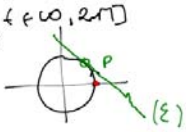
Δ.17 $c(t) = (\cos(t), \sin(t)),\ t \in [0, 2\pi]$



$t = \frac{\pi}{4} \to P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

λύση

$c'(t) = (-\sin t, \cos t)$ $\qquad \vec{v} \parallel (\varepsilon)$

$\vec{v} = c'(\pi/4) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$(\varepsilon): \langle x(s), y(s)\rangle = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle + s\cdot\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle,\ s \in \mathbb{R}$

Δ.19 $\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right\rangle = \langle 3x^2 + 3y, 10y^9 + 3x\rangle$

$P(3,0)$ $\qquad \vec{v} \parallel \varepsilon$ $\qquad \vec{v} = \left\langle -\frac{\partial f}{\partial y}(P), \frac{\partial f}{\partial x}(P)\right\rangle$

$\varepsilon: \langle x(t), y(t)\rangle = \langle 3, 0\rangle + t\langle -9, 27\rangle$

$\nabla f = \langle 2x + 3z, 4y, 3x\rangle$

$\vec{v} = \nabla f(P) = \langle 3, 8, 3\rangle$

$\vec{v} \perp \varepsilon \Rightarrow \langle x-1, y-2, z-\frac{1}{3}\rangle \cdot \langle 3, 8, 3\rangle = 0$

$3x + 8y + 3z = 20$

Δ.20 $\nabla f_1 = \langle 2x, 2y, 2z\rangle$

$\nabla f_2 = \langle 2x, 2y-2, 0\rangle$

$\vec{v} = \vec{\nabla} f_{1(P)} \times \vec{\nabla} f_{2(P)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 2\sqrt{2} \\ 2 & 0 & 0 \end{vmatrix} =$

$2\cdot(2\sqrt{2}\cdot\vec{j} - 2\vec{k}) = \langle 0, 4\sqrt{2}, -4\rangle$

Δ.21

$c(t): \mathbb{R} \to \mathbb{R}^2$

$t \to (x_1(t), x_2(t))$

α) $x_2(t) = e^{x_1(t)}$ $\qquad x_1(t) = t$

$x_2(t) = e^t$

β) $\dfrac{x_1^2(t)}{(1/2)^2} + x_2^2(t) = 1 \Rightarrow x_2(t) = \cos t$

$\sin^2$ $\quad \cos^2$

$x_1(t) = \frac{1}{2}\sin t$

γ) $\langle x(t), y(t), z(t)\rangle = \langle 0, 0, 0\rangle + t\langle a, b, c\rangle$

Δ.24 $c'(t) = \langle 1, \sin t + t\cos t, \cos t - t\cdot\sin t\rangle$

$t_1: c(t_1) = \langle 0, 0, 0\rangle \to t_1 = 0$

$t_2: c(t_2) = \langle \pi, 0, -\pi\rangle \Rightarrow t_2 = \pi$

$s = \int_0^{\pi} \sqrt{2 + t^2}\, dt$

Δ.26

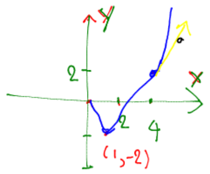
$c(t): \mathbb{R} \to \mathbb{R}^3$

$t \to \left(\int t\, dt, \int e^t dt, \int t^2 dt\right)$

$t \to \left(\frac{t^2}{2} + c_1, e^t + c_2, \frac{t^3}{3} + c_3\right)$

$\overset{t=0}{\Rightarrow} (0, -5, 1) = (c_1, 1 + c_2, c_3) \Leftrightarrow \begin{cases} c_1 = 0 \\ c_2 = -6 \\ c_3 = 1 \end{cases}$

Δ.27



$s'(t) = (2t, 3t^2-3, 0)$

Εφ. ευθεία $t=2$:

$(x(t'), y(t'), z(t')) = (4, 2, 0) + t' \cdot (4, 9, 0)$

$t' = t - 2$

$t' = 1 \to (8, 11, 0)$

Δ.27

$T(t) = (x(t), y(t), z(t)),\ t \in [a,b]$ ⊛

$|T(t)| = |\sigma'(t)| \cdot \left|\frac{1}{|\sigma'(t)|}\right| = 1$

$\boxed{T(t) \cdot T(t) = 1}$ ①

① $\Rightarrow \frac{d(T(t) \cdot T(t))}{dt} = 0 \overset{⊛}{\Leftrightarrow} (x^2(t) + y^2(t) + z^2(t))' = 0$

$\Leftrightarrow 2(x(t) \cdot x'(t) + y(t) \cdot y'(t) + z(t) \cdot z'(t)) = 0$

$\Leftrightarrow T(t) \cdot T'(t) = 0$

$$T'(t) = \left(\frac{\sigma'(t)}{|\sigma'(t)|}\right)' = \frac{\sigma''(t)}{|\sigma'(t)|} + \sigma'(t)\left(\frac{1}{|\sigma'(t)|}\right)'$$

$$\left(\frac{1}{|\sigma'(t)|}\right)' = \left(\left(\sigma_1'^2(t) + \sigma_2'^2(t) + \sigma_3'^2(t)\right)^{-\frac{1}{2}}\right)' =$$

$$= -\frac{1}{2}\frac{1}{|\sigma'(t)|^3} \cdot 2(\sigma_1'(t) \cdot \sigma_1''(t) + \sigma_2'(t) \cdot \sigma_2''(t) + \sigma_3'(t)\sigma_3''(t))$$

$$= -\frac{\sigma'(t) \cdot \sigma''(t)}{|\sigma'(t)|^3}$$

$s'(t) = (-\sin t, \cos t, 1)$

Συν. μήκους, $s \geq 0$.

$$L(u) = \int_0^u |s'(t)|\,dt = \int_0^u \sqrt{\sin^2 t + \cos^2 t + 1}\,dt =$$

$$= \int_0^u \sqrt{2}\,dt = \sqrt{2} \cdot u$$