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SMOOTHING THE DISPLACEMENT FIELD FOR EDGE-BASED MOTION ESTIMATION

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In this article we present a method for estimating the two-dimensional velocity field on moving edges. We assume that the 2D velocity field is an affine transformation of the point coordinates. This model takes into account many different types of motion, and allows us to obtain a recursive relation on the 2D velocity field. We use this relation to determine the optimal smoother based on the measure of the normal component of the velocity vector. The well-known two-filter formula [1] is used.

1. INTRODUCTION

Motion estimation in a sequence of images is an important challenge in image processing and scene analysis. Two approaches can be considered: a 2D region-based and an edge-based. In this paper we are interested in edge-based estimation. Indeed, edges may constitute relevant features for motion robust estimation. The points on the edges usually correspond usually to orientation discontinuities between different surfaces or to object boundaries. It can be assumed that the observed 2D edges or boundaries are the geometrical projection of the 3D edges on the image plane. The estimated velocity field is not dense, but there are many reasons which allows us to consider that this velocity field is more significant than the region-based one. The principal reason is that in these points the apparent motion (or optical flow) can be considered as the projection on the image plane of the 3D velocity field of a moving scene. A.Verri and T.Poggio [8] have shown that the apparent and the real velocity field are very close where the image gradient is sufficiently strong. They conclude that to recover the 3D velocity field, edge-based algorithms seem more suitable than algorithms based on spatial and temporal derivatives of the image brightness. In the following we suppose that edges correspond to features in the scene.

The first operation of an edge-based motion estimator is the edge detection. In order to realize this operation we use J.Canny's method [2], as it is implemented by R.Deriche [3]. The result of this operation is the localization of the edge points, and the estimation of the orientation of the edge segment at each point.

In order to estimate the displacement vector, it is also necessary to determine, independently of the method used for the estimation, connections between edge points. In this paper we use a simple method to test connections and to link edge points. At the end of this operation a link of the points which constitute an entire edge is determined, the orientation of the edge at each point is obtained and the normal component of the displacement vector is estimated.

The result of the feature extraction processing is a set of lists of points belonging in different edge elements. An edge is described by a list of points $\mathbf{p}_k$ with 2D coordinates (x_k, y_k),

$$\{\mathbf{p}_k : k = 1, 2, \dots, N\}$$

for an edge containing N points.

Concerning motion, only one component of the displacement vector can be measured from edge positions in successive images. This is the perpendicular to the edge component. It is the well-known aperture problem. To measure the normal component a displacement on the perpendicular direction from the first contour to the second is considered. This measurement may introduce errors if the edge is not locally a straight line [7].

If $\mathbf{w}_k = [\, u_k \;\; v_k \,]^T$ is the 2D velocity vector at point $\mathbf{p}_k$, and $\mathbf{n}_k$ the normal vector at the same point, with $\| \mathbf{n}_k \| = 1$, then the normal component of the velocity vector is given by $\mathbf{n}_k^T \mathbf{w}_k$. If $w_k^\perp$ is the measured normal component, then we can write

$$w_k^\perp = \mathbf{n}_k^T \mathbf{w}_k + z_k$$

where z_k is a random noise, here supposed to be zero-mean and white with variance equal to R_k.

To estimate the other component of the displacement vector some smoothness constraints must be used. E.Hildreth [4] proposed a regularization method which search for a compromise between the closeness on the data and the smoothness of the displacement vector. E.Hildreth [4] proposed to minimize the following criterion

$$\sum_{k=1}^{N-1} \| \mathbf{w}_{k+1} - \mathbf{w}_k \|^2 + \alpha^2 \sum_{k=1}^{N} (w_k^\perp - \mathbf{n}_k^T \mathbf{w}_k)^2$$

and to solve for $\{\mathbf{w}_k : k = 1, 2, \ldots, N\}$ using the conjugate gradient algorithm. Here we propose to use an optimal smoother, also optimizing a quadratic criterion, and based on the same measures, but in another type of smoothing, which is presented in the following Section. The resulting smoother is presented in Section 3.

