COMMUNICATION IN A FLUCTUATING CHANNEL
MODELS AND USE OF EXPLICIT OR IMPLICIT DIVERSITY

G. JOURDAIN - G. TZIRITAS

CEPHAG - Equipe de Recherche Associée au CNRS
BP 46 - 38402 SAINT MARTIN D'HERES - Tél. (76)44.82.45

ABSTRACT

When the transmission is fluctuating (with respect to the elementary transmission duration) diversity techniques enable to improve the performance.

In a first part the Rayleigh model has been tested in long distance submarine acoustics transmission and we present some experimental results of explicit frequential diversity. In the general delay and Doppler dispersive case, we present some results of implicit diversity in order to decrease the error probability and we show the consequences of this concept on to the signal design problem. We have applied this study particularly in two cases : the multipath transmission and the fast-fading modulation case.

INTRODUCTION

In many propagation cases and particularly in submarine acoustics, moreover the additive noise, the emitted signal is distorded by the transmission. The generally used model representing the propagation channel is a linear filter and the experimental results are often in good agreement with it ; but the fluctuating aspect can be considered in several ways, either by means of a random model or by a time varying model or by the two aspects. This modelization is directly related to the time scale of interest.

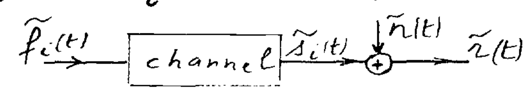
Various increasingly sophisticated models are considered here with respect to the elementary transmission duration. We are interested in the binary symetric case and the aim is, given a channel characterization, to use this knowledge in order to decrease the reception error probability.

Some experimental results using a simple channel model (Rayleigh) are given, by means of an explicit diversity to improve the performance. After that some more complex models are studied in which the implicit diversity plays the same part. Some theoretical results are given about the multipath propagation and the temporal modulation case.

I - THE MODEL AND HYPOTHESIS

The emitted and received signals are usually bandpass signals round a center frequency ν_0 ; so the transmission problem is written here in terms of complex amplitudes relative to ν_0 (denoted by $\sim$). The observation is :

$$H_i : \tilde{r}(t) = \tilde{s}_i(t) + \tilde{n}(t),\ i = 0,1,\ t\ [0,T] \quad (1)$$



where $\tilde{s}_i(t)$ is the transmitted signal through the linear channel which is, in the more general case, characterized by a bitemporal response $\tilde{H}(t,\zeta)$.

$$\tilde{s}_i(t) = \int \tilde{f}_i(t-\zeta)\, \tilde{H}(t,\zeta)\, d\zeta \quad (2)$$

In the same way, $\tilde{n}(t)$ is the complex amplitude of the centered white gaussian additive noise with a power spectral density N_0. The emitted signals have the same energy E and duration T_0 :

$$E_i = E = \int_0^{T_0} |\tilde{f}_i(t)|^2\, dt \quad (3)$$

The emitted signals $\tilde{f}_i(t)$ are supposed orthogonal and also are the received signals supposed and the two hypothesis are equiprobable : this is the "binary symetric case".

1) <u>The "ideal" or "coherent" channel</u>. In this case :

$$\tilde{H}(t,\zeta) = \alpha\, \delta(\zeta) \text{ and } \tilde{s}(t) = \alpha \tilde{f}_i(t) \ (\alpha \text{ known}) \quad (4)$$

The optimal receiver includes the matched filters to $\tilde{f}_i(t)$ and the output comparison. The error probability is well known :

$$P(e) = \mathrm{erfc}_* E/N_0 ;\ \mathrm{erfc}_*(\lambda) = \frac{1}{\sqrt{2\pi}} \int_\lambda^{+\infty} \exp - \frac{u^2}{2}\, du \quad (5)$$

The signal energy is the only parameter occuring in the performance (given the signal orthogonality).

This case is obviously illusive ; a little more realistic model is the "incoherent" channel in which the signal phasis is no longer preserved during the transmission.

2) A more accurate and usually encountered model is the Rayleigh model in which the $\tilde{s}_i(t)$ phasis is uniformly distributed and the $\tilde{s}_i(t)$ complex amplitude is a gaussian random variable (r.v.) :

$$\tilde{H}(t,\zeta) = \alpha\, e^{i\varphi}\delta(\zeta) \qquad (6)$$

There is not a true dispersive channel yet ; from one T-bit to another the values of the r.v. α and φ are different.

