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DISCRETE REALIZATION FOR RECEIVERS DETECTING SIGNALS OVER RANDOM DISPERSIVE CHANNELS. PART I: RANGE-SPREAD CHANNEL

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Abstract. We propose a new suboptimal receiver for detecting signals transmitted over random range-spread channels. This receiver is composed of a filter matched to the transmitted signal, a sampler which is in general non periodic and a discrete-time quadratic filter. To determine sampling, we exploit the form and the properties of the scattering function. We give numerical results in a specific case of the scattering function and the transmitted signal. We show the important role of transmission diversity and we clarify the role of the parameter giving the ratio of the signal's bandwidth to the coherence bandwidth of the channel.

Zusammenfassung. Wir schlagen einen neuen suboptimalen Empfänger zur Detektion von Signalen vor, die über Kanäle mit zufällig veränderlicher Laufzeit gesendet werden. Dieser Empfänger besteht aus einem speziell an das Sendesignal angepaßten Filter, einer Abtastschaltung und einem Quadrierer. Die Abtastung erfolgt im allgemeinen nicht in äquidistanten Intervallen; zur Festlegung der Abtastrate verwenden wir die Form und die Eigenschaften der Streufunktion. Für einen speziellen Fall der Streufunktion und des Sendesignals wird ein Zahlenbeispiel gegeben. Wir zeigen die Wichtigkeit der Diversity der Übertragung auf und diskutieren, welche Rolle der Parameters soielt, der das Verhältnis der Bandbreiten des gesendeten Signals under der Kohärenzbandbreite des Übertragungskanals bestimmt.

Résumé. Nous proposons un nouveau récepteur sous-optimal pour détecter des signaux ayant traversé une voie de transmission aléatoire et dispersive en retard. Ce récepteur comprend le filtre adapté au signal émis, un échantillonneur, en général non périodique, et un filtre numérique quadratique. Pour déterminer l'échantillonnage, nous exploitons la forme et les propriétés énergétiques de la fonction de diffusion. Nous donnons des résultats numériques pour un cas particulier de fonction de diffusion et de signal émis. Nous montrons le rôle important de la diversité de transmission et nous mettons en évidence le paramètre qui donne le rapport entre la largeur de bande du signal émis et la largeur de bande de la voie de transmission.

Keywords. Range spread channel, scattering function, detection, binary communication, diversity, suboptimal receiver, non periodic sampling, quadratic filter, error probability.

1. Introduction

The problem of detecting signals over random dispersive channels has been widely studied in the literature [1, 2, 3]. Such random phenomena occur naturally in ionospheric, seismic, tropospheric, or undersea communication as well as in radar astronomy and biological transmission. The signal received at the output of the channel can normally be modelled as random and would include an additive random noise term. The general double-spread (range and doppler) channel along with the special cases of range and doppler spreading considered separately are discussed by H.L. Van Trees [1].

It is well known that the standard structure of the optimal receivers is an estimator correlator structure. One determines the conditional mean estimate of the random signal (infinite dimensional) and correlates it with the observed signal. The infinite dimensionality is the crux of the problem. In this formulation, the dimension of the observation space is infinite, thus making the determination of the optimal estimator, and therefore the optimal receiver very difficult.

A reasonable approach to reducing the dimensionality consists in choosing a finite set of eigenfunctions as a basis for a subspace of the original observations space. Eigenfunctions are selected which allow the representation of most of the energy of the random signals involved [4]. In practice, however, we have to solve for the eigenfunctions which is as difficult as the estimation part of the problem.

S. Schwartz [5] has proposed firstly a reduction of the observation space by direct sampling; consequently he uses the standard discrete-time formulation to obtain the optimal receiver. He presents a recursive formula for the likelihood ratio and an estimator-correlator structure for the receiver. Another approach has been used by S. Cambanis and E. Masry [6]. They studied sampling techniques and designs for the detection of known signals in gaussian noise. Both deterministic and random sampling schemes were used and the performances of such detectors were presented.

The objective of our work was to reduce the dimensionality of the problem by using the characterization of the channel. In this paper, a new procedure for detecting signals over random channels is presented; it uses a discrete-time version of a filtered version of the observed signal. Solutions are presented for the two special cases of range-spread (Part I) and of Doppler-spread (Part II) channels. A random time-invariant linear filter is a model for the range-spread channel, while the Doppler-spread channel is represented by a random amplitude modulation. It is shown that the pre-filtering of the observed signal and consequently the sampler were determined differently for each case. The general model of the double-spread channel has not been solved.