2. MODEL OF THE VELOCITY FIELD

In a precedent article [7] we considered some simple geometrical assumptions in order to obtain a model of the 2D velocity field in the case of a rigid 3D motion projected on the image plane. If this projection is orthographic, the 2D velocity fiels is an affine transformation of the point coordinates. G.Mailloux et al. [5] use the same model in a different domain of application concerning two-dimensional echocardiograms and heart motion. We present this model in the following, and we use it for obtaining neighbourhood relations on the velocity field.

The velocity vector $\mathbf{w}$ at a point $\mathbf{p}$ is modelized by

$$\mathbf{w} = \mathbf{t} + A\mathbf{p} \qquad (2)$$

where $\mathbf{t}$ corresponds to a translation vector and matrix A takes into consideration rotation and some deformation of the pattern of the edge. The criterion (1) takes into account only a pure translation vector (A = 0). Let us suppose for simplicity, that for the definition of the velocity the time unit is equal to the temporal sampling period. Then, if $\mathbf{p}'$ and $\mathbf{p}$ are corresponding points, in two successive frames, we have

$$\mathbf{p}' = \mathbf{t} + (I+A)\mathbf{p} \qquad (3)$$

We can assume for applicable models that eigenvalues of matrix are in modulus small in comparison with 1, and therefore matrix I+A is always supposed non-singular. Under these hypotheses, it is easy to demonstrate that a straight line is transformed by (3) into a straight line, and a polygon into a polygon. Indeed, if $(\mathbf{p}', \mathbf{p})$, $(\mathbf{p}_1', \mathbf{p}_1)$ and $(\mathbf{p}_2', \mathbf{p}_2)$ are corresponding points and $(\mathbf{p}, \mathbf{p}_1, \mathbf{p}_2)$ are aligned, we have

$$\det [\, \mathbf{p}' - \mathbf{p}_1' \;\; \mathbf{p}_1' - \mathbf{p}_2' \,] = \det(I+A) \det [\, \mathbf{p} - \mathbf{p}_1 \;\; \mathbf{p}_1 - \mathbf{p}_2 \,] = 0$$

which means that points $(\mathbf{p}', \mathbf{p}_1', \mathbf{p}_2')$ are aligned.

We also remark that, if $\mathbf{t} = 0$, and A is a positive-definite matrix, then the motion modelized by (2) is a dilation. If A is a negative-definite matrix, then the motion modelized by (2) is a contraction. In conclusion, we can say that the model proposed here can take into consideration several different types of motion.

Let us note the elements of matrix A as following

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

For two successive points $\mathbf{p}_{k+1}$ and $\mathbf{p}_k$ we can write, in accordance with (2),

$$\mathbf{w}_{k+1} - \mathbf{w}_k = A(\mathbf{p}_{k+1} - \mathbf{p}_k)$$

From this last equation, and considering separately the two components of the velocity vector, we can write for component u, taking into account four successive points

$$\begin{bmatrix} u_{k+1}-u_k & x_{k+1}-x_k & y_{k+1}-y_k \\ u_k-u_{k-1} & x_k-x_{k-1} & y_k-y_{k-1} \\ u_{k-1}-u_{k-2} & x_{k-1}-x_{k-2} & y_{k-1}-y_{k-2} \end{bmatrix} \begin{bmatrix} -1 \\ a_1 \\ b_1 \end{bmatrix} = 0 \quad (4)$$

We also can write similar equations for the components v. A consequence of (4) is that the determinant of the above matrix must be zero,

$$\begin{vmatrix} u_{k+1}-u_k & x_{k+1}-x_k & y_{k+1}-y_k \\ u_k-u_{k-1} & x_k-x_{k-1} & y_k-y_{k-1} \\ u_{k-1}-u_{k-2} & x_{k-1}-x_{k-2} & y_{k-1}-y_{k-2} \end{vmatrix} = 0$$

We can then write

$$D_{1,k}(u_{k+1}-u_k)-D_{2,k+1}(u_k-u_{k-1})+D_{1,k+1}(u_{k-1}-u_{k-2}) = 0$$

with $D_{1,k} = \begin{vmatrix} x_k-x_{k-1} & y_k-y_{k-1} \\ x_{k-1}-x_{k-2} & y_{k-1}-y_{k-2} \end{vmatrix}$

and $D_{2,k+1} = \begin{vmatrix} x_{k+1}-x_k & y_{k+1}-y_k \\ x_{k-1}-x_{k-2} & y_{k-1}-y_{k-2} \end{vmatrix}$

If the three points p_k, p_{k-1} and p_{k-2} are not aligned, then $D_{1,k} \neq 0$, and we can write

$$u_{k+1}=(1+\frac{D_{2,k+1}}{D_{1,k}})u_k-\frac{D_{1,k+1}+D_{2,k+1}}{D_{1,k}}u_{k-1}+\frac{D_{1,k+1}}{D_{1,k}}u_{k-2} \quad (5)$$

which is an autoregressive relation on the velocity. The same relation is valid for the other component of the velocity vector.