3) The most general case occurs when $\tilde{H}(t,\zeta)$ is a random function. The channel is then usually supposed WSSUS (Wide Sense Stationary Uncorrelated Scatterers), so that it is statistically characterized by a scattering function $\tilde{S}(\nu,\zeta)$ which expresses the channel dispersive action versus the time delay ζ and frequency ν. The received instantaneous power is then $E\{|\tilde{s}_i(t)|^2\}$ and the received signal to noise ratio is now :

$$\frac{\overline{E_r}}{N_0} = \frac{1}{N_0}\int_0^T E\{|\tilde{s}_i(t)|^2\}\, dt = \frac{\overline{E_r}}{N_0}\iint \tilde{S}(\nu,\zeta)\, d\nu\, d\zeta = M\,\frac{E}{N_0} \qquad (7)$$

N.B. : The Rayleigh case (6) corresponds to an "ideal" scattering function :

$$\tilde{S}(\nu,\zeta) = M\,\delta(\nu)\,\delta(\zeta) \quad \text{with } M = E\{|\alpha|^2\} \qquad (8)$$

Some various techniques for estimating this scattering function have been studied and two examples in horizontal long distance submarine propagation are given below : the first corresponds to a quasi non dispersive channel ($\sim$Rayleigh) and the second is a time and frequency-spread case [11].

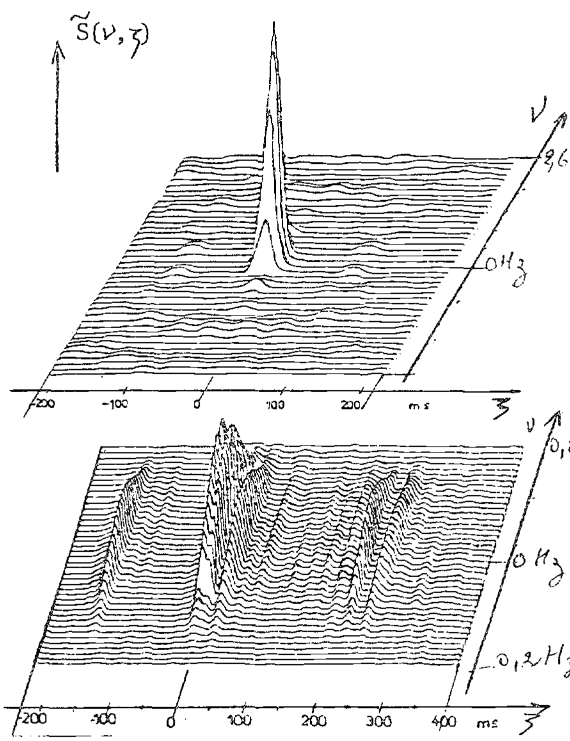


FIGURE 1
Different experimental scattering functions

II - EXPLICIT DIVERSITY USED WITH A RAYLEIGH MODEL

1) Theoretical study of the Rayleigh model (cf(8))
With orthogonal signals (e.g. FSK transmission), the optimal receiver is well known (matched filter followed by an envelope detector). The error probability is now :

$$P(e) = \frac{1}{2 + \overline{E_r}/N_0} \qquad (9)$$

It is yet only dependent on the received energy (and not on the signal form, given the $\tilde{s}_i(t)$ orthogonality). In order to decrease this error probability, the use of "explicit" diversity techniques is well known (either frequential or temporal or spatial diversity). PIERCE [6] has given for K fold-diversity with identical SNR $\eta = \frac{\overline{E_r}}{N_0}\cdot\frac{1}{K}$ on each path :

$$P(e) = \frac{1}{(2+\eta)^K}\sum_{k=0}^{K-1}\frac{(k+K-1)!}{k!\,(K-1)!}\left(\frac{1+\eta}{2+\eta}\right)^k \qquad (10)$$

In this case, given a transmitting energy by binary digit and supposing the same SNR η on each diversity path, [10] has shown that an optimal number K_0 exists (at least when $\frac{\overline{E_r}}{N_0} > 3$):

$$K_0 \# \frac{1}{3}\cdot\frac{\overline{E_r}}{N_0}, \text{ so that } \eta \# 3 \text{ on each path.} \qquad (11)$$

2) Some experimental results in long distance submarine acoustics [3]
Some results about the bit error probability are given to illustrate together the channel model and the formula (10). They come from an FSK transmission in a long distance submarine acoustics propagation ($\sim$100 km) between a submarine and a fixed some hundred meters dipped transducer. The digit duration is about 1 sec. The frequential diversity uses center frequencies distant of 100 Hz. The received SNR and error probability are both estimated. The figure 2 gives some idea of the obtained results with a 1st, 2nd or 3rd order diversity processing. The full lines correspond to the formula (10) with K = 1, 2 and 3. In this case the Rayleigh model is seen to be in good agreement with the results and the diversity paths are well independent one from another.

