CS578 - Speech Signal Processing

Lecture : Harmonic and Quasi-Harmonic Models of Speech

George P. Kafentzis

University of Crete, Computer Science Dept., Speech Signal Processing Lab
kafentz@csd.uoc.gr
(based on work from Prof. Stylianou and Dr. Pantazis)

Univ. of Crete

Outline

Background

Mentioning just a few works for speech analysis...

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- Multi-Band Excitation Vocoder (Griffin et al.1988 [1])
  - $S(\omega) = H(\omega)E(\omega)$
  - $E(\omega)$ is represented by an f_0, a V/UV decision for each harmonic, and the phase of each voiced harmonic
  - Parameters are estimated by comparing the original vs the synthetic speech spectrum
  - Voiced portion is synthesized in time domain while unvoiced part is synthesized in frequency domain

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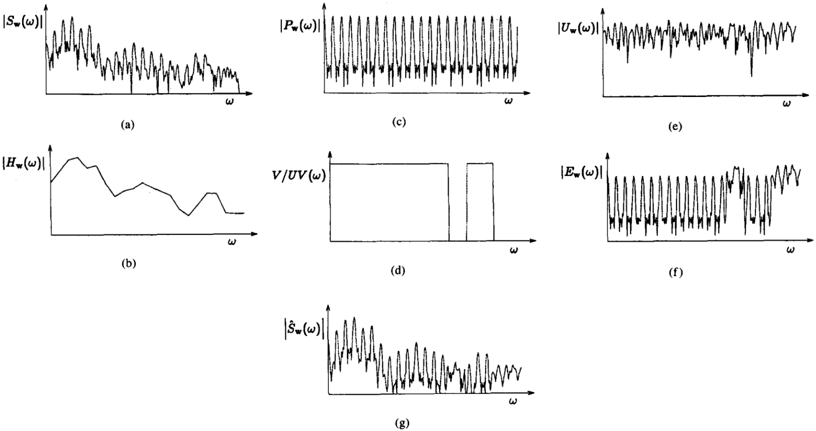

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- Sinusoids + band-pass random signals (Abrantes et al.1991 [2])
  - Completely avoids V/UV decision
  - Harmonically related sinusoids model the voiced parts
  - Random band-pass signals model the unvoiced parts
    - White noise filtered by a group of band-pass filters (filterbank) with center frequencies $k\omega_s$

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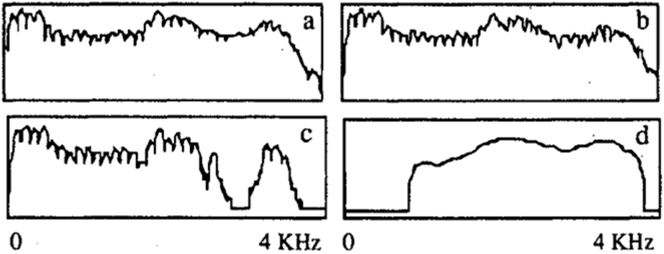

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- Periodic + Aperiodic Decomposition (Yegnayarayana et al.1995 [3])
    - The LP residual signal is used as an approximation to the excitation
    - V/UV analysis is used
    - Frequency regions of harmonic and noise components in the spectral domain
    - An iterative algorithm is proposed which reconstructs the aperiodic component in the harmonic regions
    - The periodic component is obtained by subtracting the reconstructed aperiodic component signal from the residual signal.

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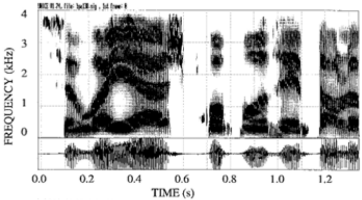

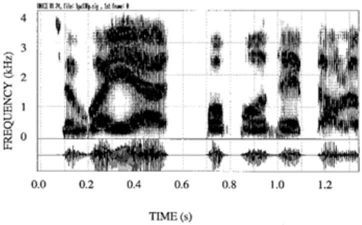

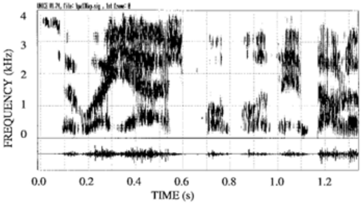

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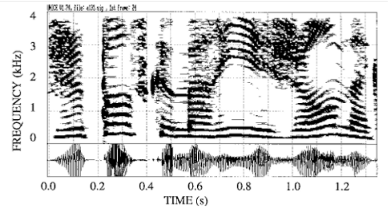

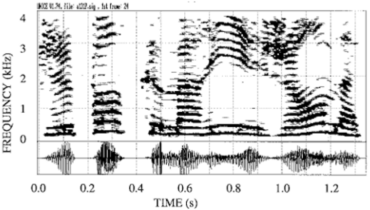

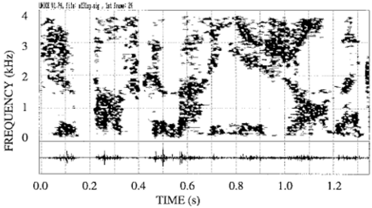

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Decomposing speech into (quasi)periodic and non-periodic part has many applications in:

