CS578- Speech Signal Processing

Lecture 5: Sinusoidal modeling and modifications

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Outline

Source-Filter[1]

- Source:

$$u(t) = Re \sum_{k=1}^{K(t)} \alpha_k(t) \exp\left[j\phi_k(t)\right]$$

where:

$$\phi_k(t) = \int_0^t \Omega_k(\sigma) d\sigma + \phi_k$$

- Filter: $h(t, \tau)$ with Fourier Transform (FT):

$$H(t, \Omega) = M(t, \Omega) \exp\left[j\Phi(t, \Omega)\right]$$

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Output speech

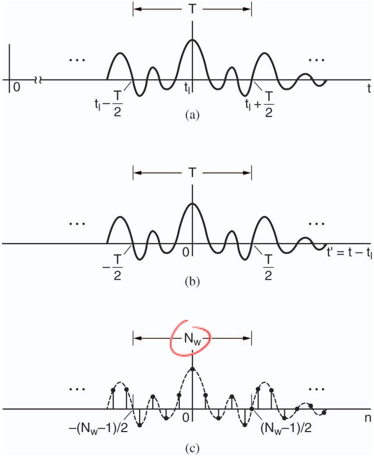
$$s(t) = Re \sum_{k=1}^{K(t)} A_k(t) \exp\left[j\theta_k(t)\right]$$

where:

$$\begin{aligned} A_k(t) &= \alpha_k(t) M\left[t, \Omega_k(t)\right] \\ \theta_k(t) &= \phi_k(t) + \Phi\left[t, \Omega_k(t)\right] \\ &= \int_0^t \Omega_k(\sigma) d\sigma + \Phi\left[t, \Omega_k(t)\right] + \phi_k \end{aligned}$$

Outline

Frame-by-Frame analysis



Stationarity Assumption

We assume stationarity inside the analysis window:

$$\begin{aligned} A_k^l(t) &= A_k^l \\ \Omega_k^l(t) &= \Omega_k^l \end{aligned}$$

which leads to:

$$\theta_k^l(t) = \Omega_k^l(t - t_l) + \theta_k^l$$

and to:

$$s(t) = \sum_{k=1}^{K^l} A_k^l \exp\left(j\theta_k^l\right) \exp\left[j\Omega_k^l(t - t_l)\right] \quad t_l - \frac{T}{2} \leq t \leq t_l + \frac{T}{2}$$

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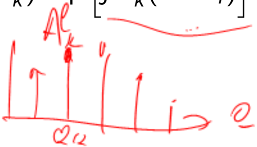

Discrete-Time Formulation

$l \rightarrow l^{th}$ frame

Steps to discrete time formula:

- Time shift: $t' = t - t_l$
- Convert to discrete time:

$$s[n] = \sum_{k=1}^{K^l} A_k^l \exp\left(j\theta_k^l\right) \exp\left(j\omega_k^l n\right) \quad -\frac{N_w - 1}{2} \leq n \leq \frac{N_w - 1}{2}$$

$y[n]$

Mean-Squared Error

Given the original measured waveform, $y[n]$ and the synthetic speech waveform, $s[n]$, estimate the unknown parameters A_k^l, ω_k^l, and θ_k^l by minimizing the MSE criterion:

$$\epsilon^l = \sum_{n=-(N_w-1)/2}^{n=(N_w-1)/2} |y[n] - s[n]|^2$$

which can be written as:

$$\epsilon^l = \sum_{n=-(N_w-1)/2}^{n=(N_w-1)/2} |y[n]|^2 + N_w \sum_{k=1}^{K^l} \left(\left| Y(\omega_k^l) - \gamma_k^l \right|^2 - |Y(\omega_k^l)|^2 \right)$$

which can be reduced further to:

$$\epsilon^l = \sum_{n=-(N_w-1)/2}^{n=(N_w-1)/2} |y[n]|^2 - N_w \sum_{k=1}^{K^l} |Y(\omega_k^l)|^2$$

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Karhunen-Loève Expansion

- Karhunen-Loève expansion allows constructing a random process from harmonic sinusoids with uncorrelated complex amplitudes.
- Estimated power spectrum should not vary "too much" over consecutive frequencies.

