ΗΥ215: Λύσεις $2^{ης}$ Σειράς ασκήσεων

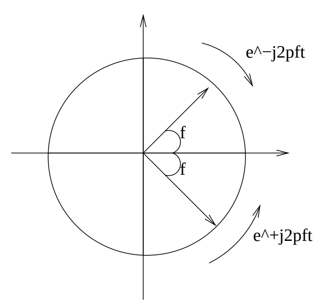
1. (α′) Ξέρουμε ότι: $\left.\begin{array}{r} \tau = -\frac{\phi}{2\pi}T \\ 0 \le \tau < T \Rightarrow \phi < 0 \end{array}\right\} \Rightarrow \left.\begin{array}{r} 0 \le -\phi < 2\pi \\ \text{Θέτω } \Phi = -\phi > 0 \end{array}\right\} \Rightarrow 0 \le \Phi < 2\pi$

   (β′) $\left.\begin{array}{r} \phi = -2\pi\,\frac{\tau}{T} \\ -\pi \le \phi < \pi \end{array}\right\} \Rightarrow -\frac{T}{2} \le -\tau < \frac{T}{2} \Rightarrow \boxed{-\frac{T}{2} < \tau \le \frac{T}{2}}$

2. (α′) $\frac{A}{2j}\,e^{j\phi}\,e^{j2\pi ft} - \frac{A}{2j}\,e^{-j\phi}\,e^{-j2\pi ft} =$

   $= \frac{A}{2}\,e^{j\phi}\,e^{j\frac{3\pi}{2}}\,e^{j2\pi ft} + \frac{A}{2}\,e^{-j\phi}\,e^{j\frac{\pi}{2}}\,e^{-j2\pi ft} =$ $\quad \boxed{\phi = \frac{\pi}{4} \text{ και } \frac{\pi}{4} + \frac{3\pi}{2} = \frac{7\pi}{4} \to -\frac{\pi}{4})}$

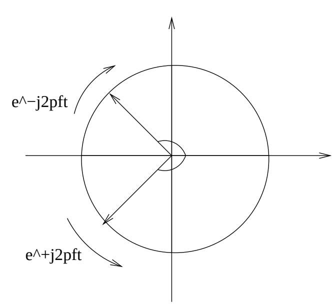
   $= \frac{A}{2}\,e^{-j\frac{\pi}{4}}\,e^{j2\pi ft} + \frac{A}{2}\,e^{j\frac{\pi}{4}}e^{-j2\pi ft}$



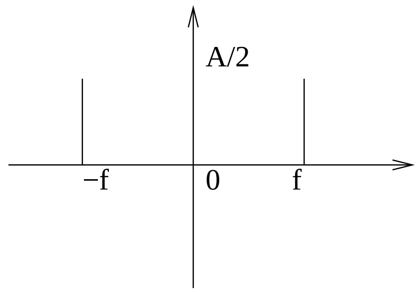
   (β′) $-A\,\cos\left(2\pi ft + \phi\right) = -\frac{A}{2}\,e^{j\phi}\,e^{j2\pi ft} - \frac{A}{2}\,e^{-j\phi}\,e^{-j2\pi ft}$

   $= \frac{A}{2}\,e^{j(\phi+\pi)}\,e^{j2\pi ft} + \frac{A}{2}\,e^{-j(\phi-\pi)}\,e^{-j2\pi ft} =$

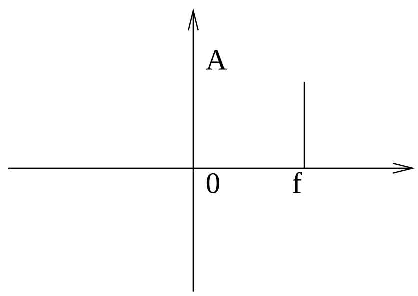
   $= \frac{A}{2}\,e^{-j\frac{3\pi}{4}}\,e^{j2\pi ft} + \frac{A}{2}\,e^{j\frac{3\pi}{4}}\,e^{-j2\pi ft}$



3. $x(t) = A\,\cos\left(2\pi f t\right)$



$$\overline{x}(t) = A\,\cos\left(2\pi f t\right) + j\,\underbrace{A\,\sin\left(2\pi f t\right)}_{\hat{x}(t)} = A\,e^{j2\pi f t}$$



Το πραγματικό σήμα έχει συμμετρικό φάσμα πλάτους, ενώ για το μιγαδικό δεν είναι συμμετρικό.

