MODIFIED LMS ALGORITHMS FOR ROBUST ADPCM

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ABSTRACT

This paper deals with the robustness of ADPCM systems versus transmission errors. To secure the stability of the decoder, it is necessary to modify the form of the LMS algorithm used to adapt the predictor. Solutions introducing soft constraints are investigated. While the leakage algorithm is proved to be not fully satisfactory, a new stabilizing algorithm is presented which allows to achieve good performances. Compared to existing methods, the main advantage of this algorithm is its low computational complexity. From a theoretical point of view, the effect of tranmission errors is decribed by a set of nonlinear recurrent equations. An analysis is carried out in the deterministic second-order case.

1. INTRODUCTION

ADPCM (Adaptive Differential Pulse Code Modulation) is a well-known technique to achieve compression of correlated signals [1]. It is indeed a satisfactory solution to code both speech and images with low complexity. Yet, in the most widespread structure using a backward adapted predictor with the LMS algorithm, it appears that the decoder may become unstable in the presence of transmission errors.To solve this problem, standard algorithms combine several means and in particular a modification of the adaptation and stability checks [2]. This last requirement makes the method substantially more complex when the order of the predictor is high and/or when two-dimensional fields are processed.

A simpler alternative solution is proposed here. It is based on a regularization of the prediction criterion leading to a class of soft-constrained LMS algorithms. According to the form of the regularization, either the usual LMS with a Leakage Factor (LF) [3] or a new algorithm called the LMS with a Stabilizing Factor (SF) [4] is obtained. In this paper, the capabilities of these two algorithms to improve the robustness versus transmission errors are compared. The SF algorithm is proved to reach the best performances.

2. ADAPTIVE PREDICTIVE CODING

The structure of the ADPCM system is presented in Figure 2.1. x_n denotes the original signal, $\tilde{x}_n$ and $\hat{x}_n$ are respectively the reconstructed and the predicted samples, e_n is the prediction error and $\tilde{e}_n$ is its quantized value. The predictor is a FIR filter driven by $\tilde{x}_n$, which is adapted in order to deal with nonstationarities of x_n. Moreover, a backward adaptation is used to have no information to transmit other than a digital code corresponding to $\tilde{e}_n$. Finally, the equations of the encoder are:

$$e_n = x_n - \mathbf{H}_{n-1}{}^{\mathrm{T}}\tilde{\mathbf{X}}_n \qquad (2.1a)$$

$$\tilde{x}_n = \mathbf{H}_{n-1}{}^{\mathrm{T}}\tilde{\mathbf{X}}_n + \tilde{e}_n \qquad (2.1b)$$

$$\mathbf{H}_n = \mathcal{A}(\tilde{\mathbf{X}}_n, \mathbf{H}_{n-1}, \tilde{e}_n) \qquad (2.1c)$$

where $\mathbf{H}_n \in \mathbb{R}^N$ is the vector of predictor weights, $\tilde{\mathbf{X}}_n \in \mathbb{R}^N$ is a vector of past values of $\tilde{x}_n$ and $\mathcal{A}$ denotes the adaptation algorithm. The equations of the decoder are similar to (2.1b) and (2.1c) but, because of transmission errors, the quantities used are distinguished by a prime from the equivalent ones at the encoder.

3. EFFECT OF TRANSMISSION ERRORS

When there is no transmission error, identical initial values at the encoder and at the decoder ensure that $(\mathbf{H}_n, \tilde{x}_n) = (\mathbf{H}_n', \tilde{x}_n')$ for every $n \geq 0$. Conversely, transmission errors lead to misadjustments between the previous variables. Subsequently, these misadjustments are denoted by Δ and their dynamical behaviour is studied. For this purpose, it is assumed that if $n > n_0$, $\Delta\tilde{e}_n = 0$, which means that errors arise before the iteration $n_0 + 1$. Then, when $n > n_0$, the following equations can be obtained from (2.1):

$$\Delta\tilde{x}_n = \mathbf{H}_{n-1}{}^{\mathrm{T}}\Delta\tilde{\mathbf{X}}_n + \tilde{\mathbf{X}}_n{}^{\mathrm{T}}\Delta\mathbf{H}_{n-1} + \Delta\tilde{\mathbf{X}}_n{}^{\mathrm{T}}\Delta\mathbf{H}_{n-1} \qquad (3.1a)$$