In the range-spread channel (discussed in Part I) range-spreading can result from a spatial distribution of a large number of scatterers or in general from a continuum of scatterers. Their distribution can be characterized by a scattering function, which is a delayed version of spatial distribution expressing the second order statistics of the scattering. The continuous-delay scattering function is represented by a finite number of discrete delays. This corresponds to a regrouping or clustering of the scatterers or continuum of scatterers. Each group is represented by its barycenter over the delay axis alone. In this way, a model of the transmission medium which simplifies the observation signal space is developed. The dimension of the observation space is now effectively finite and a solution is easily obtained. The problem has been transformed now to the multipath transmission problem presented by G. Tziritas [7]. The receiver is composed of a filter matched to the transmitted signal, of a sampler whose discrete-times are determined by the delay barycenters, and a discrete-time quadratic filter. The quadratic filter can be interpreted as a linear filter coupled with a correlator.

The Doppler-spread channel is discussed in Part II. In Part I, receivers obtained by the above described procedure for a range-spread channel are discussed. In Section 2, we will describe the channel model, its statistical characterizations by means of the scattering function. We also state the binary decision problem we want to solve. In Section 3, we discuss some important parameters of the transmission. In particular, a parameter which gives the diversity of the transmission or the number of degrees of freedom of the received signal is introduced by G. Tziritas [8]. This parameter plays an important role in sampling design. In the same paper [8], an optimal value of this parameter versus signal to noise ratio is given.

In section 4, we give the method for determining the "delay barycenters" using the medium scattering function. In Section 5, we will determine the receiver under above assumptions and we will finish (Section 6) with the numerical results obtained for a scattering function as Hanning window.

2. Channel model and detection problem

In this paper, we consider a medium composed of a large number of randomly distributed scatterers. We suppose that for all scatterers, the ratio of wave propagation velocity to scatterer's velocity is large enough in comparison with twice the product bandwidth duration of the transmitted signal, thus the scatterers are considered non-fluctuating. We also suppose that the bandwidth of the transmitted pulse is relatively small in comparison with the carrier frequency. With these assumptions, a linear time-invariant filter can be a model for the medium. Let $\tilde{f}(t)$ be the complex envelope of the transmitted signal $f(t)$, of energy E_t and duration T. Thus, if ν_0 is the carrier frequency, we have

$$\begin{aligned} &f(t)=\sqrt{2}\,\mathrm{Re}[\tilde{f}(t)\,e^{j2\pi\nu_0 t}], \quad 0\leq t\leq T,\\ &E_t=\int_0^T f^2(t)\,dt=\int_0^T |\tilde{f}(t)|^2\,dt,\\ &\tilde{f}(t)=f_c(t)-jf_s(t). \end{aligned} \tag{1}$$

The complex envelope of the signal at the output of the channel is therefore

$$\tilde{s}(t)=\int_0^L \tilde{h}(\xi)\tilde{f}(t-\xi)\,d\xi, \quad 0\leq t\leq T+L \tag{2}$$

where $\tilde{h}(\xi)$ is the random finite impulse response of the channel.

In what follows, we suppose that the expected value of $\tilde{h}(\xi)$ is zero

$$E\{\tilde{h}(\xi)\}=0, \quad \forall\xi$$

and that there exists a relationship between the real and imaginary parts of $\tilde{h}(\xi)$ which is expressed as follows

$$\begin{aligned} &E\{\tilde{h}(\xi)\tilde{h}(\zeta)\}=0, \quad \forall\xi,\zeta,\\ &E\{\tilde{h}(\xi)\tilde{h}^*(\zeta)=\tilde{K}_{\tilde{h}}(\xi,\zeta). \end{aligned} \tag{3}$$

This means that the covariance function of the real part is the same as that of the imaginary part and that the cross-covariance function between the two parts is an antisymmetrical function. In this paper, we suppose that the random function $\tilde{h}(\xi)$ is gaussian non-stationary white noise. This corresponds to uncorrelated scatterers (U.S.) and we have

$$\tilde{K}_{\tilde{h}}(\xi,\zeta)=\tilde{Q}(\xi)\delta(\xi-\zeta)$$

where $\delta(\cdot)$ is the Dirac distribution.

The function $\tilde{Q}(\xi)$ is the scattering function of the channel and illustrates its range-spreading.

