Lecture 18: Alias analysis

Unification

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Type Systems and Programming Languages

Based on slides by Jeff Foster

Introduction

- *Aliasing* occurs when different names refer to the same thing
  - Typically, we only care for imperative programs
  - The usual culprit: pointers
- A core building block for other analyses
  - For example in `*p = 3;` what does `p` point to?
- Useful for many languages
  - C – lots of pointers all over the place
  - Java – "objects" point to updatable memory
  - ML – ML has updatable references

Alias analysis

- *Alias analysis* answers the question
  Do pointers p and q alias the same address?
- Unfortunately, undecidable
  - Remember Rice's theorem: *No program can precisely decide anything interesting about arbitrary source code*
- Usual solution: allow imprecision
  - Decision problem: yes/no – undecidable
  - Approximation: yes/no/maybe – decidable

May alias analysis

- `p` and `q` *may alias* if it is possible that `p` and `q` might point to the same address
- Negative answer is precise
  - "yes" – imprecise, means `p` and `q` might alias
  - "no" – precise, means `p` and `q` never alias
- If `p` may *not* alias `q`, then a write through `p` does not affect memory pointed to by `q`
  - `*p = 3; x = *q;` means write through `p` does not affect `x`
- What is the most conservative may-alias analysis?

Must alias analysis

- `p` and `q` *must alias* if they do point to the same address
- Positive answer is precise
  - "yes" – precise, means `p` and `q` definitely alias
  - "no" – imprecise, means `p` and `q` might not alias
- If `p` must alias `q`, then a write through `p` always affects memory pointed to by `q`
  - `*p = 3; x = *q;` means `x` is `3`
- What is the most conservative must-alias analysis?

Early alias analysis

- By Landi and Ryder
- Expressed as computing alias pairs
  - E.g., `(*p, *q)` means `p` and `q` may point to the same memory
- Issues?
  - There could be many alias pairs
    - `(*p, *q)`, `(p->a, q->a)`, `(p->b, q->b)`, ...
  - What about cyclic data structures?
    - `(*p, p->next)`, `(*p, p->next->next)`, ...

Points-to analysis

- Determine the set of locations that `p` may point to
  - E.g., `(p, {&x})` means `p` may point to the location of `x`
  - To decide if `p` and `q` alias, see if their points-to sets overlap
- More compact representation
  - The same aliasing information takes less memory
  - Analysis scales better
- We must name all locations in the program
  - Pick a finite set of location names
    - ★ No problem with cyclic data structures
  - `x = malloc(...);` – where does `x` point to?
    - ★ `(x, {malloc@42})` – "the `malloc()` at line 42"

Flow-sensitivity

- An analysis is *flow-sensitive* if it computes the answer *at every program point*
  - We saw that dataflow analysis is flow-sensitive
- An analysis is *flow-insensitive* if it does not depend on the order of statements
  - We saw that type systems are flow-insensitive
- Flow-sensitive alias/points-to analysis is much more precise
- ...but also much more expensive
- Flow-insensitive alias analysis is much faster

Example

- Assume the program

```
p = &x;
p = &y;
*p = &z;
```

- Flow-sensitive analysis – solution per program point

`p = &x;`	// $(p, \{\&x\})$
`p = &y;`	// $(p, \{\&y\})$
`*p = &z;`	// $(p, \{\&y\}), (y, \{\&z\})$

- Flow-insensitive analysis – one solution

$(p, \{\&x, \&y\})$
$(x, \{\&z\})$
$(y, \{\&z\})$

A simple calculus

$$
\begin{array}{rcll}
T & ::= & T \to T \mid \mathit{Nat} \mid \mathit{Bool} \mid \mathit{Unit} \mid \mathit{Ref}\ T & \\
e & ::= & x & \text{variables} \\
 & \mid & n & \text{integers} \\
 & \mid & \text{true} \mid \text{false} & \text{booleans} \\
 & \mid & () & \text{unit} \\
 & \mid & e; e & \text{sequence} \\
 & \mid & \lambda x : T.e & \text{functions} \\
 & \mid & e\ e & \text{application} \\
 & \mid & \text{let } x = e \text{ in } e & \text{binding} \\
 & \mid & \text{if } e \text{ then } e \text{ else } e & \text{conditional} \\
 & \mid & \text{ref } e & \text{allocation} \\
 & \mid & !e & \text{dereference} \\
 & \mid & e := e & \text{assignment}
\end{array}
$$