Following the above necessary constraints, for unvoiced speech, and for a window width to be *at least* 20ms, an 100 Hz harmonic structure provides good results.

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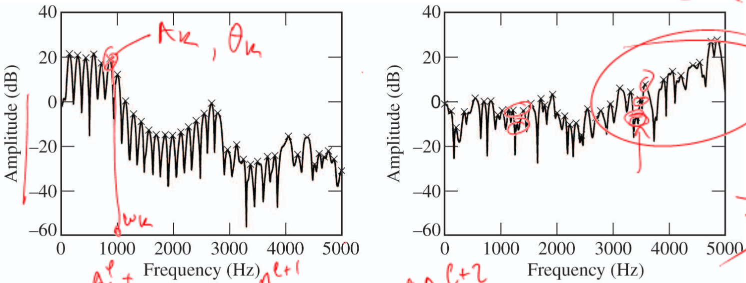
Example



40
20
0
−20
−40
−60
Amplitude (dB)
0
1000
2000
3000
4000
5000
Frequency (Hz)

(a) (b)

Implementation

- Window width be 2.5 times the average pitch period or 20 ms, whichever is larger.
- Use Hamming window, normalized to one:

$$\sum_{n=-\infty}^{\infty} w[n] = 1$$

- Use zero padding to get enough samples of the underlying spectrum (i.e., 1024-point FFT)
- Remove linear phase offset
- Refine your frequency estimates

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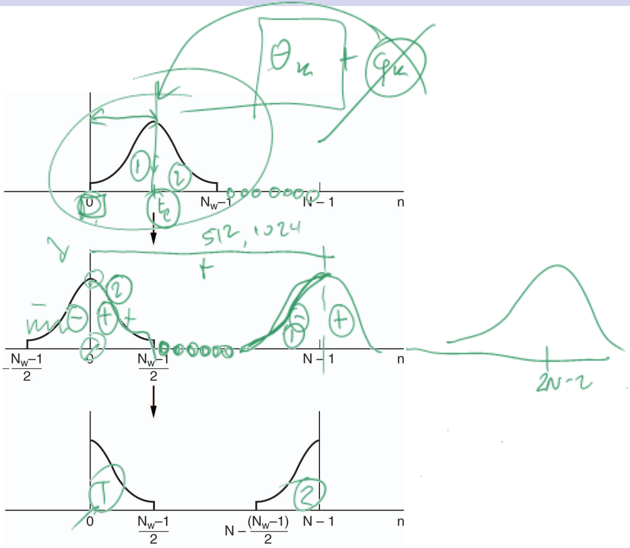
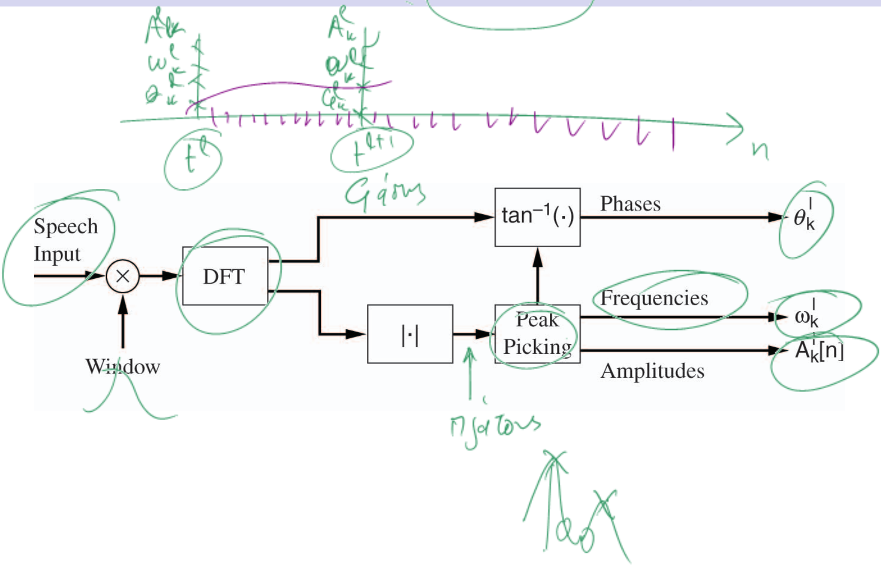
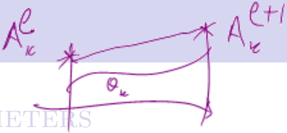
Showing the process …