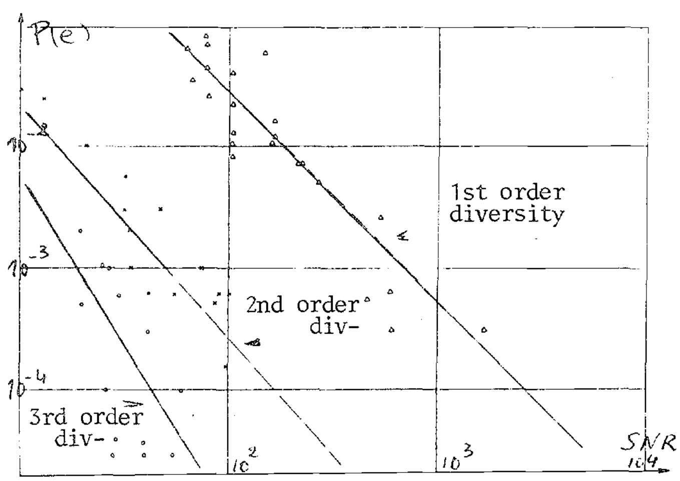


FIGURE 2
Frequential diversity results

III - OPTIMAL COMMUNICATION IN A DISPERSIVE MEDIUM IMPLICIT DIVERSITY

1) When the scattering function is time and frequency spread, the optimal receiver can be reached by means of an internal model. It is relative to the variation versus ν of $\tilde{S}(\nu,\zeta)$ and ζ-parametrized (see [8]) and it leads to a functional $\ell(\tilde{r}(t))$. The performance cannot be exactly calculated in all cases. The important point is that it depends now on $\tilde{S}(\nu,\zeta)$ and the signal $\tilde{s}_i(t)$ form. The authors have proposed :

- either borns to P(e) (Pierce and, after him, [9]). Particularly :

$$P(e) \leq 0.5 \exp \mu(1/2), \text{ with } \mu(s) = \log E\{\exp s\ell(\tilde{r}/H_0)\} \quad (12)$$

KENNEDY [5] has shown an optimum value of $\mu(1/2)$ can be obtained for different combinations of signals $\tilde{f}_i(t)$ and spread scattering functions if it is possible to manage the signal $\tilde{s}(t)$ in order to all its eigenvalues are identical and equal to $\frac{\overline{E}_r}{N_0} \cdot \frac{1}{3.07}$ (implicit diversity). In this case :

$$\mu_{opt}(1/2) = -0.1488 \frac{\overline{E}_r}{N_0} \quad (12)$$

- or approximate formulas for P(e). COLLINS [2] gives one which is not well adapted when keeping only a few approximation terms.

Recently, [7] has proposed an approximate more adapted formula given below.

$$P(e) = \frac{\Gamma(2a+2)}{\Gamma(a+1)\Gamma(a+2)} \left(\frac{1+b}{(2+b)^2}\right)^{a+1} {}_2F_1(2a+2, 1, a+2; \frac{1}{b+2}) \quad (13)$$

in which ${}_2F_1(\cdot, \cdot, \cdot ; \cdot)$ is the hypergeometric function, Γ is the Gamma function and the quantities a and b are the first and second order moments of ℓ_1/H_1 (ℓ_1 being the output of the receiver corresponding to $\tilde{f}_1(t)$) :

$$a+1 = \frac{[E\{\ell_1/H_1\}]^2}{\mathrm{Var}\{\ell_1/H_1\}} \quad ; \quad b = \frac{\mathrm{Var}\{\ell_1/H_1\}}{E\{\ell_1/H_1\}} \quad (14)$$

The advantages of this formula are :

i) it is expressed versus simple parameters (the moments) which are expressed in function of the input $\tilde{f}(t)$ signal [7] and not the ouput signal -like in KENNEDY's- So in order to decrease P(e) one acts upon a and b , and so upon the input signal : the signal design problem become easier.

ii) The quantities a and b have a simple interpretation which exhibits the implicit diversity aspect of the optimal system. In fact, if a is an integer, the formula (13) has the same form as (10) with :
a+1 = K ("implicit diversity number") and
b is the SNR on each "implicit diversity path"

It is also shown that the optimal value of b is also about 3 (with any large enough SNR). This is to be related to the values given by KENNEDY (and also the explicit diversity results). The fig. 3 (below) compares the minimal born given by (12) and the minimal P(e) given by (13) (both when optimizing the parameters).

iii) This approximate formula becomes an exact one in the interesting ideal multipath propagation case (see below).

We have particularly studied the ideal multipath case and also the opposite case which is the only Doppler spread case. These two particular cases correspond to the two axes sections of fig. 1b.