The scattering function is a non-negative function which characterizes the spreading of the signal energy. Thus the mean power at the channel output is given by

$$E\{|\tilde{s}(t)|^2\}=\int_0^L \tilde{Q}(\xi)|\tilde{f}(t-\xi)|^2\,d\xi, \quad 0\leq t\leq T+L.$$

More generally the covariance function of the signal at the output of the channel is as follows

$$\tilde{K}_{\tilde{s}}(t,u)=\int_0^L \tilde{Q}(\xi)\tilde{f}(t-\xi)\tilde{f}^*(u-\xi)\,\mathrm{d}\xi,\quad 0\leqslant t,u\leqslant T+L. \tag{4}$$

We shall study the detection and binary communication problems.

The first problem can be formulated as testing the hypothesis of signal presence in additive white noise

$$\begin{aligned} &H_1: \tilde{r}(t)=\tilde{s}(t)+\tilde{n}(t),\quad 0\leqslant t\leqslant T+L,\\ &H_0: \tilde{r}(t)=\tilde{n}(t),\quad 0\leqslant t\leqslant T+L. \end{aligned} \tag{5}$$

We suppose that the complex envelope $\tilde{n}(t)$ of the noise is gaussian, zero-mean, and white of power spectral density N_0, that is

$$\begin{aligned} &E\{\tilde{n}(t)\}=0,\quad \forall t,\\ &E\{\tilde{n}(t)\tilde{n}^*(u)\}=N_0\delta(t-u),\\ &E\{\tilde{n}(t)\tilde{n}(u)\}=0,\quad \forall t,u, \end{aligned} \tag{6}$$

where $\delta(\cdot)$ is the Dirac distribution.

In the second case, we have to test whether $\tilde{f}_1(t)$ or $\tilde{f}_0(t)$ was transmitted

$$\begin{aligned} &H_1: \tilde{r}(t)=\tilde{s}_1(t)+\tilde{n}(t),\quad 0\leqslant t\leqslant T+L,\\ &H_0: \tilde{r}(t)=\tilde{s}_0(t)+\tilde{n}(t),\quad 0\leqslant t\leqslant T+L, \end{aligned} \tag{7}$$

with the same assumptions concerning the noise $\tilde{n}(t)$.

Theoretical results concerning this subject are well-known [1]. One must solve a Fredholm integral equation, which is generally very difficult. In this paper we study principally the following scattering function

$$\tilde{Q}(\xi)=\frac{2}{L}\sin^2\frac{\pi\xi}{L},\quad 0\leqslant \xi\leqslant L, \tag{8}$$

which is normalized $\int_0^L \tilde{Q}(\xi)\,\mathrm{d}\xi=1$.

This appears to be a realistic choice for a unimodal scattering function. The envelope of the transmitted signal is constant.

$$f(t)=\sqrt{\frac{E_t}{T}},\quad 0\leqslant t\leqslant T. \tag{9}$$

Thus the average received energy is

$$\bar{E}_r=\int_0^{T+L} E\{|\tilde{s}(t)|^2\}\,\mathrm{d}t=\int_0^{T+L}\int_0^L \tilde{Q}(\xi)|\tilde{f}(t-\xi)|^2\,\mathrm{d}\xi\,\mathrm{d}t=E_t.$$

The solution for the optimum receiver, corresponding to the transmitted signal (9) in the channel characterized by the scattering function (8) is not known. In this paper we give a suboptimal receiver which is based on a heuristical representation of the scattering function. We discuss this point in Section 4. In the following section, we present certain important parameters of the transmission.

3. Transmission parameters

We introduce firstly the "coherence bandwidth" of the channel according to A. Ishimaru [9]. Let us consider the transmission of two waves at different frequencies ν_1 and ν_2, and observe the output at the same time. As we separate the frequencies, the correlation of the two output signals decreases. The separation frequency $\Delta\nu = |\nu_1 - \nu_2|$, at which the correlation function almost disappears or decreases to a specified level, is called "coherence bandwidth". In fact, one can prove that for the input signals at frequencies ν_1 and ν_2, the covariance of the output signals $\tilde{s}_1(t)$ and $\tilde{s}_2(t)$, at the same instant t, is given by

$$\tilde{K}_{\tilde{s}_1\tilde{s}_2}(t, t) = e^{-j2\pi\Delta\nu t} \int_0^L \tilde{Q}(\xi)\, e^{j2\pi\Delta\nu\xi}\, d\xi.$$

It is obvious that the Fourier transform of $\tilde{Q}(\xi)$ decreases as $\Delta\nu$ increases and the bandwidth of $\tilde{Q}(\xi)$ determines the coherence bandwidth of the channel.

