2 \tag{2}$$

The parameter $\alpha$ may be determined from the admitted error probability of first kind, that is

$$P_e = \Pr\{\hat{S}^2 > (1+\alpha)\sigma^2 | \text{homogeneous area}\} \tag{3}$$

It can be shown that $N\hat{S}^2/\sigma^2$ is distributed according to $\chi^2_{N-1}$, under the hypothesis of homogeneous data.

If it is decided that the whole area is homogeneous, the smoothed image is

$$\hat{x}(0,0) = \hat{\mu} = \frac{1}{N} \sum_{(i,j)\in\Omega} x(i,j) \tag{4}$$

If the distribution of pixel values is a mixture of two probality laws, according to the model of piecewise constant intensity corrupted by an additive white Gaussian noise, we can write the probability density function of the corresponding random variable as follows:

$$p(x) = \frac{P_0}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu_0)^2}{2\sigma^2}} + \frac{P_1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma^2}}$$

The method of moments may be used to estimate the parameters of this function. We have the following equations:

$$\begin{array}{rcccl} P_0 + P_1 & = & 1 & & \\ P_0\mu_0 + P_1\mu_1 & = & c_1 & = & \frac{1}{N}\sum_{(i,j)\in\Omega} x(i,j) \\ P_0\mu_0^2 + P_1\mu_1^2 & = & c_2 & = & \frac{1}{N}\sum_{(i,j)\in\Omega} x^2(i,j) - \sigma^2 \\ P_0\mu_0^3 + P_1\mu_1^3 & = & c_3 & = & \frac{1}{N}\sum_{(i,j)\in\Omega} x^3(i,j) - 3\sigma^2 c_1 \end{array}$$

Thus, the first three moments of the data are needed. From the above equations, it follows that $(\mu_0, \mu_1)$ are solutions of an equation of second degree

$$\mu_0 + \mu_1 = \frac{c_3 - c_1 c_2}{c_2 - c_1^2} = \beta, \quad \mu_0\mu_1 = \frac{c_1 c_3 - c_2^2}{c_2 - c_1^2} = \gamma$$

Then, we have (for $\mu_1 > \mu_0$)

$$\hat{\mu}_1 = \frac{\beta + \sqrt{\beta^2 - 4\gamma}}{2}, \quad \hat{\mu}_0 = \frac{\beta - \sqrt{\beta^2 - 4\gamma}}{2}$$

and

$$\hat{P}_0 = \frac{\hat{\mu}_1 - c_1}{\hat{\mu}_1 - \hat{\mu}_0}, \quad \hat{P}_1 = \frac{c_1 - \hat{\mu}_0}{\hat{\mu}_1 - \hat{\mu}_0}$$

Having the estimates of the parameters of the bimodal probability density function, a Bayesian approach can be used to obtain a threshold of discrimination between classes $\mathcal{C}_0$ and $\mathcal{C}_1$. The resulting threshold is

$$T = \frac{\hat{\mu}_0 + \hat{\mu}_1}{2} + \frac{\sigma^2}{\hat{\mu}_1 - \hat{\mu}_0} \ln \frac{P_0}{P_1}$$

Thus,

$$\begin{array}{llll} \text{if} & x(0,0) > T, & \text{then} & \hat{x}(0,0) = \hat{\mu}_1 \\ & \text{else,} & & \hat{x}(0,0) = \hat{\mu}_0. \end{array} \tag{5}$$

The above test treats image samples independently of any connectivity consideration. In order to decrease the risk of errors, the above decision is relaxed, if it is not locally consistent. For the results presented in Section 4 the decision is considered as locally consistent, if at least two points of the 8-point neighbourhood belong to the same class as the central point.

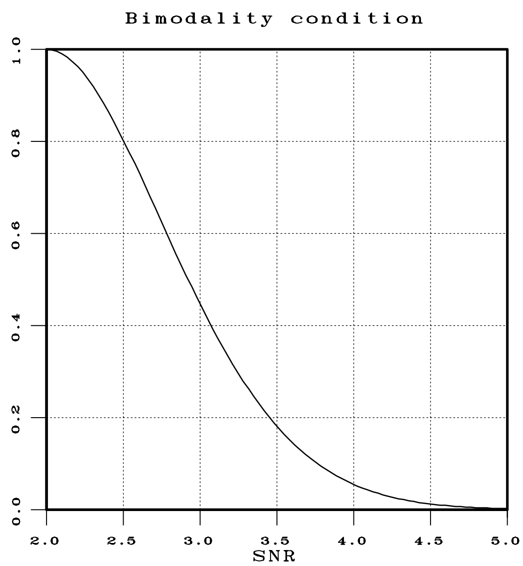


Figure 1: The minimum SNR for a bimodal distribution as a function of the ratio of *a priori* probabilities.