3. ESTIMATION OF THE VELOCITY FIELD

We propose to use the autoregressive relation given in the precedent section for estimating the 2D velocity field. Let us consider the equation (5) and write the autoregressive relation for the two velocity components

$$u_{k+1} = \beta_k u_k + \beta_{k-1} u_{k-1} + \beta_{k-2} u_{k-2}$$
$$v_{k+1} = \beta_k v_k + \beta_{k-1} v_{k-1} + \beta_{k-2} v_{k-2}$$

The identification of coefficients $\{\beta_k\}$ is obvious according to (5); they depend on edge line curvature. Of course the above model is not perfect and we have to take into acount a model noise. We designate ξ_k the state vector given by

$$\xi_k = [\, u_k \ u_{k-1} \ u_{k-2} \ v_k \ v_{k-1} \ v_{k-2} \,]^T$$

The state equation according to the above recursive relations is given below

$$\xi_{k+1} = \begin{bmatrix} \Phi_{k+1|k} & 0 \\ 0 & \Phi_{k+1|k} \end{bmatrix} \xi_k + \omega_k \quad (6)$$

where the noise vector ω_k is zero-mean with covariance matrix Q_k and the transition matrix $\Phi_{k+1|k}$ is

$$\Phi_{k+1|k} = \begin{bmatrix} \beta_k & \beta_{k-1} & \beta_{k-2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The equation of measurement is given by

$$y_k = [\, c_{1k} \ 0 \ 0 \ c_{2k} \ 0 \ 0 \,] \ \xi_k + z_k$$

where y_k is the measured projection of the velocity vector on the perpendicular to the contour vector and z_k is a measurement noise which is assumed zero-mean with variance R_k. We have

$$[\, c_{1k} \ \ c_{2k} \,]^T = n_k$$

n_k being the normal vector. We can write

$$(c_{1k}, c_{2k}) = (f_x(x_k,y_k), f_y(x_k,y_k))$$

if the equation of the contour is known: $f(x,y)=0$. The system and measurement noise are assumed to be independent and independent between different points.

The problem to solve is the following: given the observation of $\{y_k \,;\, 1 \le k \le N\}$ on the edge, how to estimate the field of $\{(\, u_k \,,\, v_k \,) : 1 \le k \le N\}$. This is a smoothing problem and we propose to use the two-filter smoothing formula [1], which gives the optimal solution with a quadratic criterion

$$\hat{\xi}_{k|N} = P_{k|N}\,(P^{-1}_{f,k|k-1}\hat{\xi}_{f,k|k-1} + P^{-1}_{b,k|k}\hat{\xi}_{b,k|k})$$

$$P^{-1}_{k|N} = P^{-1}_{f,k|k-1} + P^{-1}_{b,k|k}$$

where $\hat{\xi}_{f,k|k-1}$ is a forwards optimal prediction and $\hat{\xi}_{b,k|k}$ is a backwards optimal filtering, both based on the same state vector equation (6). $P_{f,k|k-1}$ and $P_{b,k|k}$ are the corresponding error covariance matrices. The initial covariance matrix, at k=1, for the forwards filter, and at k=N, for the backwards filter, are assumed sufficiently great in the above formula. A similar approach using a different state equation is presented in [6] where some results concerning the motion of simulated edges are given.

4. CONCLUSION

We have introduced a model for the 2D velocity field, which is adaptable in many domains of applications, and many types of 2D motion or 3D motion projected on the image plane. We have shown how this model may be used to estimate the 2D velocity field on points belonging on edges detected from a sequence of images. The evaluation of the performance of this method in natural images is currently under investigation.

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