Let us now consider a pulse of duration $T(9)$ transmitted over the random medium. The pulse is delay spread. For the scattering function in (8) the "broadening" is L and the "coherence bandwidth" approximately $1/L$. Thus, L/T is the ratio of signal bandwidth to "coherence bandwidth" or the diversity of the transmission and the ratio T/L measures the signal duration's spreading. In [8], a more convenient approach to the transmission diversity is introduced, which takes into account the transmitted signal. This approach allows us to characterize the transmission, that is the channel and the transmitted signal simultaneously in their interaction. The transmission diversity is defined as follows, for a range-spread channel (U.S.) with $\tilde{Q}(\xi)$ as scattering function.

$$a = \frac{1}{\int_0^L \int_0^L \tilde{Q}(\xi)\tilde{Q}(\zeta)|\tilde{\rho}_{\tilde{f}}(\xi-\zeta)|^2\, d\xi\, d\zeta}, \tag{10}$$

where $\tilde{\rho}_{\tilde{f}}(\cdot)$ is the normalized correlation function of the transmitted signal $\tilde{f}(t)$ ($\tilde{\rho}_{\tilde{f}}(0) = 1$).

For the scattering function in (8) and the transmitted signal in (9) we obtain, with $\alpha = T/L$

$$\begin{aligned} \frac{1}{a} &= 1 - \frac{1}{\alpha}\left(\frac{2}{3} - \frac{5}{\pi^2}\right) + \frac{1}{\alpha^2}\left(\frac{1}{6} - \frac{1}{\pi^2}\right), \quad \alpha \geq 1, \\ \frac{1}{a} &= \frac{2}{3}\alpha - \frac{\alpha^2}{6} + \frac{1}{\pi^2}\left(1 + \frac{1}{2\alpha}\right) - \frac{3}{4\pi^4\alpha^2}(1 - \cos 2\pi\alpha) - \frac{1}{4\pi^3\alpha}\left(\frac{1}{\alpha} - 1\right)\sin 2\pi\alpha, \quad \alpha \leq 1. \end{aligned} \tag{11}$$

We give in Fig. 1 diversity transmission, a, versus the ratio $\alpha = T/L$. One can observe that for $\alpha \to \infty$, transmission diversity tends to 1 ($a \to 1$). If α is small ($T/L \ll 1$), then the transmission diversity is approximately like L/T. To illustrate the role of the scattering function, we superimpose the same parameter, a, for the same transmitted signal (9) and a constant in $[0, L]$ scattering function (with approximately the same coherence bandwidth as for (8))

$$\tilde{Q}(\xi) = \frac{1}{L}, \quad 0 \leq \xi \leq L.$$

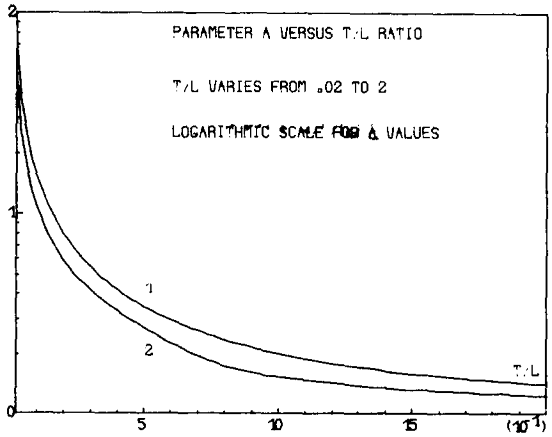

Fig. 1. Transmission diversity, a, versus T/L ratio for a transmitted signal of constant envelope and for a scattering function 1: constant; and 2: as in (8).

In this case, we obtain the transmission diversity, a, with $\alpha = T/L$ as follows

$$\frac{1}{a} = 1 - \frac{2}{3\alpha} + \frac{1}{6\alpha^2}, \quad \alpha \geq 1,$$

$$\frac{1}{a} = \frac{2\alpha}{3} - \frac{\alpha^2}{6}, \quad \alpha \leq 1.$$

In Fig. 2, we also give the optimal value of the T/L ratio versus signal to noise ratio, for the transmission diversity in (11), using the method introduced in [8]. In the same reference [8] we see the error probability, which corresponds to the optimal choice of the T/L ratio.