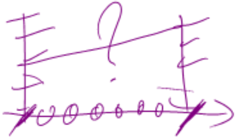
Block diagram of the Analysis System



Outline



1. Sinusoidal Speech Model
2. Estimation of Sinewave Parameters
   - Voiced Speech
   - Unvoiced Speech
   - The Analysis System
3. Synthesis
   - Linear Amplitude Interpolation
   - Cubic Phase Interpolation
4. Examples
5. Sound Examples
6. Shape Invariant Time-Scale Modifications
   - The Model
   - Parameters Estimation
   - Synthesis
   - Sound Examples
7. Shape Invariant Pitch Modifications
8. Acknowledgments
9. References



$$\frac{d\theta(t)}{dt} = \omega(t)$$

$\theta(t) = at + \beta$

$\omega(t) = a$

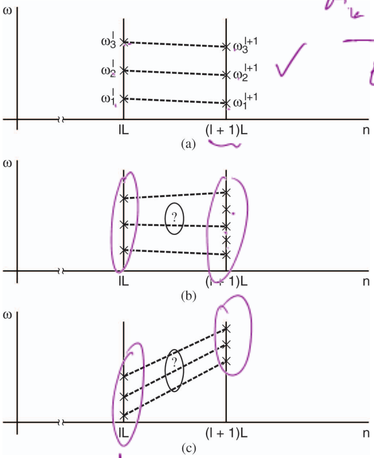
$\theta(t) = at^2 + \beta t + \gamma$

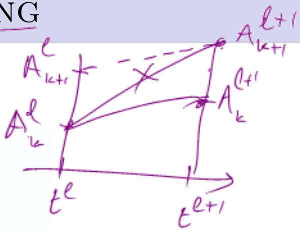