- Speech modification
- Speech coding
- Pathologic voice detection (i.e., HNR ...)
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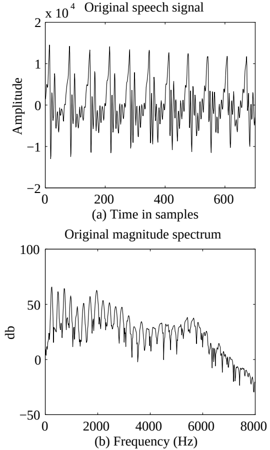
Motivation for HNM



Brief overview of HNM

- HNM (Stylianou 1995 [4]) is a pitch-synchronous harmonic plus noise representation of the speech signal.
- Speech spectrum is divided into a low and a high band delimited by the so-called *maximum voiced frequency*
- The *lower* band of the spectrum (below the maximum voiced frequency) is represented solely by harmonically related sine waves.
- The *upper* band is modeled as a noise component modulated by a time-domain amplitude envelope.
- HNM allows high-quality copy synthesis and prosodic modifications.

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HNM in equations

- Harmonic part:

$$h(t) = \sum_{k=-L(t)}^{L(t)} A_k(t) e^{j2\pi k f_0(t) t}$$

  where $A_k(t)$ and $f_0(t)$ are the instantaneous complex amplitude and real frequency, respectively

- Noise part:

$$n(t) = e(t) \left[v(\tau, t) \star g(t) \right]$$

  where $e(t), v(\tau, t), g(t)$ are a time envelope, an estimation of the PSD (filter), and white gaussian noise, respectively

- Speech:

$$s(t) = h(t) + n(t)$$

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Models for periodic part

- HNM_1: Sum of exponential functions without slope

$$h_1[n] = \sum_{k=-L(n_a^i)}^{L(n_a^i)} a_k(n_a^i) e^{j2\pi k f_0(n_a^i)(n-n_a^i)}$$

- HNM_2: Sum of exponential function with complex slope

$$h_2[n] = \Re \left\{ \sum_{k=1}^{L(n_a^i)} A_k(n) e^{j2\pi k f_0(n_a^i)(n-n_a^i)} \right\}$$

where

$$A_k(n) = a_k(n_a^i) + (n - n_a^i) b_k(n_a^i)$$

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Models for periodic part

- HNM_3: Sum of sinusoids with time-varying real amplitudes

$$h_3[n] = \sum_{k=0}^{L(n_a^i)} a_k(n) \cos(\varphi_k(n))$$

where

$$\begin{aligned} a_k(n) &= c_{k0} + c_{k1}\,(n - n_a^i)^1 + \cdots + c_{kp}\,(n - n_a^i)^{p(n)} \\ \varphi_k(n) &= \epsilon_k + 2\pi\, k\zeta\,(n - n_a^i) \end{aligned}$$

where $a_k(n)$, $\phi_k(n)$ are real functions of discrete time and $p(n)$ is the order of the amplitude polynomial, which is, in general, a time-varying parameter.

Residual (Noise) part

The non-periodic part is just the *residual* signal obtained by subtracting the periodic-part (harmonic part) from the original speech signal in the time-domain

$$r[n] = s[n] - h[n]$$

where $h[n]$ is either $h_1[n], h_2[n]$, or $h_3[n]$ (harmonic part of HNM_1, HNM_2, and HNM_3, respectively).

Outline

Initial fundamental frequency

- Get an initial estimation of fundamental frequency f_0 [5]
- Determine the voicing of the frame using normalized error over first four harmonics:

$$E = \frac{\int_{0.7f_0}^{4.3f_0} (|S(f)| - |\tilde{S}(f)|)^2}{\int_{0.7f_0}^{4.3f_0} |S(f)|^2}$$

  where $\tilde{S}(f)$ is a synthetic DFT-based spectrum using the initial f_0 estimation
- If $E < T$, where T an appropriate threshold (e.g. -15 dB), then frame is voiced, else it is labeled as unvoiced

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Maximum Voiced Frequency - MVF

- The MVF F_M is determined frame-wise from the speech spectrum
- Starting from the frequency f_c of the maximum spectral peak, A_m, in $[f_0/2, 3f_0/2]$, spectral peak values are collected around that maximum peak, along with their frequencies
- The range of collection is $R_{search} = [f_c - f_0/2,\ f_c + f_0/2]$
- Determine peak frequencies f_i in R_{search}, and the corresponding amplitudes, $A(f_i)$ and cumulative amplitudes $A_c(f_i)$
- Cumulative amplitude $A_c(f)$ is the sum of all spectral peak values from previous valley to following valley