Type system

$$[\textsc{T-Var}]\frac{x : T \in \Gamma}{\Gamma \vdash x : T} \qquad [\textsc{T-Nat}]\frac{}{\Gamma \vdash n : Nat}$$

$$[\textsc{T-True}]\frac{}{\Gamma \vdash \mathsf{true} : Bool} \qquad [\textsc{T-False}]\frac{}{\Gamma \vdash \mathsf{false} : Bool}$$

$$[\textsc{T-Unit}]\frac{}{\Gamma \vdash () : Unit} \qquad [\textsc{T-Seq}]\frac{\Gamma \vdash e_1 : Unit \quad \Gamma \vdash e_2 : T}{\Gamma \vdash (e_1; e_2) : T}$$

$$[\textsc{T-Lam}]\frac{\Gamma, x : T \vdash e : T'}{\Gamma \vdash \lambda x : T.e : T \to T'} \qquad [\textsc{T-App}]\frac{\Gamma \vdash e_1 : T \to T' \quad \Gamma \vdash e_2 : T}{\Gamma \vdash (e_1\ e_2) : T'}$$

Type system (cont'd)

$$[\textsc{T-Let}]\ \frac{\Gamma \vdash e_1 : T_1 \qquad \Gamma, x : T_1 \vdash e_2 : T_2}{\Gamma \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 : T_2}$$

$$[\textsc{T-If}]\ \frac{\Gamma \vdash e : Bool \qquad \Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 : T}$$

$$[\textsc{T-Ref}]\ \frac{\Gamma \vdash e : T}{\Gamma \vdash \mathsf{ref}\ e : Ref\ T} \qquad [\textsc{T-Deref}]\ \frac{\Gamma \vdash e : Ref\ T}{\Gamma \vdash !e : T}$$

$$[\textsc{T-Assign}]\ \frac{\Gamma \vdash e_1 : Ref\ T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 := e_2 : Unit}$$

Label flow analysis

- A way to compute points-to information
- We extend references with labels
  - $e ::= \ldots \mid \text{ref}^r\ e \mid \ldots$
  - A label r identifies this particular allocation instruction
    - ★ Like *malloc*@42 identifies a point in the program
    - ★ Drawn from a finite set of labels
  - For now, the programmers add these labels
- Goal of points-to analysis: find the set of labels a pointer may refer to
  - For example:

$$
\begin{array}{l}
\textbf{let } x = \text{ref}^{R_x}\ 0 \textbf{ in} \\
\textbf{let } y = x \textbf{ in} \\
\quad y := 3 \quad (*\ y \textit{ may point to } \{R_x\}\ *)
\end{array}
$$

Type-based alias analysis

- We will build an alias analysis using the type system
  - Similar to OCaml's *type inference*
- We use *labeled types* in the analysis
  - Extend reference types with labels: $T ::= \ldots \mid Ref^r\ T \mid \ldots$
  - To find the location at a pointer dereference $!e$ or assignment $e := \ldots$
    - Find the type T of e (which must be a reference)
    - We look at the reference type to decide which location might be accessed

Type system (with labels)

$$[\text{T-REF}]\ \frac{\Gamma \vdash e : T}{\Gamma \vdash \mathsf{ref}^r\ e : Ref^r\ T}$$

$$[\text{T-DEREF}]\ \frac{\Gamma \vdash e : Ref^r\ T}{\Gamma \vdash !e : T}$$

$$[\text{T-ASSIGN}]\ \frac{\Gamma \vdash e_1 : Ref^r\ T \quad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 := e_2 : Unit}$$

Example

- In the previous program

```
let x = ref^{R_x} 0 in
let y = x in
  y := 3
```

- x has type $Ref^{R_x}\ Nat$
- y has the same type as x
- Therefore, at the assignment expression, we know which location y points to