$\omega(t) = 2at + \beta$

$\Rightarrow \theta(t) = at^3 + \beta t^2 + \gamma t + \delta$

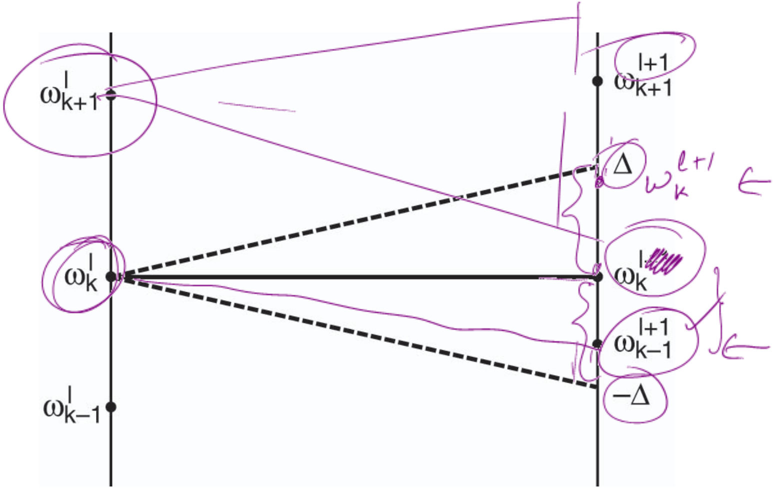
$\omega(t) = 3at^2 + 2\beta t + \gamma$

Problem of frequency matching

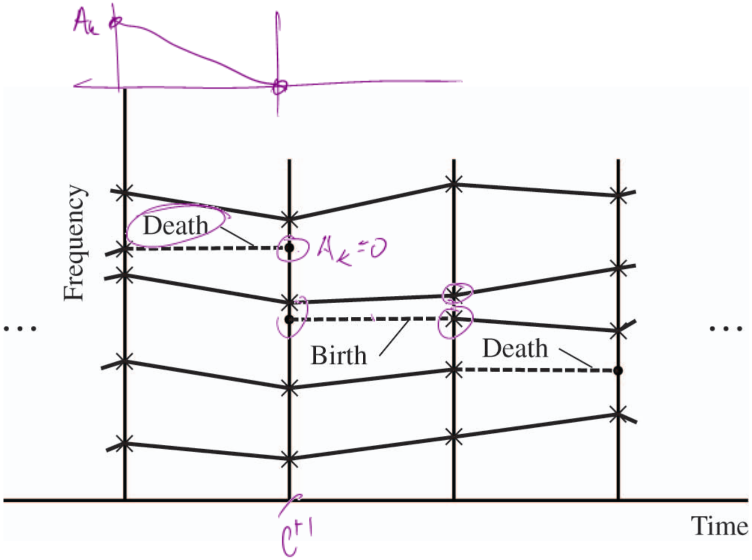




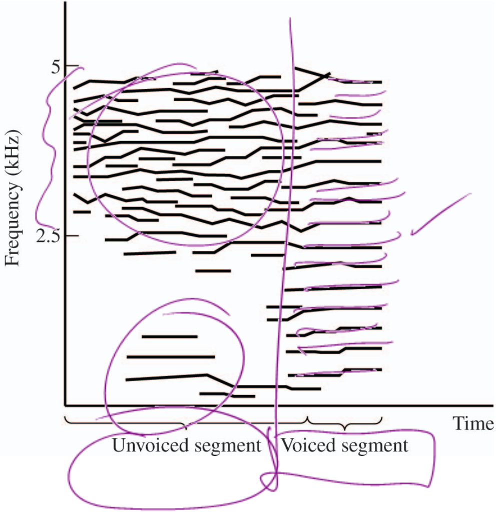
Frame-to-Frame Peak Matching



The birth/death process



A birth/death process in speech

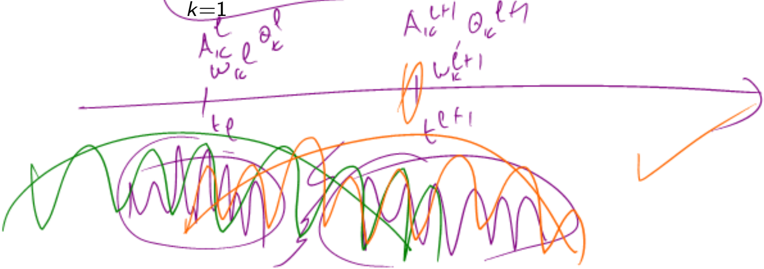


Why not ...

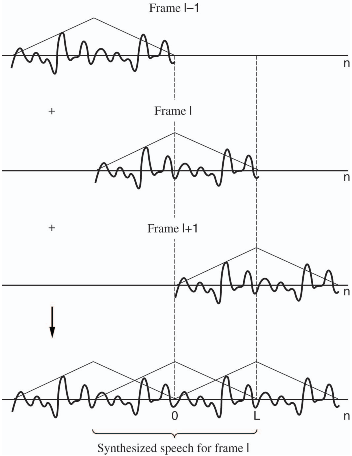
OLA

Why not to estimate the original speech waveform on the lth frame, directly as:

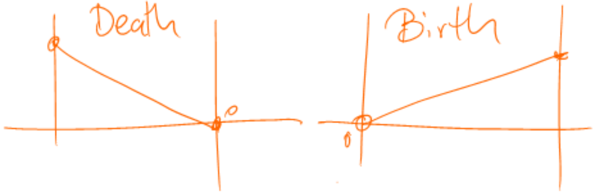
$$s[n] = \sum_{k=1}^{K^l} A_k^l \cos\left(n\omega_k^l + \theta_k^l\right), \quad n = 0, 1, 2, \cdots, L-1$$