$$\Delta\mathbf{H}_n = \mathcal{Q}_n\Delta\tilde{\mathbf{X}}_n + \mathcal{B}_n\,\Delta\mathbf{H}_{n-1} + \mathbf{C}_n \qquad (3.1b)$$

where $\mathcal{Q}_n = \left(\frac{\partial\mathcal{A}}{\partial\tilde{\mathbf{X}}}\right)_{\tilde{\mathbf{X}}_n}$, $\mathcal{B}_n = \left(\frac{\partial\mathcal{A}}{\partial\mathbf{H}}\right)_{\mathbf{H}_{n-1}}$ and $\mathbf{C}_n$ is a vector depending on second or higher order terms in $\Delta\tilde{\mathbf{X}}_n$ and $\Delta\mathbf{H}_{n-1}$. The problem is to get $(\Delta\tilde{x}_n, \Delta\mathbf{H}_n) \underset{n\to+\infty}{\to} (0, \mathbf{0})$. In general, the analysis is difficult for two reasons. The first one is the nonlinearity of system (3.1). The second one is the stochasticity of these equations since they are dependent on $\tilde{\mathbf{X}}_n$, $\mathbf{H}_{n-1}$ and $\tilde{e}_n$ which themselves are functions of the random signal x_n.

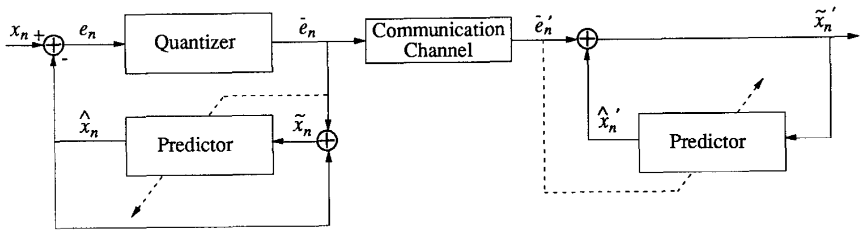

fig. 2.1: Structure of the ADPCM system

4. FAILURE OF THE LMS ALGORITHM

A possible solution to adapt the predictor weights is the LMS algorithm [5] :

$$\mathcal{A}(\tilde{\mathbf{X}}_n, \bar{e}_n, \mathbf{H}_{n-1}) = \mathbf{H}_{n-1} + \mu\bar{e}_n\tilde{\mathbf{X}}_n\,,\ \mu > 0\,. \quad (4.1)$$

Thus equation (3.1b) becomes

$$\Delta\mathbf{H}_n = \mu\bar{e}_n\,\Delta\tilde{\mathbf{X}}_n + \Delta\mathbf{H}_{n-1}\,. \quad (4.2)$$

Let us now consider a signal x_n, such that

$$x_n = \tilde{\mathbf{H}}^{\mathrm{T}}\mathbf{X}_n \quad (4.3)$$

where $\tilde{\mathbf{H}} \in \mathbb{R}^N$ belongs to the border of the stability domain of linear predictors of order N when x_n is non vanishing. Such a signal is a predictable process like a tone in telephony or a uniform area in an image. Further, let us assume that the algorithm has converged at the encoder when $n > n_0$ (by choosing n_0 large enough) and that the quantizer allows to get $\bar{e}_n = 0$. Then, $\tilde{x}_n = x_n$ and equations (3.1a) and (4.2) lead to

$$\Delta\tilde{x}_n = (\tilde{\mathbf{H}} + \Delta\mathbf{H}_{n_0})^{\mathrm{T}}\Delta\tilde{\mathbf{X}}_n + \Delta\mathbf{H}_{n_0}{}^{\mathrm{T}}\tilde{\mathbf{X}}_n \quad (4.4a)$$
$$\Delta\mathbf{H}_n = \Delta\mathbf{H}_{n_0}\,. \quad (4.4b)$$

Therefore, there exists a constant misadjustment on $\mathbf{H}_n$. Moreover, $\Delta\tilde{x}_n$ is the output of an IIR linear filter whose recursive part is $\tilde{\mathbf{H}} + \Delta\mathbf{H}_{n_0}$. Because of the form of $\tilde{\mathbf{H}}$, this filter can be either stable or unstable according to the direction of $\Delta\mathbf{H}_{n_0}$. This means that $\Delta\tilde{x}_n$ is not surely bounded.

