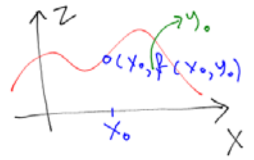
Δ.27

$$\lim_{(x,y)\to(0,0)} e^y \frac{\sin x}{x} = \lim_{y\to 0} e^y \cdot \lim_{x\to 0} \frac{\sin x}{x}$$

L'Hospital

$$= 1 \cdot \lim_{x\to 0} \frac{\cos x}{1} = 1 \cdot \cos(0) = 1$$

Δ.35



$f : \bar{U} \subseteq \mathbb{R} \to \mathbb{R}$

$(x,y_0) \to f(x,y_0)$

$$\boxed{\text{ε3.24:} \quad z - f(x_0,y_0) = \frac{\partial f}{\partial x}(x_0,y_0)\cdot(x - x_0)}$$

3D: $\langle x(t), y(t), z(t)\rangle = \langle x_0, y_0, f(x_0,y_0)\rangle + t\langle 1, 0, \frac{\partial f(x_0,y_0)}{\partial x}\rangle$

Δ.43

$F(x,y) = \sin(x^2) + y^2$

$f(x) = \sin(x^2)$

$f(x) = f_1 \circ f_2(x) = f_1(f_2(x))$

$f_1(x) = \sin x$

$f_2(x) = x^2$

$f'(x) = f_1'(f_2(x)) \cdot f_2'(x) = \cos(x^2)\cdot 2x$

$\frac{\partial F(x,y)}{\partial x} =$

Δ.47

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = \frac{\partial}{\partial x_1}\left(\frac{\partial f}{\partial x_2}\right) =$$

$$= \frac{\partial}{\partial x_1}\left(5x_1^2 x_2^4\right) = 10x_1 x_2^4$$

$f : \mathbb{R}^3 \to \mathbb{R}^2$

$(x,y,z) \quad (x+y+z^2,\ x\cdot y\cdot z^2)$

$Df \to 2\times 3$ πίνακας

$$Df = \begin{pmatrix} 1 & 1 & 2z \\ yz^2 & xz^2 & 2xyz \end{pmatrix}$$

$$Df(0,0,0) = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$f(x,y,z) = x^2 y^2 z^2 \quad x^2y^2+z^2$

$$\frac{\partial^3 f}{\partial x \partial y \partial z} = \frac{\partial^2}{\partial x\, \partial y}(2x^2y^2z)$$

$$= \frac{\partial}{\partial x}(4x^2 y z) = 8xyz$$

$$\frac{\partial^3 f}{\partial z \partial x \partial y} = \cdots = 8xyz$$