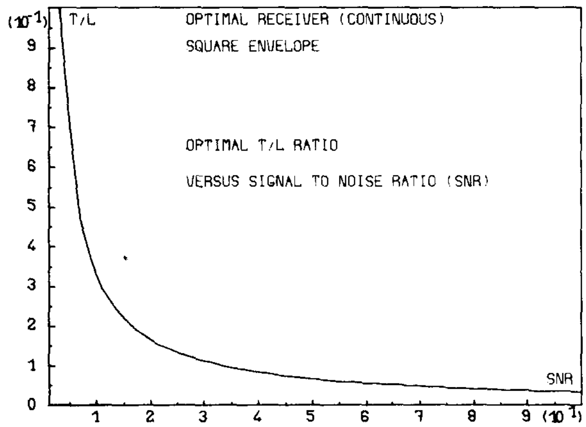

Fig. 2. Optimal T/L ratio versus signal to noise ratio.

4. Quantification of the scattering function

The sampling design is determined by the form of the scattering function. The scattering function may be thought of as the representation, on the delay axis, of a continuum of scatterers distributed in the space. We are looking for a method to represent this continuum with a finite number of single (of course, non fluctuating) scatterers. If one considers the scatterers in the space with a "mass" equal to the reflected power, then a reasonable representation seems to be through a finite number of appropriate barycenters. It is obvious that one can realize this choice by means of the delay-axis. In fact, the scattering function represents the reflected power. Then one can divide the area of the scattering function into a finite number of subareas of equal surfaces, and represent each subarea by its barycenter. Using the normalization of the scattering function, we have to determine the x_i ($i=1,\ldots,N-1$), such that

$$\int_0^{x_i} \tilde{Q}(\xi)\,\mathrm{d}\xi = \frac{i}{N}; \quad i=1,\ldots,N-1 \tag{12}$$

and the ξ_i, which are the barycenters, by

$$\xi_i = N\int_{x_{i-1}}^{x_i} \tilde{Q}(\xi)\,\mathrm{d}\xi; \quad i=1,\ldots,N; \quad x_0=0, x_N=L. \tag{13}$$

Thus the scattered function is replaced by

$$\tilde{Q}_p(\xi) = \frac{1}{N}\sum_{i=1}^{N} \delta(\xi-\xi_i) \tag{14}$$

where $\delta(\cdot)$ is the Dirac distribution.

We shall construct the optimal receiver, as if the scattering function were that of (14).

The corresponding received signal is as follows

$$\tilde{s}_p(t) = \sum_{i=1}^{N} Z_i\tilde{f}(t-\xi_i), \tag{15}$$

where the complex random variables Z_i are zero-mean and gaussian with:

$$\begin{aligned} &E\{Z_iZ_j^*\} = \frac{1}{N}\delta_{ij}, \quad \forall i,j,\\ &E\{Z_iZ_j\} = 0, \quad \forall i,j, \end{aligned} \tag{16}$$

where δ_{ij} is Kronecker δ, that is

$$\delta_{ij}\begin{cases} =0, & \text{if } i=j,\\ =1, & \text{if } i\neq j. \end{cases}$$

The optimal receiver for signals in (15) is given in [7] and is discussed in the following section as a suboptimal receiver for the scattering function in (8). We shall see that the scattering function's quantification gives the sampling choice.

5. Receiver using sampling

The receiver for the N-path signal $\tilde{s}_p(t)$ in (15), which corresponds to the case of factorizable covariance, is treated in [7]. In reality, the covariance function of $\tilde{s}_p(t)$ is as follows

$$\tilde{K}_{\tilde{s}_p}(t,u) = \frac{1}{N}\sum_{i=1}^{N} \tilde{f}(t-\xi_i)\tilde{f}^*(u-\xi_i). \tag{17}$$

Thus we can use the signals $\tilde{f}(t-\xi_i)$; $i=1,\ldots,N$ as basis of the observation signal space. The optimum receiver for the covariance function in (17) is composed of a matched filter to $\tilde{f}(t)$ (with $\tilde{f}^*(T-t)$, $0\leq t\leq T$, as impulse response), a sampler at $T+\xi_i$ $(i=1,\ldots,N)$ and a quadratic filter, which will be determined in what follows.

At the output of the sampler we obtain the signal vector $\tilde{\boldsymbol{s}}=[\tilde{s}_1,\ldots,\tilde{s}_i,\ldots,\tilde{s}_N]^{\mathrm{T}}$, with

$$\tilde{s}_i=\frac{1}{E_t}\int_0^{T+L}\tilde{s}(t)\tilde{f}^*(t-\xi_i)\,\mathrm{d}t;\quad i=1,\ldots,N \tag{18a}$$

and the noise vector $\tilde{\boldsymbol{n}}=[\tilde{n}_1,\ldots,\tilde{n}_i,\ldots,\tilde{n}_N]^{\mathrm{T}}$, with

$$\tilde{n}_i=\frac{1}{E_t}\int_0^{T+L}\tilde{n}(t)\tilde{f}^*(t-\xi_i)\,\mathrm{d}t;\quad i=1,\ldots,N. \tag{18b}$$