Another example

- Consider the program

  $\textbf{let}\ x = \text{ref}^{R}\ 1\ \textbf{in}$
  $\textbf{let}\ y = \text{ref}^{R}\ 2\ \textbf{in}$
  $\textbf{let}\ w = \text{ref}^{R_w}\ 0\ \textbf{in}$
  $\textbf{let}\ z = \textbf{if}\ \text{true}\ \textbf{then}\ x\ \textbf{else}\ y\ \textbf{in}$
  $\quad z := 3$

- Here, x and y both have type $Ref^{R}\ Nat$
  - They must have the same type because of the if
- At assignment, we write to location R
  - We do not know which location this is exactly, x or y
  - But we know it cannot affect w

And another example

- Another program

$$\textbf{let } x = \text{ref}^{R}\ 0 \textbf{ in}$$
$$\textbf{let } y = \text{ref}^{R_y}\ x \textbf{ in}$$
$$\textbf{let } z = \text{ref}^{R}\ 2 \textbf{ in}$$
$$y := z$$

  - Both x and z have the same label
    - They must have the same type because of the pointed type of y
  - We do not know whether y points to x or y

Things to notice

- We have a finite set of labels
  - At most one label for each occurence of a ref in the program
  - A label may represent more than one run-time locations
- Whenever two labels "meet" in the type system, they must be the same
  - Can you see where this happens in the type-rules?
- The system is flow-insensitive
  - Types don't change after assignment

Type inference

- In practice, the programmer does not write the labels
  - We need to infer them
- Given an unlabeled program that satisfies the standard type system, is there a labeling that satisfies the labeled type system?
  - That labeling is the analysis result

Checking vs. inference

- Type checking
  - The programmer annotates the program with types
  - Typing checks that the annotations are correct
  - It is "obvious" how to check
- Type inference
  - The programmer does not annotate the program
  - Typing tries to discover correct types
  - It is not "obvious", requires more work to check
- Consider the type-system of C
  - C requires type annotations only at function types and local variable declarations
    - ⋆ $3 + 4$ does not need a type annotation
  - Trade-off: programmer annotations vs. computed types

A type inference algorithm

- A standard approach in type inference
  - Type the program by introducing *variables* at any point when an annotation is missing
    - We will use *label variables* ρ here
    - Now *r* may be either a constant *R* or a variable ρ
- Typing the unlabeled program does two things
  - Introduces label variables in all Ref types
  - Creates *constraints* among labels
- Solve the constraints to find a labeling
  - No solution means no valid labeling: type error
  - Alias analysis solution always exists: everything aliases

Step 1: Introduce labels

- Problem 1: What label to assign to the reference at [T-REF]?
- Solution: Introduce a fresh, unknown variable

$$[\textsc{T-Ref}]\frac{\Gamma \vdash e : T \qquad \rho - \text{fresh}}{\Gamma \vdash \text{ref } e : Ref^{\rho}\ T}$$

- Why a variable and not a constant?

Step 1: Introduce labels (cont'd)

- Problem 2: What type to give to function arguments?
  - Type language T uses labeled reference types $Ref^{\rho}\ T$
  - But the programmer uses unlabeled types $Ref\ T$
- Solution:
  - Use two type languages
    - Standard $S ::= S \to S \mid Nat \mid Bool \mid Unit \mid Ref\ S$
    - Labeled $T ::= T \to T \mid Nat \mid Bool \mid Unit \mid Ref^{\rho}\ T$
  - Annotate type S with fresh labels to get a T
    - We write this as $T = \text{fresh}(S)$

$$[\text{T-LAM}]\ \frac{\Gamma, x : T \vdash e : T' \qquad T = \text{fresh}(S)}{\Gamma \vdash \lambda x : S.e : T \to T'}$$

Step 2: Generate constraints

- Problem 3: Some rules implicitly require types to be equal
- Solution: Make this explicit using *equality constraints*
  - We write equality constraints as premises $T_1 = T_2$
  - Each such premise is not checked, instead produces a constraint
  - We solve all generated constraints together after typing
- Rule [T-IF] requires both branches to have the same type

$$[\textsc{T-If}]\ \frac{\begin{array}{c}\Gamma \vdash e : Bool \\ \Gamma \vdash e_1 : T_1 \\ \Gamma \vdash e_2 : T_2 \\ T_1 = T_2\end{array}}{\Gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 : T_1}$$