A simple solution: OLA



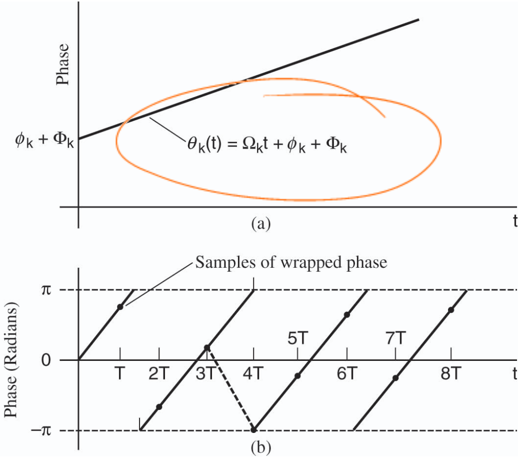
Amplitude Interpolation



Linear Interpolation:

$$A_k^l[n] = A_k^l + \left(A_k^{l+1} - A_k^l\right)\left(\frac{n}{L}\right) \quad n = 0, 1, 2, \cdots, L-1$$

Phase wrapped



Cubic Phase model

$$\theta(t) = \zeta + \gamma t + \alpha t^2 + \beta t^3$$

About the phase derivative

Assuming that vocal tract is slowly varying, and since:

$$\theta(t) = \int_0^t \Omega(\sigma) d\sigma + \phi + \Phi[t, \Omega(t)]$$

$$\dot{\theta}(t) \approx \Omega(t)$$

So:

$$\begin{aligned} \dot{\theta}^l &\approx \Omega^l \\ \dot{\theta}^{l+1} &\approx \Omega^{l+1} \end{aligned}$$

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Four constraints for phase polynomial

There are four constraints

$$
\begin{aligned}
\theta(0) &= \theta^l \\
\dot{\theta}(0) &= \Omega^l \\
\theta(T) &= \theta^{l+1} + 2\pi M \\
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\end{aligned}
$$

and ... five unknowns (don't forget M)

We need one more constraint!

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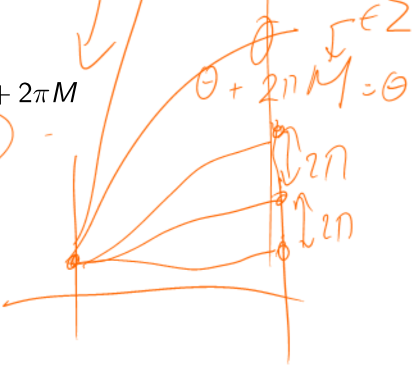

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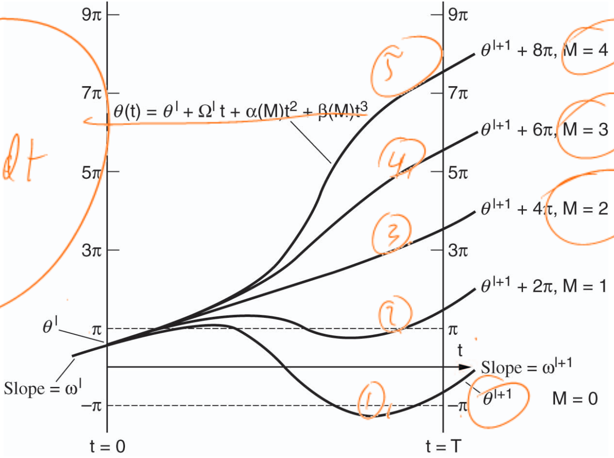
How to choose M



ESTIMATING M

- Find M that minimizes the criterion:

$$f(M) = \int_0^T \left[\ddot{\theta}(t; M)\right]^2 dt$$

- Using continuous variable:

$$x^* = \frac{1}{2\pi}\left[(\theta^l + \Omega^l T - \theta^{l+1}) + (\Omega^{l+1} - \Omega l)\frac{T}{2}\right]$$

