Δ.9

$\frac{\partial f}{\partial x}(x,y) = 3x^2 + 3y$

$\frac{\partial f}{\partial y}(x,y) = 3y^2 + 3x$

$\frac{\partial^2 f}{\partial x^2}(x,y) = 6x$

$\frac{\partial^2 f}{\partial y^2}(x,y) = 6y$

$\frac{\partial^2 f}{\partial x \partial y}(x,y) = 3$

Κρ. σημεία: $\frac{\partial f}{\partial x} = 0$ και $\frac{\partial f}{\partial y} = 0$

$\Rightarrow \begin{cases} 3x^2 + 3y = 0 \\ 3y^2 + 3x = 0 \end{cases} \Rightarrow \begin{cases} y = -x^2 \\ x^4 + x = 0 \end{cases}$

$\Rightarrow \begin{cases} y = -x^2 \\ \text{και} \\ \begin{Bmatrix} x = 0 \\ \text{ή} \\ x^3 = -1 \end{Bmatrix} \end{cases} \Rightarrow \begin{cases} (x,y) = (0,0) \\ \text{ή} \\ (x,y) = (-1,-1) \end{cases}$

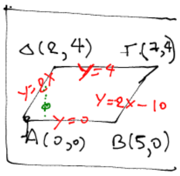
$(0,0)$: $\Delta = -9 < 0$ σημείο σαγματικό

$(-1,-1)$: $\Delta = 27 > 0$, $\frac{\partial^2 f}{\partial x^2}(-1,-1) = -6 < 0$ T.M.

Δ.17

D: ΑΒΓΔ,

$f(x,y) = -x^2 - y^2 + x + y + 2$



D°: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \Rightarrow \begin{cases} -2x + 1 = 0 \\ -2y + 1 = 0 \end{cases}$ (Κρ. σημεία)

$(x,y) = \left(\frac{1}{2}, \frac{1}{2}\right) \in D$

$\partial D^\circ = AB \cup B\Gamma \cup \Gamma\Delta \cup \Delta A$ $\quad f\left(\frac{1}{2}, \frac{1}{2}\right) = 2{,}5$

AB: $c(t): [0,1] \to \mathbb{R}^2$ $\quad \langle x(t), y(t) \rangle = A + t\,\overrightarrow{AB}$

$t \to (5t, 0)$ $\quad t \in [0,1]$

$F(t) = f \circ c(t) = f(5t, 0) = -25t^2 + 5t + 2, \; t \in [0,1]$

$F'(t) = 0 \Rightarrow -50t + 5 = 0 \Rightarrow t = 0.1 \in [0,1]$

$t = 0.1 \to (0.5, 0)$ $\quad f(0.5, 0) = 2.25$

A: $f(0,0) = 2$ $\quad$ B: $f(5,0) = -18$

$B\Gamma$: $c(t) = (5,0) + t \cdot (2,4) = (5 + 2t, 4t)$, $t \in [0,1]$ (B, $\overrightarrow{B\Gamma}$)

$F(t) = f(5 + 2t, 4t) = \ldots$

Δ.20

$\begin{cases} 6x = \lambda \cdot 2x \\ 10y = \lambda \cdot 2y \\ x^2 + y^2 = 1 \end{cases} \Rightarrow \begin{cases} x = 0 \text{ ή } \lambda = 3 & (1) \\ y = 0 \text{ ή } \lambda = 5 & (2) \\ x^2 + y^2 = 1 & (3) \end{cases}$

$x = 0$ (1)
- $y = 0$ → $0 = 1$, αδύνατο
- $\lambda = 5$ (2) → $y = +1$ ή $y = -1$ (3) → $(0,1)$ ή $(0,-1)$

απορ.

$\lambda = 3$ (1)
- $y = 0$ → $(-1,0)$, $(1,0)$ (3)
- $\lambda = 5$ αδύνατο (2)

$f(-1,0) = 3$

$f(1,0) = 3$

ολικό ελάχιστο

$f(0,1) = 5$

$f(0,-1) = 5$

ολικό μέγιστο

Δ.21

$\begin{cases} 2\pi r h = \lambda \cdot (4\pi r + 2\pi h) & (1) \\ \pi r^2 = \lambda \cdot 2\pi r & (2) \end{cases} \Rightarrow$

$2\pi r^2 + 2\pi r h = c$ (3)

(1) (2) → (απαλείφω το λ) → 2×2 σύστημα

$\begin{cases} r \cdot h = 2\lambda \cdot r + \lambda h \\ (r=0) \to V(r,h) = 0 \text{ ελάχιστο} \quad \lambda = r/2 \end{cases} \Rightarrow \begin{cases} r \cdot h = r^2 + \frac{r \cdot h}{2} \end{cases}$

$2\pi r^2 + 2\pi r \cdot h = c$ → (3)

$\begin{cases} h = 2r \\ 2\pi r^2 + 4\pi r^2 = c \end{cases} \to \begin{cases} r = \sqrt{\frac{c}{6\pi}} \end{cases}$ $\quad V(r,h) = \ldots$ $\quad h = 2\sqrt{\frac{c}{6\pi}}$ μέγιστο

Δ.24

β' Τρόπος λύσης:

→ υπολογίζω τομή επιπέδων

$(1) \to x = 10 - 3z$

$(2) \to 2(10 - 3z) + y + z = 1$ $\Rightarrow \begin{cases} x = 10 - 3z \\ y = -19 + 5z \end{cases}$ $(z = t)$ $z \in \mathbb{R}$

$\langle x(t), y(t), z(t) \rangle = \langle 10, -19, 0 \rangle + t \cdot \langle -3, 5, 1 \rangle$ $\quad t \in \mathbb{R}$ (3)

$d(x,y,z) = x^2 + y^2 + z^2 \overset{(3)}{\Longrightarrow}$

$\Rightarrow d(t) = (10 - 3t)^2 + (-19 + 5t)^2 + t^2$, $t \in \mathbb{R}$

$d'(t) = 0 \to \ldots \; t = \ldots \begin{cases} d(t) = \ldots \\ (3)\; (x,y,z) = \ldots \end{cases}$

$\boxed{\Delta.26}$

$$\begin{cases} 2x+y=\lambda\, 4x^3 & (1) \\ 2y+x=\lambda\, 2y & (2) \\ x^4+y^2=1 & (3) \end{cases} \Rightarrow \begin{cases} (2x+y)\cdot 2y = 4x^3\cdot(2y+x) \\ x^4+y^2=1 \\ \left(\lambda = \dfrac{2y+x}{2y}\right) \quad y\neq 0 \end{cases}$$

(*) Αν $y=0 \overset{(2)}{\Rightarrow} x=0 \overset{(3)}{\Rightarrow} 0=1$ αδύνατο

$$\begin{cases} 4x^4+4x^3y-4xy-2y^2=0 \\ x^4+y^2=1 \end{cases}$$