Step 2: Generate constraints (cont'd)

- Rule [T-Assign] requires that the assigned value has the same type as the pointer

$$[\text{T-Assign}]\ \frac{\Gamma \vdash e_1 : \mathit{Ref}\ T_1 \quad \Gamma \vdash e_2 : T_2 \quad T_1 = T_2}{\Gamma \vdash e_1 := e_2 : \mathit{Unit}}$$

- We assume that e_1 always has a pointer type
  - That is always true
  - We assume the program typechecks with standard types

Step 2: Generate constraints (cont'd)

- Rule [T-App] requires the formal and actual arguments to have the same type

$$[\text{T-App}]\;\frac{\Gamma \vdash e_1 : T_1 \to T' \quad \Gamma \vdash e_2 : T_2 \quad T_1 = T_2}{\Gamma \vdash (e_1\ e_2) : T'}$$

- Again, we assume e_1 has a function type
  - As before, this is always true
  - Because the program typechecks with standard types

Step 3: Solve the constraints

- After applying the type rules, we are left with a set of equality constraints
  - $T_1 = T_2$
- We solve these constraints using rewriting
- Each rewriting step simplifies a constraint into simpler constraints
- $C => C'$ rewrites the set C of all constraints to constraints C'

Step 3: Solve the constraints (cont'd)

- $C \cup \{Nat = Nat\} => C$
- $C \cup \{Bool = Bool\} => C$
- $C \cup \{Unit = Unit\} => C$
- $C \cup \{T_1 \to T_2 = T'_1 \to T'_2\} => C \cup \{T_1 = T'_1\} \cup \{T_2 = T'_2\}$
- $C \cup \{Ref^{\rho_1}\ T_1 = Ref^{\rho_2}\ T_2\} => C \cup \{T_1 = T_2\} \cup \{\rho_1 = \rho_2\}$
- $C \cup \{$mismatched constructors$\} =>$ error
  - Cannot happen if we start with a program that typechecks with standard types
- This algorithm always terminates
- When no further reduction applies, we have only label equalities

Last step: Use solution to add constants

- Compute the sets of labels that are equal
  - Using union-find
- Create a constant label R for each equivalence class of label variables
- Two pointers alias if their types refer to the same constant label

Example

Program

```
let x = ref 1 in
let y = ref 2 in
let z = ref 3 in
let w = if true then x else y in
  w := 42
```

Variable types:

x : $Ref^a\ Nat$
y : $Ref^b\ Nat$
z : $Ref^c\ Nat$
w : $Ref^a\ Nat$

- Typing annotates each `ref` expression with a variable a, b, c
- Typing the if creates equality constraint $Ref^a\ Nat = Ref^b\ Nat$
- Solving the constraint gives $a = b$
- Two equivalence classes: $\{a, b\}$ and $\{c\}$
  - Create two constants R_1 and R_2 for the equivalence classes

Example (cont'd)

Annotated program

$\textbf{let}\ \ x\ = \mathsf{ref}^{R_1}\ 1\ \textbf{in}$
$\textbf{let}\ \ y\ = \mathsf{ref}^{R_1}\ 2\ \textbf{in}$
$\textbf{let}\ \ z\ = \mathsf{ref}^{R_2}\ 3\ \textbf{in}$
$\textbf{let}\ \ w = \textbf{if}\ \mathsf{true}\ \textbf{then}\ x\ \textbf{else}\ y\ \textbf{in}$
$\quad w := 42$

Variable types:

$x : Ref^{R_1}\ Nat$
$y : Ref^{R_1}\ Nat$
$z : Ref^{R_2}\ Nat$
$w : Ref^{R_1}\ Nat$

- The assignment writes to one of the locations labeled by R_1
- Result: `x`, `y` and `w` may alias either of the first two allocated locations, but `z` cannot
  - May alias: their types have the same location label

Steensgaard's Analysis

- Flow-insensitive
- Inter-procedural
  - Can analyze multiple functions together
- Context-insensitive
  - Does not discriminate between different calls to the same function
- Unification-based
  - Analysis named after Bjarne Steensgaard (1996)
  - In practice: implementation for C handles type casts, etc.
- Properties
  - Very scalable
    - What is its complexity?
  - Imprecise