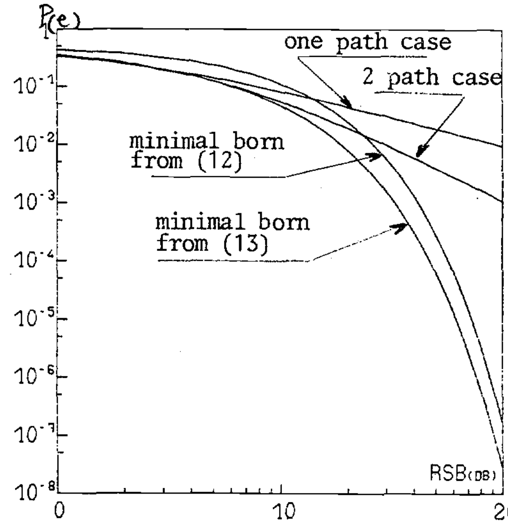


FIGURE 3
Output error probabilities

2) Ideal Multipath Propagation - N paths [4]

The scattering function is :

$$\tilde{S}(\nu,\zeta) = \sum_{i=1}^{N} q_i \, \delta(\zeta - \zeta_i) \quad ; \quad \sum_{i=1}^{N} q_i = 1$$

$q_i \overline{E}_r$ is the transmitted power over the path i. On each path the fading is a Rayleigh one and they are independent from one path to another. The important parameter is here the bandwith signal : the optimal $\tilde{f}(t)$ is a signal the correlation function of it is zero for the path-delays ζ_i. In this case we know the exact error probability [8] and when all the q_i are identical, this is exactly the formula (13) -or (10)- with a+1 = N.

On the above fig. 3 the error probability for one path and for the optimal two-paths case are plotted : as long as $\frac{\overline{E}_r}{N_0} < 3$, P(e) is identical to the proposed born for one path (but inferior to the KENNEDY's born).

For two-paths P(e) and the born are identical as long as $\frac{\overline{E}_r}{N_0} < 6$. When $\frac{\overline{E}_r}{N_0}$ increases, the optimal performance would be reached if there would be more propagation paths in order to distribute $\overline{E}_r$.

3) Temporal modulation propagation

The scattering function is supposed of the type

$$\tilde{S}(\nu,\zeta) = \frac{2k}{(2\pi\nu)^2 + k^2} \delta(\zeta)$$

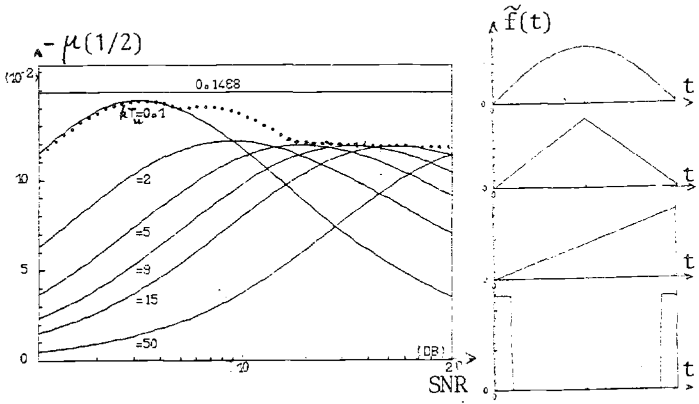
(LORENTZIAN Doppler spreading measured by k)

The main signal parameter is now its duration or rather an "effective" duration. Different signal envelopes have been tested (see fig. 4 below) for which the parameter $\mu(1/2)$, that represents a performance measure, has been calculated. It has been pointed out [1] that, with respects to the effective duration T_u given by [5] :

$$T_u = [\int_R |\tilde{f}|^4 dt]^{-1}$$

all these forms are equivalent. $-\mu(1/2)$ has been plotted fig. 4 below versus $\overline{E_r}/N_0$ and for different products kT_u (k = channel doppler spread ; T_u = effective signal duration).

A maximum of $-\mu(1/2)$ is still found for $\frac{E_r}{N_0} \cdot \frac{1}{N} \sim 3.4$ with $N = 1/kT_u$ (implicit diversity of the system). The last form (n° 4) (corresponding to dotted line in fig. 4a) is more adapted yet to the channel modulation : the farther the two signal impulses are, the best the performance is. This means that these two impulses become more decorrelated versus the channel correlation time $1/k$: so the temporal diversity appears really in this signal form.



4a) 4b)

FIGURE 4

a) Evolutions of $-\mu(1/2)$
b) Different envelopes $\tilde{f}(t)$

V - CONCLUSION

The efficacity of the diversity techniques in a dispersive medium has been well exhibited here : "explicit" diversity when a Rayleigh model is satisfying over the transmitted bit duration, "implicit" diversity when the time variation is faster. In submarine acoustics, the above conditions were in a good agreement with a Rayleigh model. But other cases have been studied in which the temporal modulation was more important. An optimal communication system necessarily includes a previous knowledge of the channel, and a previous choice of some system parameters or configuration. For example, our submarine acoustics experience leads to take in account for the channel model -so for the optimal receiver- the following parameters (and probably in the decreasing order) : the SNR, the multipath effect, the reverberation, the modulation effect -according to these parameters importance and time evolution, one or other model- and optimal receiver, will be pertinent.

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