Simultaneous Segmentation and Modelling of Signals based on an Equipartition Principle

Costas Panagiotakis and Georgios Tziritas

Computer Science Department, University of Crete, Heraklion, Greece
{cpanag, tziritas}@csd.uoc.gr

Abstract

We propose a general framework for simultaneous segmentation and modelling of signals based on an Equipartition Principle (EP). According to EP, the signal is divided into segments with equal reconstruction errors by selecting the most suitable model to describe each segment. In addition, taking into account change detection on signal model an efficient signal reconstruction is also obtained. The model selection concerns both the kind and the order of the model. The proposed methodology is very flexible on different error criteria and signal features.

I. Introduction

As most signals are non-stationary, the problems of segmentation and model selection are very important in pattern recognition and signal processing applications. In many of these applications it could be interesting to have a uniform representation according to an appropriate quality measure. The objective is the partition of the feature sequence into "homogenous" segments with uniform characteristics according to a predefined criterion.

We have adopted such an approach for 3-D modelling and non articulated motion tracking [1], polygonal approximation [2] and key frames detection [3] leading to the curve equipartition problem (EP). More formally, the equipartition problem (EP) can be formulated as follows [4]. Let $c : [a, b] \to \mathbb{R}^m$ be a continuous, injective curve in an m-dimensional space, which represent the signal in a predefined feature space. Let us further suppose that a distance $d(t, t')$ is a measure of how dissimilar are the corresponding instances. We search a sequence of feature points that are uniformly distributed according to this distance.

Piecewise stationary signal segmentation and reconstruction can be solved by first detecting the segments and then applying the modelling [5]. Otherwise, signal segmentation and reconstruction can be solved simultaneously based on energy optimization methods [6]. These methods compute the time segments and the approximations per segment by minimizing an energy function. The optimization problem is solved using level sets algorithms applying successfully in piecewise stationary signals. Another class of segmentation methods is based on Bayesian inference [7], [8]. These algorithms consist of defining appropriate prior distributions for the unknown signal parameters (including the change points between the different segments) and estimating these unknown parameters from their posterior distributions. Finally, these methods provide a possibility of change for each frame of given signal. The method proposed in [7] was applied in speech data. In [8], the segmentation procedure allows joint segmentation of signals recorded by different sensors using astronomical time series data.

Most of the above mentioned approaches address the signal segmentation problem focusing either on a restricted signal content (e.g. piecewise stationary signals) under predefined and specific modelling or minimizing specific energy criteria. On the contrary, in this paper, the most appropriate model from a set of models is selected to describe each segment. We have applied our method to polynomial, Fourier and wavelet modelling, however any other signal modelling can be considered. Moreover, the proposed method can be used under different energy criteria. A preliminary version of the proposed method is presented in [9], where the most appropriate model between polynomial and Fourier series is selected to describe the whole signal instead of each signal segment description that is presented in this work.

II. Problem formulation

Let us consider a discrete-time signal $f(t)$, $t \in \{0, 1, \cdots, T-1\}$, where T denotes the number of samples. The goal is to automatically and simultaneously segment and reconstruct each segment of the given signal using the most appropriate model for each time segment. It is possible that the given signal can be assumed as piecewise stationary, where the segments have undetermined variable lengths, that admit a sparse representation in different bases. The proposed method simultaneously solves the segmentation and modelling problem, selecting for each detected segment a compact representation so that the signal is divided into segments with equal reconstruction errors and number of coefficients.

Let $g(t)$ be a model representation of $f(t)$. Let N be the number of the time segments $[0, s_1] \cup [s_1, s_2] \cup \cdots \cup [s_{N-1}, T-1]$, where $s_i \leq s_{i+1} \in \{0, \ldots, T-1\}, i \in \{1, \ldots, N-1\}$ be the end time of $(i-1)$ segment and the start time of i segment $(s_0 = 0, s_N = T-1)$. The error between the segment $[s_i, s_{i+1}]$ of $f(t)$ and the corresponding segment of $g(t)$ is formulated as a distance $d(s_i, s_{i+1})$. Then the global approximation error of $f(t)$ by $g(t)$ can be defined as the maximum error between the segments of $f(t)$ and their corresponding segments of $g(t)$,

$$E(f, g) = \max_{i \in \{0,1,\cdots,N-1\}} d(s_i, s_{i+1}) \qquad (1)$$

This error criterion has been successfully used on polygonal approximation problem [2].

The goal of the proposed method is to simultaneously detect changes and select the best model for each segment, so that the global approximation error $E(f, g)$ is minimized. We assert that a near optimal solution of the segmentation problem is achieved, when the approximation errors per segment are equal, as the error is shared between all the segments,

$$\epsilon = d(0, s_1) = d(s_1, s_2) = \cdots = d(s_{N-1}, T-1) \qquad (2)$$

Arguments that the approximation error ϵ is minimum or close to minimum have been given in [2], [4], [9].