The covariance matrix of the signal vector has as elements the

$$\begin{aligned}E\{\tilde{s}_i\tilde{s}_j^*\}&=\frac{1}{E_t^2}\int_0^{T+L}\int_0^{T+L}\tilde{K}_{\tilde{s}}(t,u)\tilde{f}^*(t-\xi_i)\tilde{f}(u-\xi_j)\,\mathrm{d}t\,\mathrm{d}u\\&=\frac{1}{E_t^2}\int_0^{T+L}\int_0^{T+L}\int_0^{L}\tilde{Q}(\xi)\tilde{f}(t-\xi)\tilde{f}^*(t-\xi_i)\tilde{f}^*(u-\xi)\tilde{f}(u-\xi_j)\,\mathrm{d}t\,\mathrm{d}u\,\mathrm{d}\xi\\&=\int_0^{L}\tilde{Q}(\xi)\tilde{\rho}_{\tilde{f}}(\xi-\xi_j)\tilde{\rho}_{\tilde{f}^*}(\xi-\xi_i)\,\mathrm{d}\xi\end{aligned} \tag{19}$$

with $\tilde{\rho}_{\tilde{f}}(\cdot)$ the normalized correlation function of $\tilde{f}(t)$.

The covariance matrix of the noise vector has as elements the

$$\begin{aligned}E\{\tilde{n}_i\tilde{n}_j^*\}&=\frac{1}{E_t^2}\int_0^{T+L}\int_0^{T+L}N_0\delta(t-u)\tilde{f}^*(t-\xi_i)\tilde{f}(u-\xi_j)\,\mathrm{d}t\,\mathrm{d}u\\&=\frac{N_0}{E_t}\tilde{\rho}_{\tilde{f}}(\xi_j-\xi_i).\end{aligned} \tag{20}$$

If $\tilde{\boldsymbol{r}}=[\tilde{r}_1,\ldots,\tilde{r}_i,\ldots,\tilde{r}_N]^{\mathrm{T}}$ is the observed vector, with

$$\tilde{r}_i=\frac{1}{E_t}\int_0^{T+L}\tilde{r}(t)\tilde{f}^*(t-\xi_i)\,\mathrm{d}t. \tag{21}$$

Then the detection problem can be formulated as follows

$$\begin{aligned}&H_1:\ \tilde{\boldsymbol{r}}=\tilde{\boldsymbol{s}}+\tilde{\boldsymbol{n}}\\&H_0:\ \tilde{\boldsymbol{r}}=\tilde{\boldsymbol{n}}\end{aligned} \tag{22}$$

as well as the binary communication problem. The signal and noise vectors are independent. Let $\boldsymbol{K}_1$ be the covariance matrix of $\tilde{\boldsymbol{r}}$ under hypothesis H_1 (signal + noise covariance matrix) and $\boldsymbol{K}_0$ be the covariance matrix of $\tilde{\boldsymbol{r}}$ under hypothesis H_0 (only noise covariance matrix). The quadratic filter of the detector realizes the following processing [1-2]

$$l=\tilde{\boldsymbol{r}}^{\dagger}(K_0^{-1}-K_1^{-1})\tilde{\boldsymbol{r}} \tag{23}$$

where $\tilde{\boldsymbol{r}}^{\dagger}$ is the transposed conjugate of $\tilde{\boldsymbol{r}}$.

The matrix $K_0^{-1} - K_1^{-1}$ is non Toeplitz. If one wants a causal filter realization, one must determine the Cholesky decomposition of this matrix [10]. Thus the matrix $K_0^{-1} - K_1^{-1}$ is decomposed as a product of a lower and an upper triangular matrix

$$K_0^{-1} - K_1^{-1} = \Lambda^{\dagger}\Lambda \tag{24}$$

with Λ being the lower triangular matrix. The upper triangular matrix is the transposed conjugate of Λ, because $K_0^{-1} - K_1^{-1}$ is a Hermitian matrix. Thus the obtained statistic l is given by

$$l = \|\Lambda \tilde{r}\|^2 \tag{25}$$

where $\|\cdot\|$ is the euclidean norm, that is $\|x\|^2 = \sum_{i=1}^{N} |x_i|^2$ for every complex vector x.

The statistic l is then compared with a threshold, which is determined by the costs and a priori probabilities in a Bayes test, and the desired false alarm probability in a Neyman-Pearson test.