5. USE OF SOFT CONSTRAINED LMS ALGORITHMS

To prevent the divergence mentioned in the previous Section, an approach of regularization can be followed. The criterion of minimization is the following one:

$$J(\mathbf{H}) = \mathrm{E}\{(x_n - \mathbf{H}^{\mathrm{T}}\tilde{\mathbf{X}}_n)^2\} + \alpha(\mathbf{H} - \mathbf{H}_f)^{\mathrm{T}}(\mathbf{H} - \mathbf{H}_f) + \beta(\mathbf{H} - \mathbf{H}_f)^{\mathrm{T}}\mathrm{E}\{\tilde{\mathbf{X}}_n\tilde{\mathbf{X}}_n{}^{\mathrm{T}}\}(\mathbf{H} - \mathbf{H}_f) \quad (5.1)$$

where $\mathbf{H}_f \in \mathbb{R}^N$, $\alpha \geq 0$ and $\beta \geq 0$. The minimum can be reached by the stochastic gradient technique yielding the following update equation:

$$\mathbf{H}_n = (1 - \mu\alpha)\mathbf{H}_{n-1} + \mu[x_n - (1 + \beta)\mathbf{H}_{n-1}{}^{\mathrm{T}}\tilde{\mathbf{X}}_n + \beta\mathbf{H}_f{}^{\mathrm{T}}\tilde{\mathbf{X}}_n]\tilde{\mathbf{X}}_n + \mu\alpha\mathbf{H}_f\,. \quad (5.2)$$

On condition that $\mathbf{H}_n$ is convergent, its asymptotic mean value can be specified:

$$\mathrm{E}\{\mathbf{H}_n\}\underset{n\to+\infty}{\longrightarrow}[(1 + \beta)\mathbf{I} + \alpha\mathbf{R}^{-1}]^{-1}[\tilde{\mathbf{H}} + (\beta\mathbf{I} + \alpha\mathbf{R}^{-1})\mathbf{H}_f] \quad (5.3)$$

where $\mathbf{R} = \mathrm{E}\{\tilde{\mathbf{X}}_n\tilde{\mathbf{X}}_n{}^{\mathrm{T}}\}$ and $\tilde{\mathbf{H}}$ is the best mean square estimate of the vector of weights. The above relation shows that the use of α and β leads to a bias in the estimation of the coëfficients. However this bias may be reduced owing to $\mathbf{H}_f$ which appears as an *a priori* estimate of $\tilde{\mathbf{H}}$.

When a backward adaptation is used for the ADPCM system, the quantities appearing in (3.1b) are

$$\mathbf{A}_n = \mu\{[\bar{e}_n - \beta(\mathbf{H}_{n-1} - \mathbf{H}_f)^{\mathrm{T}}\tilde{\mathbf{X}}_n]\mathbf{I} - \beta\tilde{\mathbf{X}}_n(\mathbf{H}_{n-1} - \mathbf{H}_f)^{\mathrm{T}}\} \quad (5.4a)$$
$$\mathbf{B}_n = (1 - \mu\alpha)\mathbf{I} - \mu\beta\tilde{\mathbf{X}}_n\tilde{\mathbf{X}}_n{}^{\mathrm{T}} \quad (5.4b)$$
$$\mathbf{C}_n = -\mu\beta\{\tilde{\mathbf{X}}_n\Delta\mathbf{H}_{n-1}{}^{\mathrm{T}}\Delta\tilde{\mathbf{X}}_n + [\tilde{\mathbf{X}}_n{}^{\mathrm{T}}\Delta\mathbf{H}_{n-1} + (\mathbf{H}_{n-1} - \mathbf{H}_f)^{\mathrm{T}}\Delta\tilde{\mathbf{X}}_n + \Delta\mathbf{H}_{n-1}{}^{\mathrm{T}}\Delta\tilde{\mathbf{X}}_n]\Delta\tilde{\mathbf{X}}_n\}\,. \quad (5.4c)$$

Two cases are of particular interest. The first one is the LMS with a Leakage Factor (LF) which is obtained when $\alpha \neq 0$ and $\beta = 0$. The second new algorithm, which is also considered, corresponds to $\alpha = 0$ and $\beta \neq 0$ and is the LMS with a Stabilizing Factor (SF).

6. ANALYSIS OF A SECOND-ORDER RECURRENCE

6.1. Background

For the reasons given in Section 3, the general form of the system defined by (3.1) and (5.4) seems untractable. Nevertheless, some analytical conclusions can be drawn for this nonlinear problem when it reduces to a deterministic second-order recurrence.