III. Signal Modelling

In this work, we have used three types of bases, Fourier, polynomial and wavelet to reconstruct the signal segments. The most suitable basis provides a sparse representation for a class of signal segment and effectively captures the structure inherent in the class [10]. For example, it holds that the Fourier basis sparsifies smooth signals, and wavelet bases sparsify piecewise smooth signals, while polynomial basis sparsifies polynomial signals. The model coefficients in each segment can be used for signal summarization.

We give now the formulation for the models used for a segment of duration $0, \ldots, T_0$. The signal $f(t)$ can be reconstructed from Fourier coefficients representing the harmonic components:

$$f(t) = \frac{1}{T_0} \sum_{k=0}^{T_0-1} w_k \cdot e^{2\pi i \frac{kt}{T_0}}.$$

As it concerns polynomial models, the least square method is used for the estimation of the polynomial coefficients. The squared error

$$\sum_{t=0}^{T_0-1} (f(t) - \sum_{k=0}^{R} c_k (\frac{t}{T_0})^k)^2$$

is minimized, where R and $\{c_1, ..., c_R\}$ denote the polynomial degree and the polynomial coefficients, respectively.

A widely used representation in signal processing is the discrete wavelet transform, where the initial signal is decomposed into subbands. The resulting representation for a signal of duration $T = 2^L$ and an l-level decomposition could be noted as $a(k, 0), 0 \leq k < 2^{L-l}$, for the low-pass approximation and

$$a(k, m), 0 \leq k < 2^{L-l+m-1}, 1 \leq m < l,$$

for the high-pass components. The importance of each coefficient depends on its energy $a(k, m)^2$. In this paper, the Haar wavelet with a three-level decomposition was used, providing perfect reconstruction of step functions, using the subset of highest in energy wavelet coefficients.

A. Model selection

This section describes the proposed method for selecting the most appropriate model for signal segment modelling. Let u, v, where $0 \leq u < v \leq T-1$ be the start and the end of a time segment of the given signal $f(t)$. Let $g_1(t)$ be the reconstruction of $f(t)$, $t \in \{u, ..., v\}$ using the S most important Fourier coefficients. Let $g_2(t)$ be the reconstruction of $f(t)$, $t \in \{u, ..., v\}$ using a polynomial of $2 \cdot S - 1$ degree. Let $g_3(t)$ using the $2 \cdot S$ most important (highest in energy) wavelet coefficients. Therefore the number of parameters is the same $(2 \cdot S)$ for the three kinds of model.

The most suitable basis provides a sparse representation for a class of signal segment. S is a user-defined parameter of the proposed method. It holds that we can easily include many models/bases and also adapt the per segment number of parameters.

Let $d_j(u,v), j \in \{1,2,3\}$ be the approximation error between $g_j(t)$ and $f(t)$. Then, $d(u,v)$ is the minimum error among the three admitted representations

$$d(u,v) = \min_j d_j(u,v). \tag{3}$$

$m(u,v) = \arg\min_j d_j(u,v)$ corresponds to the method giving the minimum error. According to the EP problem constraints [2], $d(u,v)$ should satisfy isolation ($d(u,v) = 0 \Leftrightarrow u = v$) and symmetry ($d(u,v) = d(v,u)$) properties. These properties are satisfied by the difference in mean energy of signal $f(t)$ and $g_j(t), j \in \{1,2,3\}$.

$$d_j(u,v) = \frac{1}{v-u+1} \left| \sum_{t=u}^{v} f^2(t) - \sum_{t=u}^{v} g_j^2(t) \right|. \tag{4}$$

Therefore, having the best fitting parameters for each possible representation, the model selected is that of best approximation of the whole signal energy. This holds because the different representations aim the same objective, the interpretation of the whole signal energy. In each case, if the error is low, it means that the segmentation is good and the content description of the signal by the proposed descriptors is valid. Moreover, it means that the segments are homogenous in content, since they can be described by a small number of coefficients that are related to the signal content.

B. Iso-Level Algorithm (ILA)

The signal segmentation and reconstruction is done simultaneously using the Iso-Level Algorithm (ILA). The input of the ILA is the number of segments N. In addition, it needs the matrix $d(u,v), u,v \in \{0,1,\cdots,T-1\}$ of distortions. It is a recursive method with computation cost $O(N \cdot T^2)$. The detailed description of ILA can be found in [2], [4]. The number of segments N can be given by the user or can be estimated automatically by terminating the EP algorithm, when the estimated "distortion" exceeds a predefined error [3], [9].

IV. Experimental Results

In this section, experimental results using the proposed algorithm are presented. We have tested the proposed algorithm on a data set consisting of different types of signals, like physiologic signals, human motion signals and synthetic signals, in order to show that our method can be used under any type of signal without any constraint. In electrocardiogram (ECG) signals irregular states like arrhythmia, tachycardia, bradycardia, etc, could be detected. Human motion tracking data have been used for human action recognition [11] using data from athletics meeting. An important task for human action recognition is to segment the human motion signal into homogenous in content segments that correspond to sequential human actions.