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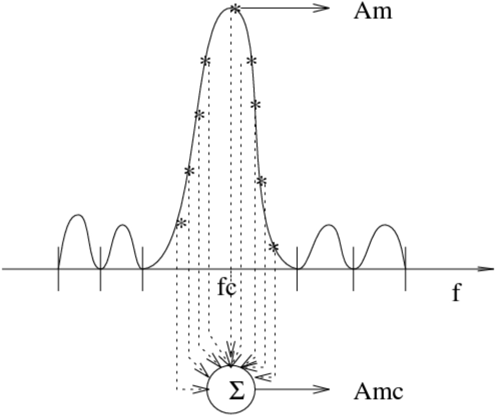

Fig. 1. Cumulative amplitude definition

Maximum Voiced Frequency - MVF

- Compute the average cumulative amplitude for all f_i: $\bar{A}_c(f_i)$
- Pass f_c through the *voicing test* (see next slide)
- Search for the maximum spectral peak in $[f_c + f_0/2,\ f_c + 3f_0/2]$, and find new f_c
- Repeat the steps until $f_c \leq f_s/2$.
- Determine voiced and unvoiced spectral areas
- Maximum voiced frequency M_F is the maximum frequency of the last voiced spectral area.

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- If

$$\frac{A_c}{\bar{A}_c(f_i)} > 2$$

or

$$|A - \max\{A(f_i)\}| > 13 \text{ dB}$$

then

  - if f_c is *really* close to the closest harmonic lf_0, then
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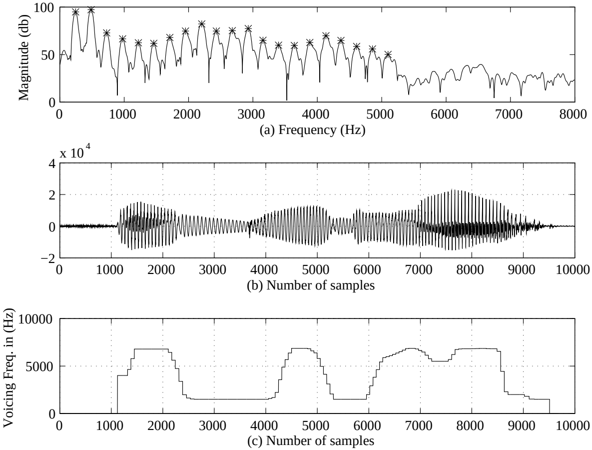
Maximum voiced frequency example

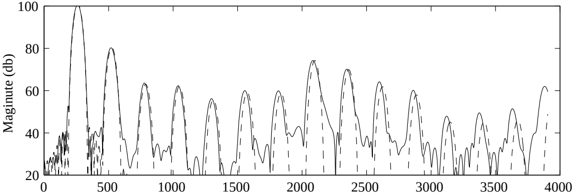


Fundamental frequency refinement

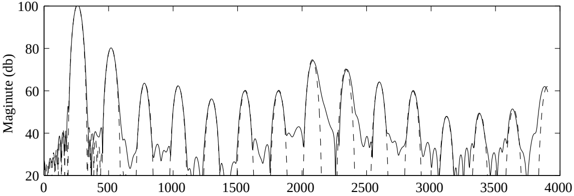
Using the initial f_0 value and the L detected voiced frequencies f_i, then the refined fundamental frequency, $\hat{f}_0$ is defined as the value that minimizes the error:

$$E(\hat{f}_0) = \sum_{i=1}^{L} |f_i - i \cdot \hat{f}_0|^2$$

Refinement frequency example



(a) Frequency in Hz



(b) Frequency in Hz

Amplitudes and phases estimation

Having f_0 estimated for voiced frames, amplitudes and phases are estimated by minimizing the criterion:

$$\epsilon = \sum_{n=n_a^i-N}^{n_a^i+N} w^2[n](s[n] - \hat{h}[n])^2$$

where $n_a^i = n_a^{i-1} + P(n_a^{i-1})$, and $P(n_a^{i-1})$ denotes the pitch period at n_a^{i-1}.

- for HNM_1 and HNM_2, this criterion has a quadratic form and is solved by inverting an over-determined system of linear equations.
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Reformulate the error function - for HNM_1

Cost function:

$$\epsilon(a_{-L}, ..., a_L, f_0) = \frac{1}{2}\sum_{n=-N}^{N}(e[n])^2 = \frac{1}{2}\mathbf{e}^h\mathbf{e}$$

where

$$e[n] = w[n](s[n] - h[n])$$

or

$$\mathbf{e} = \begin{bmatrix} e[-N], & e[-N+1], & \dots & e[N] \end{bmatrix}^T$$

Reformulate the error function - for HNM_1

$$\epsilon(\mathbf{a}) = \frac{1}{2}(\mathbf{s} - \mathbf{Ea})^h \mathbf{W}^2 (\mathbf{s} - \mathbf{Ea})$$

where

$$\mathbf{a} = \begin{bmatrix} a_{-L}, & \dots & a_0, & \dots & a_L \end{bmatrix}^T$$

and

$$\mathbf{E} = \begin{bmatrix} e^{j2\pi(-L)\hat{f}_0(-N)/f_s}, & \dots & e^{j2\pi L\hat{f}_0(-N)/f_s} \\ e^{j2\pi(-L)\hat{f}_0(-N+1)/f_s}, & \dots & e^{j2\pi L\hat{f}_0(-N+1)/f_s} \\ \vdots & \vdots & \vdots \\ e^{j2\pi(-L)\hat{f}_0 N/f_s}, & \dots & e^{j2\pi L\hat{f}_0 N/f_s} \end{bmatrix}^T_{(2L+1\times 2N+1)}$$