This particular case corresponds to a predictor of order 1 and a constant signal $x_n = X$. Additionally, it is assumed that the quantization error is negligible so that $\bar{e}_n = e_n$. Ideas about the hard nonlinearity introduced by the quantizer can be found in [6]. It is also considered that $0 \leq H_f \leq 1$ which is not a strong assumption since $\tilde{H} = 1$.

6.2. LF algorithm

Characteristics at the encoder

Under the previous assumptions, equation (5.2) yields:

$$H_n = [1 - (\delta + \mu X^2)]H_{n-1} + \mu X^2 + \delta H_f \qquad (6.1)$$

with $\delta = \mu\alpha$.

Therefore, if the following condition of convergence is satisfied:

$$-1 < 1 - (\delta + \mu X^2) < 1 \qquad (6.2)$$

the asymptotic values of H_n and e_n are:

$$H_\infty = \frac{\mu X^2 + \delta H_f}{\mu X^2 + \delta} \qquad (6.3a)$$

$$e_\infty = (1 - H_\infty)X\,. \qquad (6.3b)$$

Form of the recurrence

To simplify the notations, let us define $\chi_n = \frac{\Delta\tilde{x}_n}{X}$ and $K_n = \Delta H_n$. Then , (3.1) and (5.4) lead to:

$$\chi_n = H_\infty \chi_{n-1} + K_{n-1} + \chi_{n-1}K_{n-1} \qquad (6.4a)$$

$$K_n = \delta(H_\infty - H_f)\chi_{n-1} + (1 - \delta)K_{n-1}\,. \qquad (6.4b)$$

The fixed points (χ, K) of this second order recurrence are (0, 0) and $\left(\frac{1 - 2H_\infty + H_f}{H_\infty - H_f},\ 1 - 2H_\infty + H_f\right)$.

Local stability

Using the Jacobian matrix of system (6.4), the previous recurrence may be linearized in the neighbourhood of the fixed point (χ, K) as follows

$$\begin{bmatrix} \chi_n - \chi \\ K_n - K \end{bmatrix} = \begin{bmatrix} H_\infty + K & 1 + \chi \\ \delta(H_\infty - H_f) & 1 - \delta \end{bmatrix} \begin{bmatrix} \chi_{n-1} - \chi \\ K_{n-1} - K \end{bmatrix}. \qquad (6.5)$$

Then, the local stability at the point (χ, K) is ensured if and only if the eigenvalues of the Jacobian are strictly lower than 1 in module. It is straightforward to show that an equivalent condition is:

$$D < 1 \qquad (6.6a)$$

$$|T| < 1 + D \qquad (6.6b)$$

where D and T denote respectively the determinant and the trace of the Jacobian.

For the first fixed point (0, 0), these inequalities associated to (6.2) and (6.3a) are easily shown to be equivalent to the four following linear inequations:

$$H_\infty < \frac{1 + H_f}{2} \qquad (6.7a)$$

$$H_f < H_\infty \qquad (6.7b)$$

$$\delta > 0 \qquad (6.7c)$$

$$2H_\infty + (1 - H_f)\delta < 2 \qquad (6.7d)$$

It follows from (6.3 b) and (6.7.a) that

$$|e_\infty| > \frac{1 - H_f}{2}|X| \qquad (6.8)$$

which is a strong limitation if H_f is not close to 1.

It may be also useful to characterize the stability domain with the help of the parameters (μ, α) as depicted in Figure 6.1. This plot shows that for μ and α given, the local stability is independent of the choice of H_f in [0, 1].

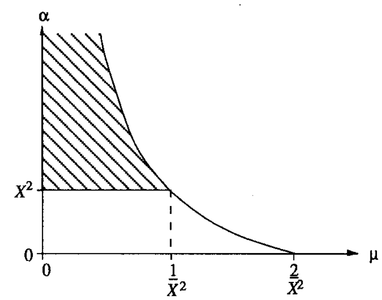

fig. 6.1: LF local stability domain at point (0,0)

For the second fixed point, it is straightforward that the stability can not be obtained when (0, 0) is stable.