- M^* is the nearest integer to x^*

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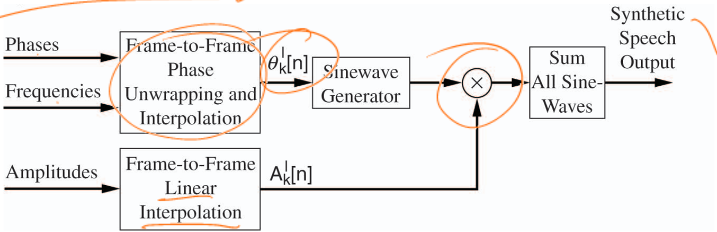
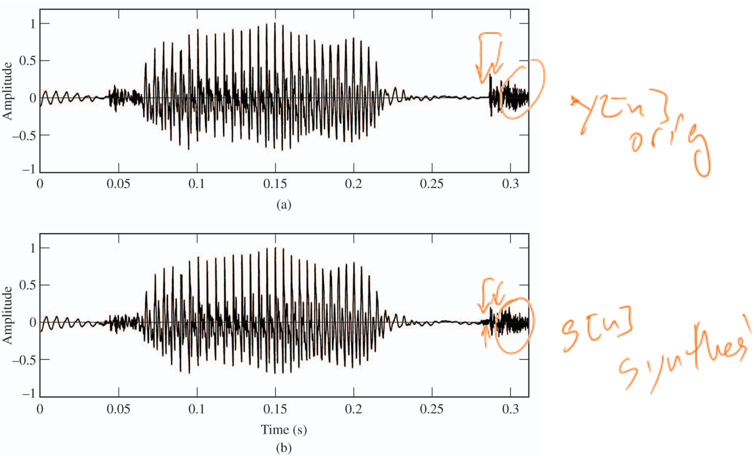
Block diagram of the Synthesis System

Analysis ω_k^l, A_k^l, θ_k^l → peak matching



Outline

Reconstruction Example



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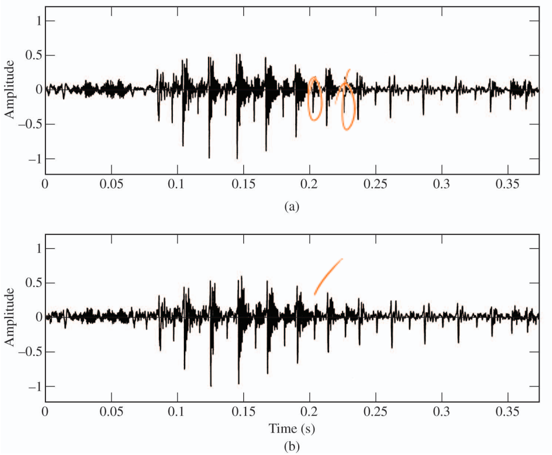
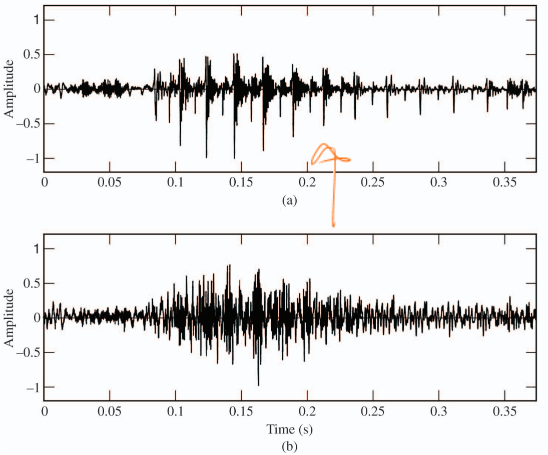


y[n]

s[n]