We summarize in Fig. 3 the receiver structure.

The receiver for the binary communication problem is obtained in a similar way and is composed of two branches as in Fig. 3.

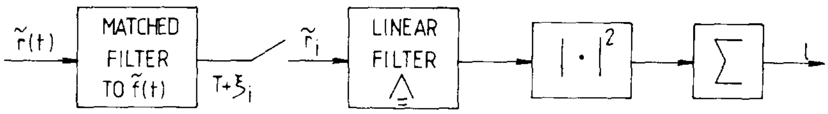

Fig. 3. The structure of the receiver using sampling.

In the following section, we study this receiver for the different values of the number of paths. We illustrate the role of parameters as the signal to noise ratio and the ratio of signal bandwidth to medium coherence bandwidth. We also discuss the existence of a value of the T/L ratio, which optimizes the transmission, that is, which minimizes the probability of error.

6. Numerical results and comments

We give numerical results for the scattering function in (8) and the transmitted signal in (9). Firstly, we consider the receiver using only one sample at the instant $T + L/2$. This corresponds to using the receiver for a Rayleigh channel in the case of a range spread channel. In Fig. 4, the error probability in the simple detection problem is given, for a signal to noise ratio equal to 10, versus the T/L ratio. The error probability is defined as follows:

$$P_E = \frac{1 - P_D + P_F}{2}$$

where P_D is the detection probability and P_F is the false alarm probability.

Here we have

$$P_F = e^{-\gamma/\sigma_0^2}, \quad P_D = e^{-\gamma/\sigma_1^2}, \qquad \gamma = \frac{\sigma_1^2\sigma_0^2}{\sigma_1^2 - \sigma_0^2} \ln \frac{\sigma_1^2}{\sigma_0^2}, \tag{26}$$

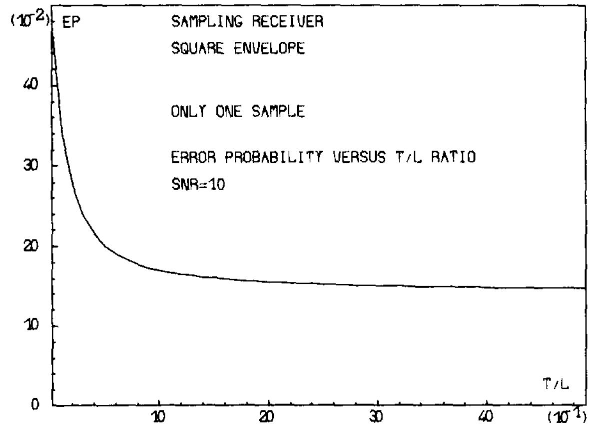

Fig. 4. The error probability for the Rayleigh receiver in the simple detection problem.

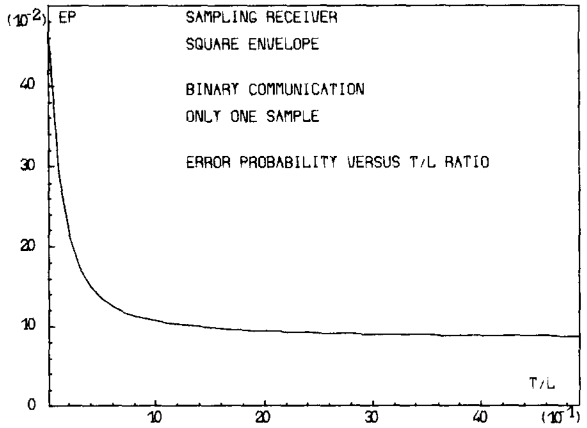

Fig. 5. The error probability for the binary symmetrical communication problem using the Rayleigh receiver.

with $\sigma_0^2 = N_0/E_t$ and

$$\sigma_1^2 = \sigma_0^2 + \int_0^L \tilde{Q}(\xi)\left|\tilde{\rho}_{\tilde{f}}\left(\xi - \frac{L}{2}\right)\right|^2 \mathrm{d}\xi$$

assuming that the two hypotheses H_1 and H_0 are equally probable.

In Fig. 5 we give the same curve for the binary symmetrical communication problem. The two transmitted signals have the same constant modulus and differ in frequency (FSK) in such a way that the received signals are orthogonal. The error probability is given by

$$Pr\{\varepsilon\} = \frac{1}{2 + (Et/N_0)\int_0^L \tilde{Q}(\xi)|\tilde{\rho}_{\tilde{f}}(\xi - L/2)|^2 \,\mathrm{d}\xi}. \tag{27}$$

We consider in the following only the simple detection problem with the above definition of error probability. We calculate these probabilities using the method of approximating the distribution of probability for positive definite quadratic forms developed in [11]. We give in Fig. 6 the evolution of error probability versus the number of samples for a signal to noise ratio equal to 100 and for different values for the T/L ratio.