Least Squares - for HNM_1

- Setting:

$$\frac{\partial \epsilon(\mathbf{a})}{\partial \mathbf{a}} = 0 \Longrightarrow \mathbf{E}^h \mathbf{W}^2 \mathbf{E} \mathbf{a} - \mathbf{E}^h \mathbf{W}^2 \mathbf{s} = 0$$

- Solution:

$$\mathbf{a}_{LS} = (\mathbf{E}^h \mathbf{W}^2 \mathbf{E})^{-1} \mathbf{E}^h \mathbf{W}^2 \mathbf{s}$$

- Properties:
  - Asymptotically efficient even when the noise is colored.
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Avoiding ill-conditioning

- For HNM_1 there is no problem if window length is twice the local pitch period
- Same thing for HNM_2
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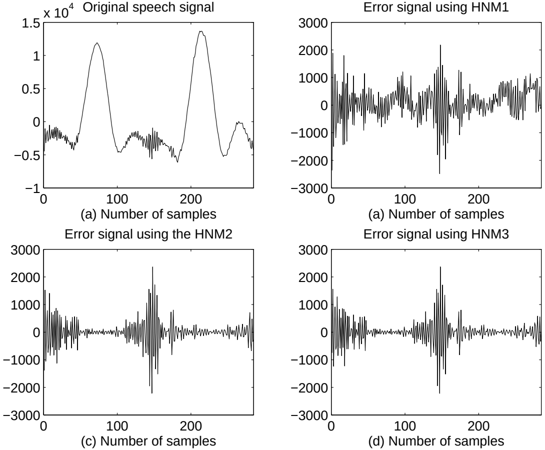
Residual Signal

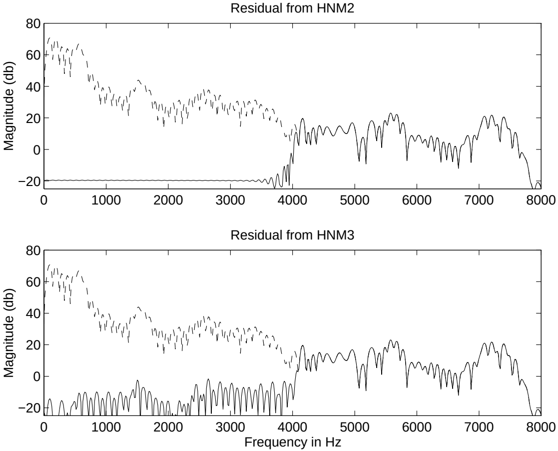
The residual signal $r[n]$ is estimated by

$$\hat{r}[n] = s[n] - \hat{h}[n]$$

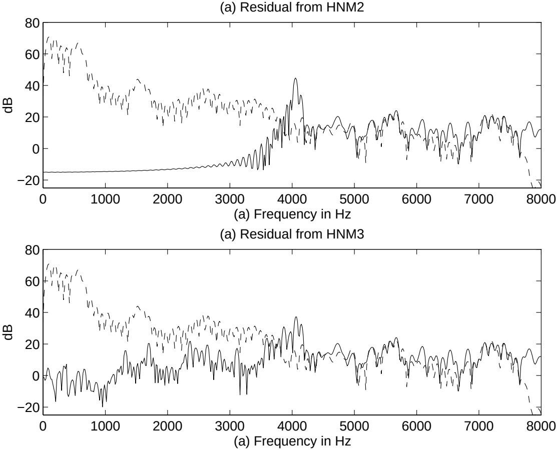
Time domain characteristics of $\hat{r}[n]$



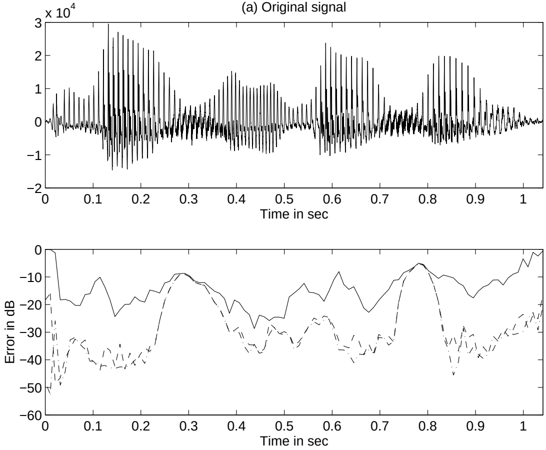
Spectral domain characteristics of $\hat{r}[n]$