Left pictures of Fig. 1 illustrate the symmetric function $m(u,v)$. Pixels of cyan color correspond to Fourier modelling selection ($m(u,v) = 1$). Pixels of yellow color correspond to polynomial modelling selection ($m(u,v) = 2$). Pixels of brown color correspond to wavelet modelling selection ($m(u,v) = 3$). Right plots of Fig. 1 illustrate the original signal (blue curves) and segmentation (black dotted lines). In order to show the efficiency of the proposed simultaneous segmentation and modelling, we have compared the SNR of the resulting reconstructed signals (red curves) with signals reconstructed using the best model without segmentation (cyan curves). We have used the same total number of coefficients in both reconstructions. The detected model changes are plotted with black dotted lines and the selection of the proposed modelling is written in the middle of each segment. We have used different values for S, like $S = 2$ (Fig. 1(d)), $S = 4$ (Fig. 1(b)) and $S = 7$ (Fig. 1(f)).

Fig. 1(b) shows a bradycardia electrocardiogram (ECG) from a defibrillator [12]. The proposed simultaneous segmentation and modelling method gives SNR=30.88, while the SNR without using segmentation was 26.86. Fig. 1(d) shows the human major axis angle ϕ [11] during a high jump sequence. In this case, the signal segmentation corresponds to several sequential phases (actions) like running ($\phi \simeq \frac{\pi}{2}$), jumping ($\frac{-\pi}{2} \leq \phi \leq \frac{\pi}{2}$), falling ($\frac{-\pi}{2} \leq \phi \leq 0$) and standing ($0 \leq \phi \leq \frac{\pi}{2}$) actions. In this case, the SNR of the proposed method increases from 18.36 (for $N = 1$) to 25.38, due to efficient segmentation and model selection.

Fig. 1(f) shows a synthetic signal. The synthetic signal consists of three type of segments: step functions, sinusoidal and polynomial. Thus, the signal modelling method should take into account different models, as the proposed method does. Most transitions are difficult to detect by sequential sliding windows based methods due to the signal gradual transitions. On the contrary, the proposed method successfully detects them, since it globally solves the problem of segmentation and modelling by minimizing a global error criterion, without any constraint on minimum or maximum segment size. In this case, we have tested

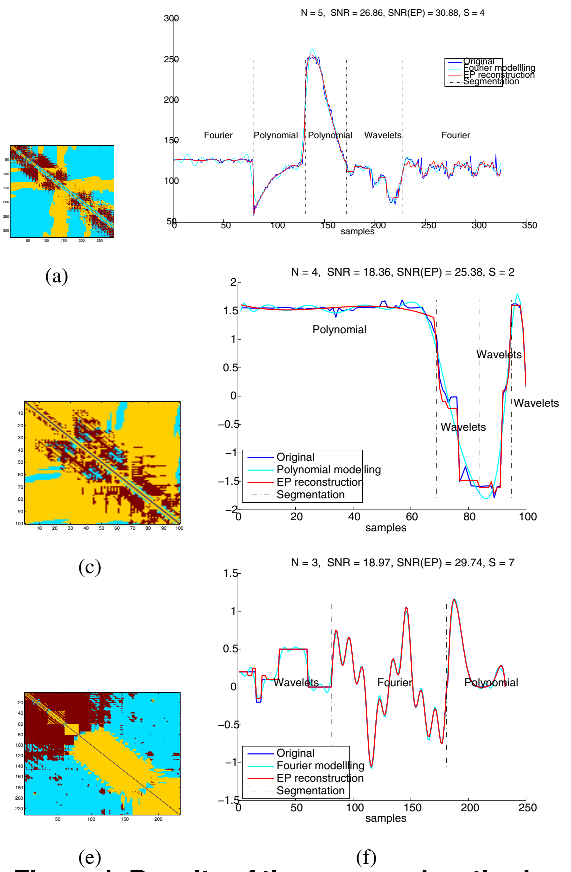

Figure 1. Results of the proposed method.

the robustness of the proposed method in presence of Gaussian white noise (with SNR from 30 db to 7 db) by measuring the change detection accuracy (in samples). The first transition is robustly detected to noise effects with mean deviation error of 6.54 samples. The mean deviation error on detecting the second transition was 15.51 samples, since this transition is difficult to be detected due to derivative continuity of signal in transition.

V. Conclusions

Consecutively, an EP based method for simultaneous time interval segmentation and modelling of signals has been described. According to the proposed method the signal is segmented into segments that give quasi-equal reconstruction errors, selecting the most suitable model to describe each segment. Moreover, the segments are *equivalent* in the content domain yielding segments described by the same number of coefficients. Experimental results on a large data set of different types of signals have been obtained to demonstrate the efficiency and the robustness of the proposed schema.

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