In Figs. 7 and 8, we give curves for a signal to noise ratio equal to 50 and 20 respectively.

In these curves, we illustrate that the optimal value of the T/L ratio, determined for the optimal receiver, gives better results for a sufficient number of samples, in the case of the suboptimal receiver, than other values of this ratio. In fact, in figure 6, we give the curves of error probability for $T/L = 0.01$, 0.0348 (optimal), 0.1. We see that $T/L = 0.0348$ becomes better than $T/L = 0.1$ after 28 samples. In Fig. 7, we give the curves of error probability for $T/L = 0.0687$ (optimal), 0.1, 0.25. We remark that the optimal T/L almost reaches the performance of $T/L = 0.1$ with 30 samples. We can make the same remark in Fig. 8, where we give the curves of error probability for $T/L = 0.1$, 0.166 (optimal), 0.25.

These results show that the receiver using sampling has almost the same behaviour for the T/L parameter as does the optimal receiver. Thus we have a confirmation of the existence of an optimal T/L ratio.

Hence we have determined a very interesting family of suboptimal receivers over range-spread channels, which are also very simple to realize. The only complexity is in the non-periodic sampling and the difficulties consist in inversing two matrices and making a Cholesky decomposition, if one wants to obtain a causal filter realization.

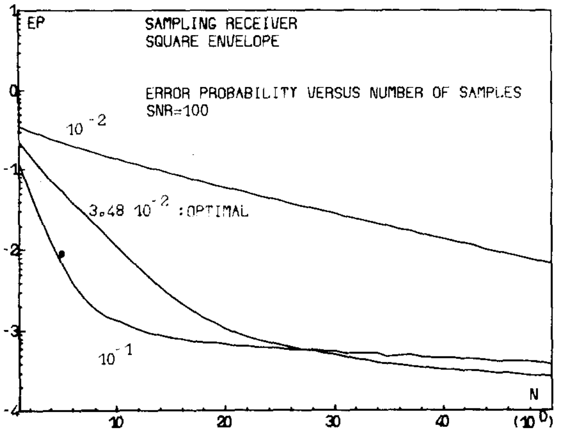

Fig. 6. Error probability of the receiver using sampling versus number of samples, for a signal to noise ratio equal to 100 and for $T/L = 0.01$, 0.0348 and 0.1.

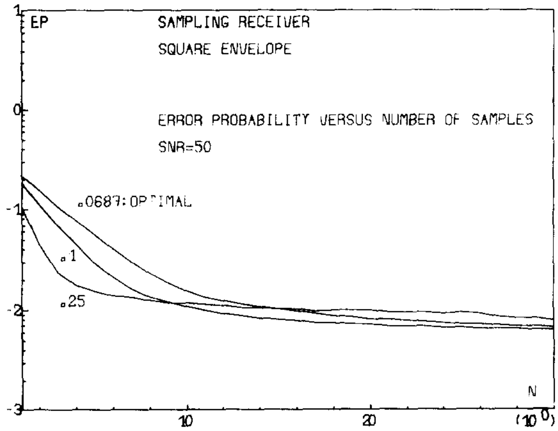

Fig. 7. Error probability of the receiver using sampling versus number of samples, for a signal to noise ratio equal to 50, and for $T/L = 0.0687$, 0.1 and 0.25.

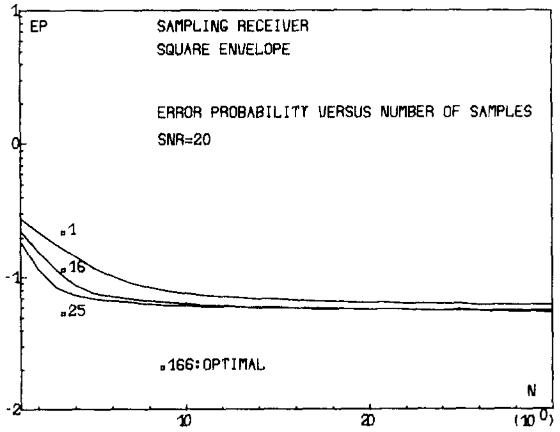

Fig. 8. Error probability of the receiver using sampling versus number of samples, for a signal to noise ratio equal to 20 and for $T/L = 0.1$, 0.166 and 0.25.

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