The test of eq.(1) is more efficient if the probability density function is bimodal. We give hereafter the condition to have a bimodal distribution in the case of a mixture of two Gaussians. Let $\rho = \frac{\mu_1 - \mu_0}{\sigma}$ be the signal to noise ratio; then the condition of bimodality is as follows

$$\rho > 2 \quad \text{and} \quad (\rho - 1)\exp\frac{1-(\rho-1)^2}{2} < \min\left(\frac{P_0}{P_1}, \frac{P_1}{P_0}\right)$$

Figure 1 shows the minimum value of the signal to noise ratio satisfying the bimodality condition as a function of the ratio of the two *a priori* probabilities.

# 3 Unsupervised classification

For cases where the Gaussian distribution hypothesis does not hold, the same approach may be used by automatic clustering of the data. In this case the variance should be firstly estimated on the basis of homogeneous data. Then, the test of eq.(1) is used with $\hat{\sigma}^2$ the estimated variance. If an area is detected as being non homogeneous, the mean values of the two classes are estimated. A squared reconstruction error criterion is used for the parameter estimation. The following cost function must be minimized

$$\mathcal{E} = \sum_{x(i,j) \leq T} (x(i,j) - \hat{\mu}_0)^2 + \sum_{x(i,j) > T} (x(i,j) - \hat{\mu}_1)^2$$

The minimization of $\mathcal{E}$ gives $\hat{\mu}_0$, $\hat{\mu}_1$ and $T$. The well known ISODATA iterative algorithm is used for minimizing $\mathcal{E}$. Smoothing is obtained in exactly the same way as for the Gaussian case (eq.(5)).

# 4 Results

Firstly we present results obtained with our method for the Gaussian case on a synthetic image of concentric disks with additive simulated Gaussian noise. Four values of $n$ are considered, namely $\{3, 4, 5, 6\}$, and two values of $\sigma$, namely $\{10, 20\}$, to be compared to the gray levels of the disks $\{90, 120, 140, 180\}$. The value of parameter $\alpha$ is obtained from tables of probabilities for the $\chi^2$ distribution and for $P_e = 0.05$ (see eq.(3)).

Figure 2 shows the noisy image ($\sigma = 20$), the smoothed image with (bottom-left) and without (top-right) local classification, and also the pixels where a mixture distribution is detected when a local classifier is used. In both cases $n = 5$, that is an area of 121 pixels is considered. The localization of central points of non homogeneous areas (bottom-right image) illustrates well the role of the signal to noise ratio, which is 1.5 for the exterior circle, 1.0 for the middle circle and 2.0 for the interior circle.

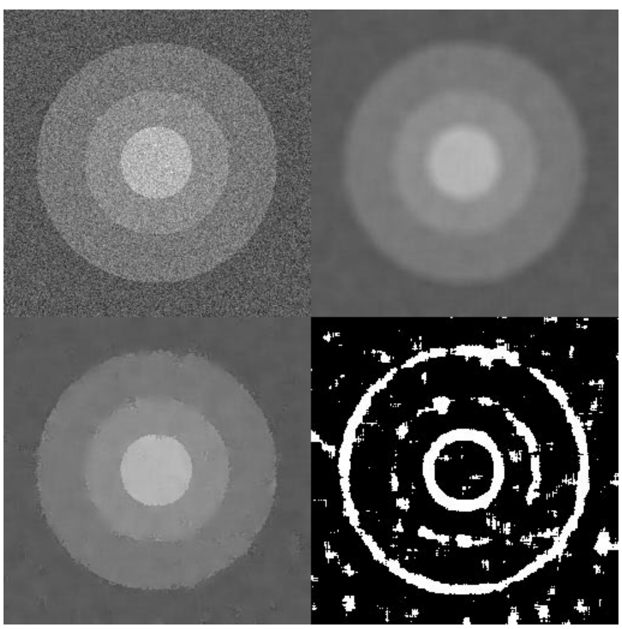

Figure 2: Result on a synthetic image

In Table 1, the enhancement factor

$$F = \frac{KL\sigma^2}{\sum_{k=1}^{K} \sum_{l=1}^{L} (x(k,l) - \hat{x}(k,l))^2}$$

is given for the different values of parameters $n$ and $\sigma$ ($KL$ being the number of image points) for the method here proposed ("a" in the Table) and for a moving average filter with equal coefficients ("b" in the Table).

| | $\sigma$ | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ |
|---|---|---|---|---|---|
| a | 10 | 18.98 | 24.21 | 28.52 | 33.04 |
| b | 10 | 9.51 | 8.23 | 7.06 | 6.11 |
| a | 20 | 19.33 | 23.28 | 25.40 | 26.77 |
| b | 20 | 24.19 | 25.36 | 24.10 | 22.07 |

Table 1: The enhancement factor for different values of the area size and noise variance.