… AND AFTER ADDING NOISE



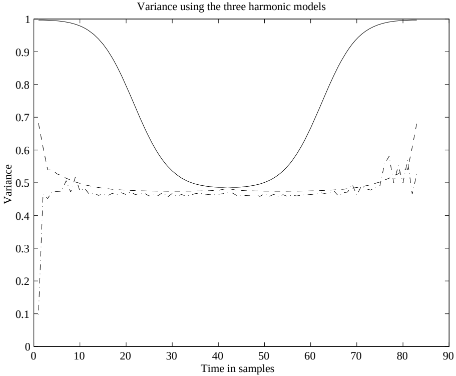
Modeling error



Variance of the residual signal

The variance of the residual signal is given as:

$$E(\mathbf{r}\mathbf{r}^h) = \mathbf{I} - \mathbf{W}\mathbf{P}(\mathbf{P}^h\mathbf{W}^h\mathbf{W}\mathbf{P})^{-1}\mathbf{P}^h\mathbf{W}^h$$



Modeling the residual signal

- Full bandwidth representation using a low-order (10th) AR filter
- Time-domain characteristics of the residual signal are modeled using deterministic functions

Outline

For all HNMs

- $n_s^i \longleftrightarrow n_a^i$
- For the periodic part: Overlap-and-Add
- For the stochastic (noise) part):
  - Instead of AR coefficients we use reflection coefficients
  - Sample-by-sample filtering of Gaussian noise using normalized lattice filtering
  - Modulation in time with a deterministic function (i.e., triangular)

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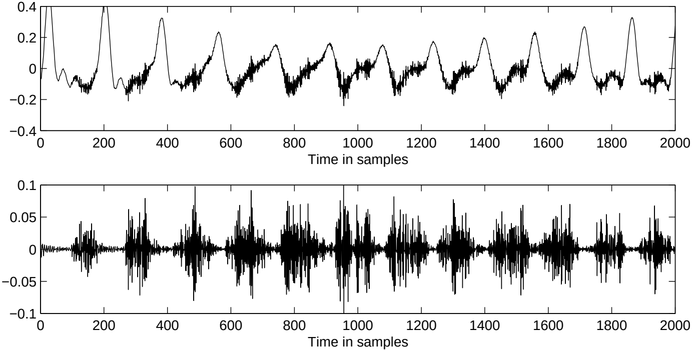
For HNM_1 specifically

for Periodic part (as an alternative to OLA)

- Direct frequency matching
- Linear amplitude interpolation
- Linear phase interpolation using average pitch value

Outline

Again on the energy modulation



So far, mainly

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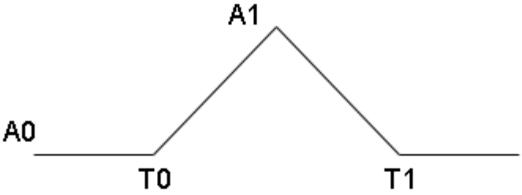

Signal envelope

There are many ways to obtain the "envelope" of a signal, as:

- Hilbert Transform (analytic signal)
- Low-pass local energy (energy envelope):

$$e[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} |r[n-k]|$$

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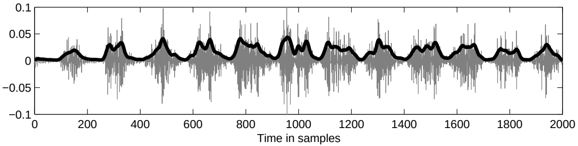
Hilbert envelope

We may also use the Hilbert envelope, computed as:

$$\tilde{e}_H[n] = \sum_{k=L-M+1}^{L} a_k e^{2\pi k (f_0/f_s) n}$$

Example of energy envelope

Example of Energy Envelope, with $N = 7$



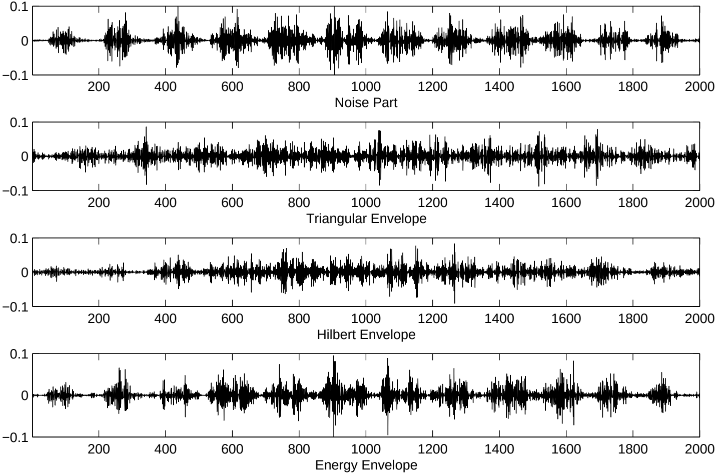
Energy envelope

The energy envelope can be efficiently parameterized with a few Fourier coefficients:

$$\hat{e}[n] = \sum_{k=-L_e}^{L_e} A_k e^{j2\pi k (f_0/f_s) n}$$

where L_e is set to be 3 to 4

Looking at time domain properties



Results from Listening test I

	Triangular	No pref.	Hilbert
Male	8 (8.3%)	43 (44.8%)	45 (46.9%)
Female	40 (41.7%)	47 (48.9%)	9 (9.4%)

	Hilbert	No pref.	Energy
Male	22 (22.9%)	47 (49.0%)	27 (28.1%)
Female	22 (22.9%)	54 (56.3%)	20 (20.8%)

	Energy	No pref.	Triangular
Male	43 (44.8%)	50 (52.0%)	3 (3.2%)
Female	16 (16.7%)	67 (69.8%)	13 (13.5%)

Table: Results from the listening test for the English sentences.