Magnitude-only Reconstruction Example



Outline

Sound Examples

	Original	Mixed	Min	Zero
Male				
Female				
Male				
Female				

Outline

Excitation model

We have seen that:

$$u(t) = \sum_{k=1}^{K(t)} \alpha_k(t) \exp\left[j\phi_k(t)\right]$$

where:

$$\phi_k(t) = \int_0^t \Omega_k(\sigma) d\sigma + \phi_k$$

Assuming voiced speech and constant frequency in the analysis window, then:

$$u(t) = \sum_{k=1}^{K(t)} \alpha_k(t) \exp\left[j(t - t_0)\Omega_k\right] \quad t \in [0, T]$$

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Speech model[2]

Then:

$$s[n] = \sum_{k=1}^{K(t)} A_k(t) \cos\left[\theta_k(t)\right]$$

where:

$$\begin{aligned} A_k(t) &= \alpha_k(t) M_k(t) \\ \theta_k(t) &= \phi_k(t) + \Phi_k(t) \end{aligned}$$

Therefore:

$$\Phi_k(t) = \theta_k(t) - (t - t_0)\Omega_k$$

Uniform time-scale, by ρ

Let's t represent the original articulation rate and t' the transformed rate:

$$t' = \rho \ t$$

Given the source/filter model:

- System parameters are time-scaled
- Excitation parameters (phase) are scaled in such a way to maintain fundamental frequency.

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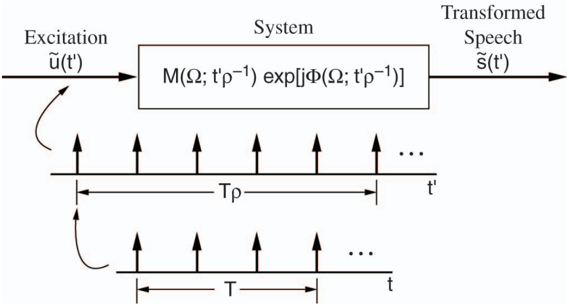
Onset-time model for time-scale



Excitation function in t'

- Time-scaled pitch period:

$$\tilde{P}(t') = P(t'\rho^{-1})$$

- Modified excitation function

$$\tilde{u}(t') = \sum_{k=1}^{K(t)} \tilde{\alpha}_k(t') \exp\left[j\tilde{\phi}_k(t')\right]$$

where:

$$\tilde{\phi}_k(t') = (t^{'}\rho^{-1} - t_0^{'})\Omega_k$$

and

$$\tilde{\alpha}_k(t') = \alpha_k(t'\rho^{-1})$$

System function parameters in t'

$$\begin{aligned}\tilde{M}_k(t') &= M_k(t'\rho^{-1}) \\ \tilde{\Phi}_k(t') &= \Phi_k(t'\rho^{-1})\end{aligned}$$

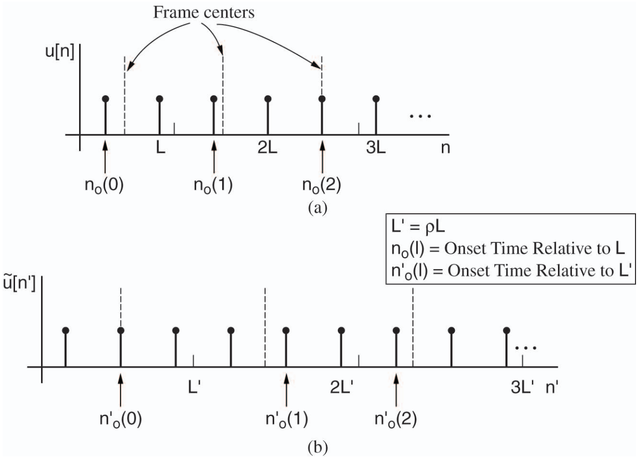
Waveform in t'

$$\tilde{s}(t') = \sum_{k=1}^{K(t)} \tilde{A}_k(t') \exp\left[j\tilde{\theta}_k(t')\right]$$

where

$$\begin{aligned}
\tilde{A}_k(t') &= \tilde{\alpha}_k(t')\tilde{M}_k(t') \\
\tilde{\theta}_k(t') &= \tilde{\phi}_k(t') + \tilde{\Phi}_k(t')
\end{aligned}$$

Onset times estimation



Estimating System Phase

Let's assume that the onset time $n_o(l)$ for the l^{th} frame is known, then:

$$\phi_k^l = \hat{n}_o(l)\omega_k^l$$

where $\hat{n}_o(l) = n_o(l) - lL$.