Domain of attraction

When μ and α are chosen according to Figure 6.1, we can assert that there exists a domain of attraction $\mathcal{D}$ belonging to the phase plane such that if $(\chi_{n_0}, K_{n_0}) \in \mathcal{D}$, $(\chi_n, K_n) \underset{n\to+\infty}{\to} 0$. The border of $\mathcal{D}$ may be determined using numerical methods [7]. The plot given in Figure 6.2 shows the form of this domain for a particular choice of H_∞, H_f and δ . From a practical point of view, χ_{n_0} and K_{n_0} are related to the transmission errors arising before n_0. When there is a single error at n_0, this dependence is simply $\chi_{n_0} = \frac{\Delta\tilde{e}_{n_0}}{X}$ and $K_{n_0} = \mu X \Delta\tilde{e}_{n_0}$. Generally, $|\Delta\tilde{e}_{n_0}| \le X$ and μX^2 is small versus 1 so that $|\chi_{n_0}| < 1$ and $K_{n_0} \cong 0$. Then, according to Figure 6.2, there is no problem to satisfy the adjustment property.

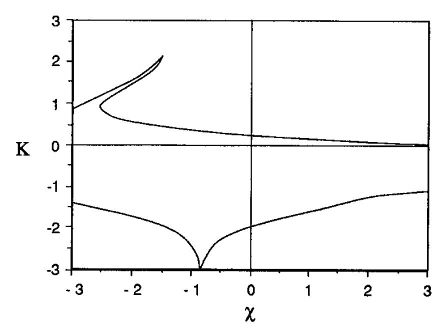

fig. 6.2: LF domain of attraction : $H_\infty = 0.4$, $H_f = 0$, $\delta = 0.1$ ($\alpha = 1.5\,X^2$, $\mu X^2 = 0.067$)

6.3. SF algorithm

A quite similar approach may be followed in this case and will be briefly reviewed.

Form of the recurrence

By setting $\delta = \mu\beta X^2$, (5.2) leads again to equation (6.1). Then, it comes from (3.1) and (5.4):

$$\chi_n = H_\infty \chi_{n-1} + K_{n-1} + \chi_{n-1} K_{n-1} \quad (6.9a)$$

$$K_n = -\delta(H_\infty - H_f)\chi_{n-1} + (1-\delta)K_{n-1} - \delta[2K_{n-1} + (H_\infty - H_f)\chi_{n-1} + K_{n-1}\chi_{n-1}]\chi_{n-1} \quad (6.9b)$$

and it is easily seen that the only fixed point of this recurrence is (0, 0).

Stability

By calculating the Jacobian, it appears that conditions (6.6) are equivalent to (6.7b), (6.7c) and (6.7d). The local stability domain of point (0, 0) is therefore larger than for the LF algorithm since the constraint (6.7a) is not necessary (see Figure 6.3). The main interest is that there is no lower bound on the prediction error.

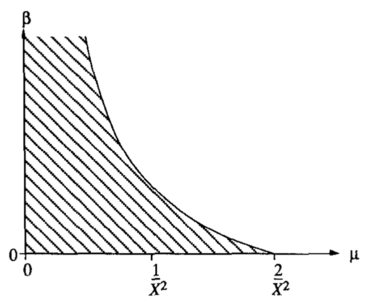

fig. 6.3 : SF local stability domain

The domain of attraction of (0, 0) is plotted in Figure 6.4.

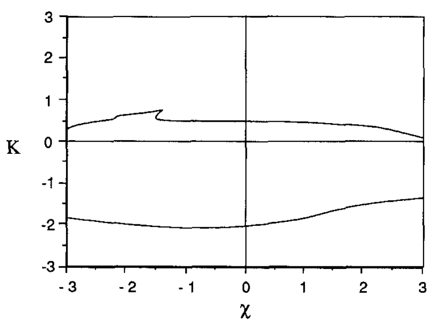

fig. 6.4: SF domain of attraction : $H_\infty = 0.8$, $H_f = 0$, $\delta = 0.02$ ($\beta = 0.25$, $\mu X^2 = 0.08$)

7. CONCLUSIONS

In an ADPCM system, transmission errors lead to misadjustments between the variables at the encoder and at the decoder, which have been modeled by nonlinear recurrent equations. As the LMS algorithm fails to ensure the adjustment of predictable processes, two constrained algorithms of low complexity were introduced. There is then a bias on the estimation of weights which is performed but it can be decreased if an *a priori* estimate of the weights is available.

In the case of a prediction of order one and a constant input, the dynamical behaviour of misadjustments has been studied. The analysis of local stability and the evaluation of domains of attraction evidence that the robustness is improved by the constrained algorithms. Compared to the LF algorithm, less restrictive stability conditions are obtained for the new SF algorithm which allows a better prediction.

Further developments are currently investigated to extend these theoretical results. Computer simulations show that similar conclusions could be drawn for speech [8] and image [9] coding.

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