4.

$$\left.\begin{array}{r}\hat{x}(t) = \frac{1}{\pi}\int\limits_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\,d\tau \\ x(\tau) = A\,\cos(2\pi f\tau)\end{array}\right\} \Rightarrow \hat{x}(t) = \frac{1}{\pi}\int\limits_{-\infty}^{\infty}\frac{\cos\left(2\pi f_0(t-\alpha)\right)}{\alpha}\,d\alpha$$

Θέτοντας: $\boxed{\begin{array}{l}\alpha = t-\tau \Rightarrow d\alpha = -d\tau \\ \tau \to -\infty\;\alpha \to +\infty \\ \tau \to \infty\;\alpha \to -\infty\end{array}}$

$$\cos\left(2\pi f_0(t-\alpha)\right) = \cos\left(2\pi f_0 t\right)\,\cos\left(2\pi f\alpha\right) + \sin\left(2\pi f_0 t\right)\,\sin\left(2\pi f\alpha\right)$$

Άρα $\hat{x}(t) = \frac{1}{\pi}\cos(2\pi f_0 t) \underbrace{\int\limits_{-\infty}^{\infty} \frac{\cos(2\pi f \alpha)}{\alpha}\,d\alpha}_{\text{περιττή προς α και 0}} + \frac{1}{\pi}\sin(2\pi f_0 t) \underbrace{\int\limits_{-\infty}^{\infty} \frac{\sin(2\pi f \alpha)}{\alpha}\,d\alpha}_{\text{άρτια και ίση με } \pi} =$

$= \sin(2\pi f_0 t)$

5. $\int\limits_0^T A\sin(2\pi f t)\,dt = \frac{A}{2\pi f}(-1)\ \cos(2\pi f t)\big|_0^T = \frac{A}{2\pi f}(-1)(\cos(2\pi) - \cos(0)) = 0$

$A\int\limits_0^T \sin(2\pi f(t-\tau))\,dt = A\int\limits_{-\tau}^{T-\tau} \sin(2\pi f \alpha)\,d\alpha = \frac{A}{2\pi f}(-1)(\cos(2\pi f(T-\tau))) - \cos(2\pi f t) =$

$= \frac{A}{2\pi f}(-1)(\cos(2\pi f \tau) - \cos(2\pi f \tau)) = 0$

Θέτοντας:

$\alpha = t - \tau \Rightarrow d\alpha = dt$
$t = 0 \Rightarrow \alpha = -\tau$
$t = T \Rightarrow \alpha = T - t$

$\int\limits_0^T |A\sin(2\pi f t)|\ dt = 2\,A\int\limits_0^{\frac{T}{2}} \sin(2\pi f t)\,dt = \frac{2A}{2\pi f}(-1)\ \cos(2\pi f t)\big|_0^{\frac{T}{2}} = \frac{A}{\pi f}(-1-1)(-1) = \frac{2}{\pi}\,A\,T$

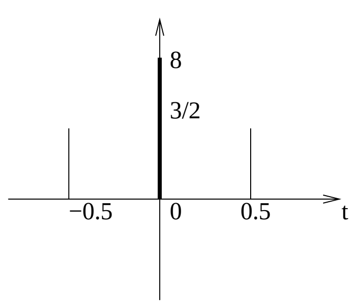
Έστω $\tau < 0$

$A\int\limits_0^T |\sin(2\pi f(t-\tau))|\ dt = 2\,A\int\limits_{\tau}^{\frac{T}{2}+\tau} \sin(2\pi f(t-\tau))\,dt = 2\,A\int\limits_0^{\frac{T}{2}} \sin(2\pi f \alpha)\,d\alpha = \frac{2}{\pi}\,A\,T$

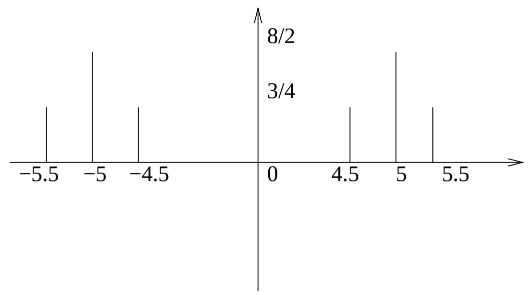
Θέτοντας:

$\alpha = t - \tau \Rightarrow d\alpha = dt$
$t \to \tau \Rightarrow \alpha = 0$
$t \to \frac{T}{2} + \tau \Rightarrow \alpha = \frac{T}{2}$

6. $x(t) = 8 + 3\,\sin(\pi t - \frac{\pi}{4}) = 8 + 3\,\cos(\pi t - \frac{\pi}{4} - \frac{\pi}{2}) = 8 + 3\,\cos(\pi t - \frac{3\pi}{4})$



$$y(t) = x(t)\ \cos(10\pi t) = 8\ \cos(10\pi t) + \tfrac{3}{2}\ \cos(11\pi t - \tfrac{3\pi}{4}) + \tfrac{3}{2}\ \cos(9\pi t + \tfrac{3\pi}{4})$$



$$z(t) = y(t)\ \cos(10\pi t) = \left[8 + 3\ \sin(\pi t - \tfrac{\pi}{4})\right]\ \tfrac{1}{2} + \tfrac{1}{2}\ \left[8 + 3\ \sin(\pi t - \tfrac{\pi}{4})\right]\ \cos(20\pi t)$$