Then, the system phase is estimated as:

$$\tilde{\Phi}_k^l = \theta_k^l - \phi_k^l$$

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Estimating Excitation Phase

Let's assume we know the onset time in the previous frame $l - 1$, then the current onset time in t', is given by:

$$n_o^{'}(l) = n_o^{'}(l-1) + J^{'}P^{l}$$

and then:

$$\tilde{\phi}_k^{l} = (n_o^{'}(l) - lL^{'})\omega_k^{l}$$

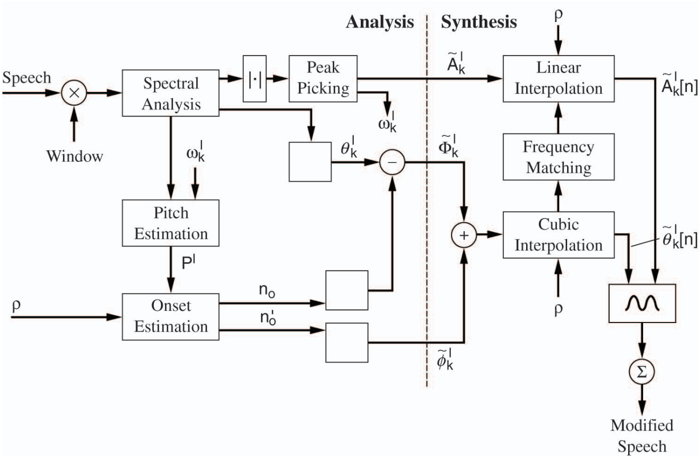
where $L^{'} = \rho L$

Synthesis

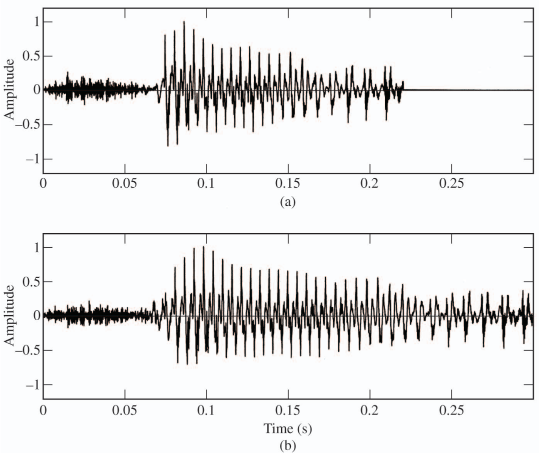
Synthesis is performed in the same way as if no modification is applied:

- Linear interpolation for amplitudes
- Cubic interpolation for phases

Block diagram for Analysis/Synthesis for Time-Scale modification



Example of Time-scale modification



Sound Examples

	0.5	0.8	Orig	1.2	1.5
Male					
Female					

	0.75	Orig	1.25
Trumpet			

Outline

Reading paper

Paper:

T. F. Quatieri and R. J. McAulay:
Shape Invariant Time-Scale and Pitch Modification of Speech
IEEE Trans. Acoust., Speech, Signal Processing, Vol.40, No.3,
pp 497-510, March 1992

Outline

Acknowledgments

Most, if not all, figures in this lecture are coming from the book:

T. F. Quatieri: Discrete-Time Speech Signal Processing, principles and practice
2002, Prentice Hall

and have been used after permission from Prentice Hall

Outline

R. J. McAulay and T. F. Quatieri, “Speech analysis/synthesis based on a sinusoidal representation,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 744–754, Aug 1986.

T. F. Quatieri and R. J. McAulay, “Shape Invariant Time-Scale and Pitch Modification of Speech,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-40, pp. 497–510, March 1992.