Results from Listening test II

	Triangular	No pref.	Hilbert
Male	10 (10.4%)	47 (49.0%)	39 (40.6%)
Female	8 (8.3%)	71 (74.0%)	17 (17.7%)

	Hilbert	No pref.	Energy
Male	11 (11.5%)	58 (60.4%)	27 (28.1%)
Female	13 (13.5%)	58 (60.4%)	25 (26.1%)

	Energy	No pref.	Triangular
Male	42 (43.7%)	48 (50.0%)	6 (6.3%)
Female	16 (16.7%)	68 (70.8%)	12 (12.5%)

Table: Results from the listening test for the French sentences.

Outline

Estimating Sinusoidal parameters

- Sinusoidal representation for a speech/signal frame:

$$x(t) = \left(\sum_{k=-K}^{K} a_k e^{j2\pi f_k t} \right) w(t)$$

- Methods:
  - FFT-based methods (i.e., QIFFT [Abe et al., 2004–05, [6] [7]])
  - Subspace methods
  - Least Squares (LS) method
- Frequency mismatch:

$$\hat{f}_k = f_k + \eta_k$$

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- QHM (de Prony 1795, Laroche [8] (1989), Stylianou 1993, Pantazis [9] (2008, 2011)):

$$x(t) = \left(\sum_{k=-K}^{K} (a_k + t b_k) e^{j2\pi \hat{f}_k t} \right) w(t)$$

  - a_k, b_k are complex numbers
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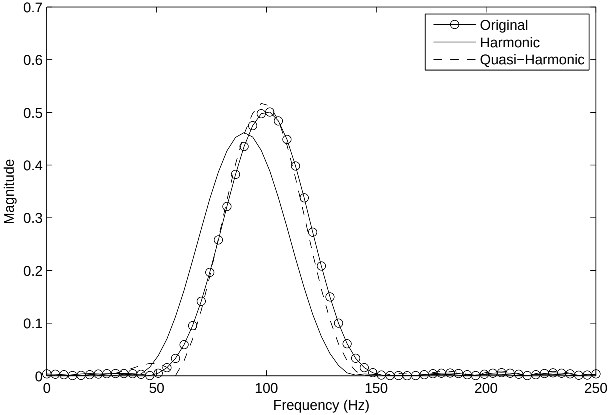
Quasi-Harmonic Model, QHM [9]

- HM versus QHM in frequency estimation - pure tone @ 100 Hz
- Given frequency for both models: 90 Hz



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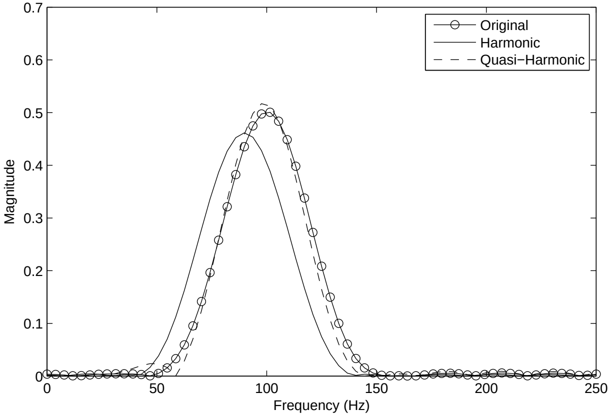

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Time domain properties:

- Inst. amplitude:
  $M_k(t) = |a_k + tb_k| = \sqrt{(a_k^R + tb_k^R)^2 + (a_k^I + tb_k^I)^2}$
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- Inst. amplitude:
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- Inst. frequency: $F_k(t) = \dfrac{1}{2\pi}\Phi'(t) = \hat{f}_k + \dfrac{1}{2\pi} \dfrac{a_k^R b_k^I - a_k^I b_k^R}{M_k^2(t)}$
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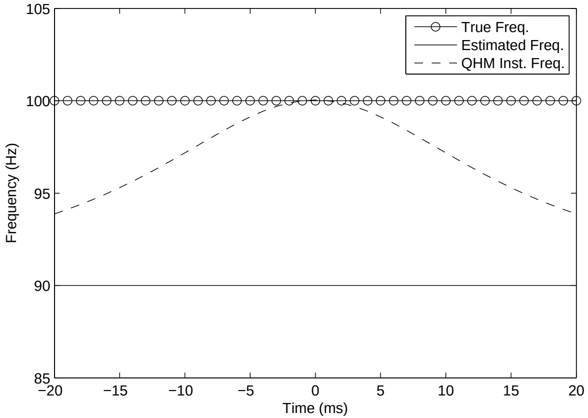
- HM vs QHM inside analysis window - pure tone @ 100 Hz:
- Given frequency for both models: 90 Hz