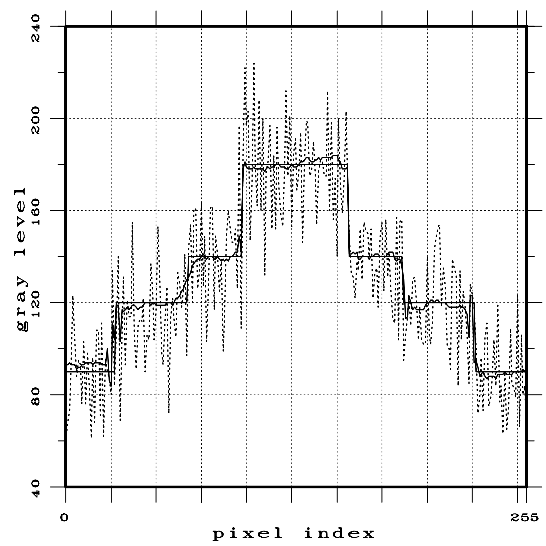


Figure 3: Pixel values along the middle row of the original, noisy and smoothed images.

In Figure 3, we plot the pixel values along the middle row of the initial, noisy ($\sigma = 20$), and smoothed ($n = 5$) images.

Figure 4 shows an original MR image (top-left), the smoothed image using our method under the Gaussian hypothesis (top-right), the smoothed image using the method presented in Section 3 (bottom-left), and the smoothed image using the method presented in [4]. Figure 5 shows a row of the original MR image, and the same row smoothed using the unsupervised clustering algorithm.

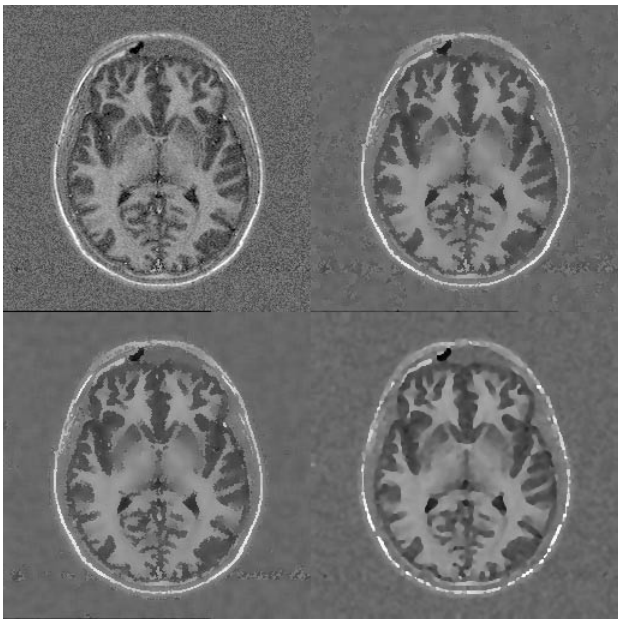

Figure 4: Result on a MR image

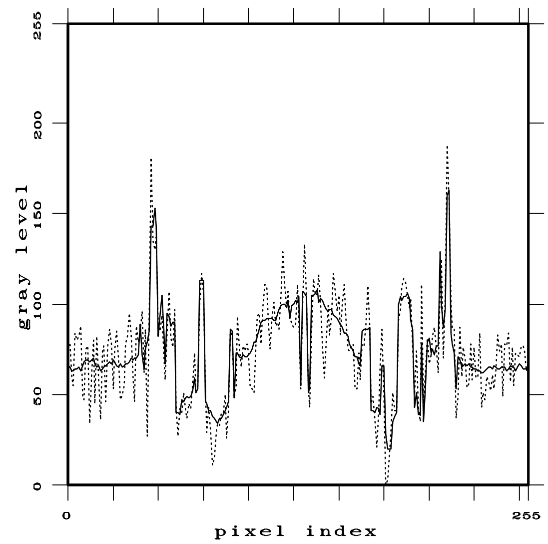


Figure 5: Pixel values along a row of the MR image

# 5 Conclusion

We have presented a statistical image enhancement method, which is very effective in both preserving edges and smoothing the intensity function, when the data are locally bimodal. For the case of an additive Gaussian noise a condition on the signal to noise ratio is given for having a bimodal distribution.

Results on synthetic images illustrate well the above statement. As the number of modes is at most two, the larger is the locally considered area, the better is the obtained enhancement factor. From this conclusion we would expect that the presented method could be improved with an adaptive variable-size local area in order to obtain at each point the more largest area with at most two modes.

As illustrated by the results obtained with real images, our method gives at least as good results as methods using anisotropic diffusion [4], but requires less computations.

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# References


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