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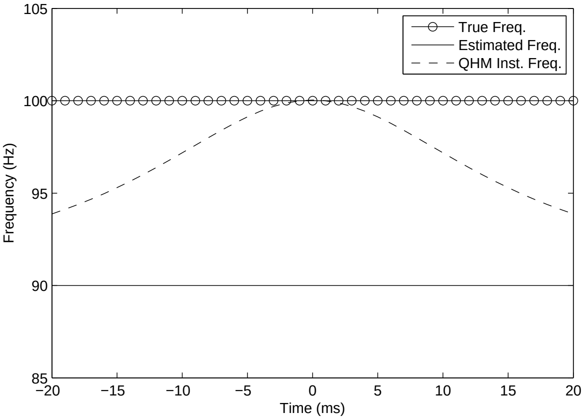

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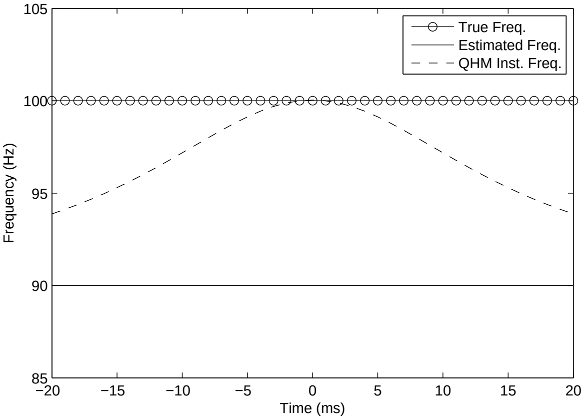

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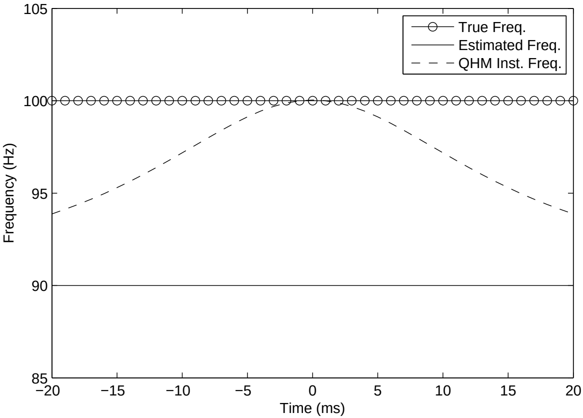

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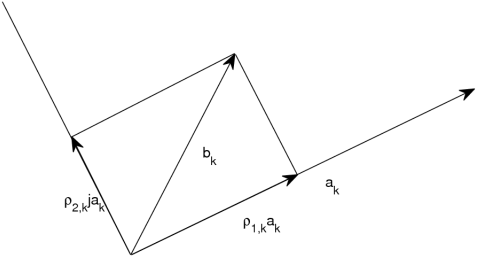
A few details of QHM

- QHM in the frequency domain:

$$X_k(f) = a_k W(f - \hat{f}_k) + j\frac{b_k}{2\pi} W'(f - \hat{f}_k)$$

- **Decomposition of** b_k: $b_k = \rho_{1,k} a_k + \rho_{2,k} j a_k$



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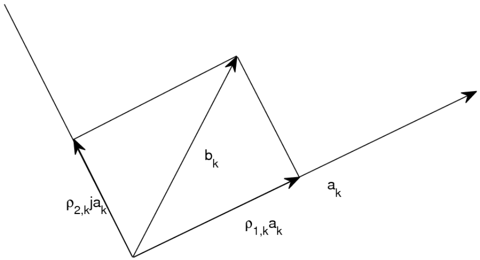

A few details of QHM

- Then replacing b_k with the decomposition

$$X_k(f) = a_k \left[W(f - \hat{f}_k) - \frac{\rho_{2,k}}{2\pi} W'(f - \hat{f}_k) + j \frac{\rho_{1,k}}{2\pi} W'(f - \hat{f}_k) \right]$$

- and taking into account the Taylor series expansion of $W(f - \hat{f}_k - \frac{\rho_{2,k}}{2\pi})$:

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- If the value of term $W''(f)$ at f_k is small, then for small values of $\rho_{2,k}$, then an **approximation of** the *kth* component of QHM is:

$$X_k(f) \approx a_k \left[W(f - \hat{f}_k - \frac{\rho_{2,k}}{2\pi}) + j \frac{\rho_{1,k}}{2\pi} W'(f - \hat{f}_k) \right]$$

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- Initially, we assumed:

$$x_k(t) = a_k \left[e^{j(2\pi(\hat{f}_k + \eta_k))t} \right] w(t)$$

- in other words, **it is suggested**:

$$\hat{\eta}_k = \rho_{2,k}/2\pi = \frac{1}{2\pi} \frac{a_k^R b_k^I - a_k^I b_k^R}{|a_k|^2}$$

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- In other words, QHM suggests a frequency correction to the input frequencies $\hat{f}_k$ (or a frequency estimator). This suggestion is however conditional on the magnitude of $\rho_{2,k}$ and the value of term $W''(f)$ at f_k
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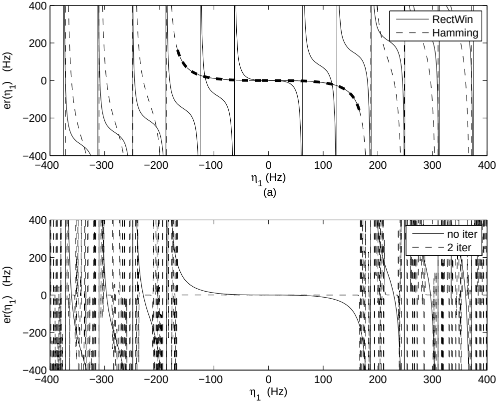
Single Sinusoid



(a)

(b)

• Iteratively, the bias can be removed when $|\eta| < B/3$, where B is the bandwidth of the squared analysis window.

Outline

ITERATIVE QHM, iQHM [10]

- This frequency updating mechanism provides frequencies which can be used in the model iteratively and result in better parameter estimation (a_k, b_k)
- This iterative parameter estimation is referred to as the *iterative QHM*

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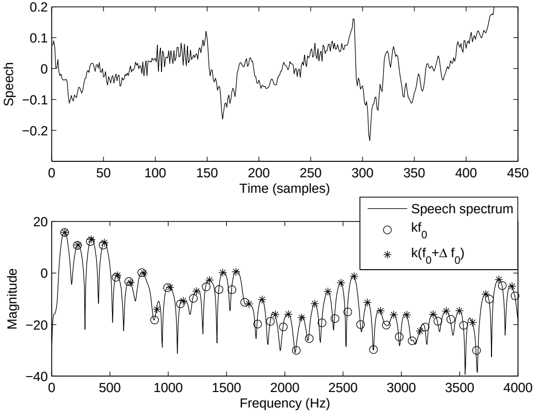
ITERATIVE QHM, iQHM [10]

HM versus iQHM in frequency estimation - speech signal:



Robustness against additive noise

- Signal contaminated by noise:

$$y(t) = \sum_{k=1}^{4} a_k e^{j2\pi f_k} + v(t)$$

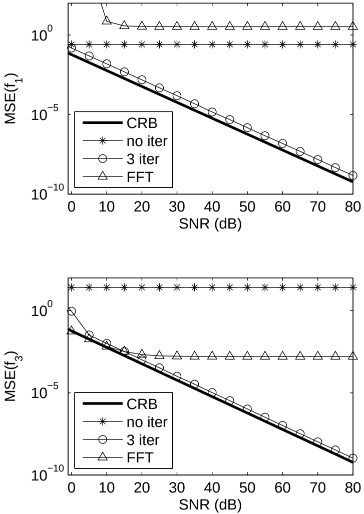
- Mean Squared Error (MSE):

$$MSE\{\hat{f}_k\} = \frac{1}{\mathrm{M}} \sum_{i=1}^{\mathrm{M}} |\hat{f}_k(i) - f_k|^2$$

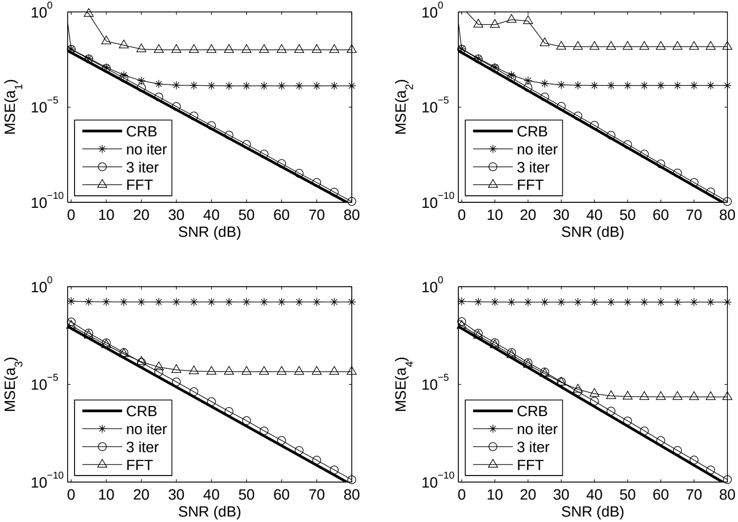
$$MSE\{\hat{a}_k\} = \frac{1}{\mathrm{M}} \sum_{i=1}^{\mathrm{M}} |\hat{a}_k(i) - a_k|^2$$

- Comparison with Cramer-Rao Bounds (CRB) and QIFFT (Abe et al. 2004)
- 10000 Monte Carlo simulations

MSE of frequencies as a function of SNR.



MSE of amplitudes as a function of SNR.



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References I

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